10 Steps toward Empowering Students Mathematically

Marian Small April, 2023

Step 1

- Students are empowered by a teacher focus on understanding the math and not just a focus on doing the math.

Step 1

- What's the difference?
- It's huge!

Step 1

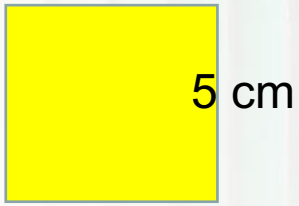
- Consider the differences in each of these examples.

Primary

- DOING: List the pairs of numbers that add to 10.
- UNDERSTANDING: If you add two numbers to make 10, can they both be less than 5?

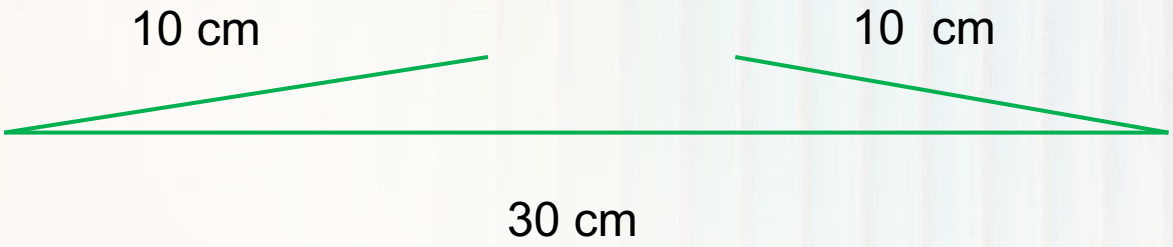
Junior

- DOING: What is the perimeter of this square?



- UNDERSTANDING: I made a shape with a perimeter of 50 cm. Could one side be 30 cm?

Junior



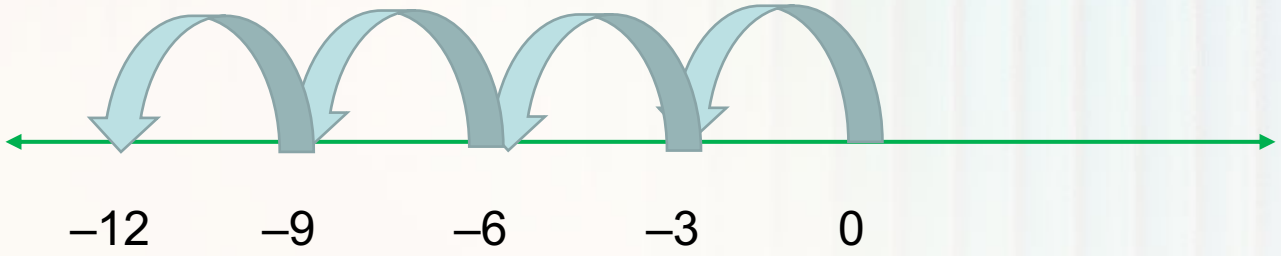
Intermediate

- DOING: What is the sign if you divide a negative number by a positive one?

- UNDERSTANDING: Draw a picture that explains why (–12) ÷ 4 must be a negative.

Intermediate

- (–12) ÷ 4



Secondary

- DOING: What is the equation of the line through (4,10) and (6,–2)?
- UNDERSTANDING: How do you know that there can only be one line that goes through (4, 10) and (6, –2)?

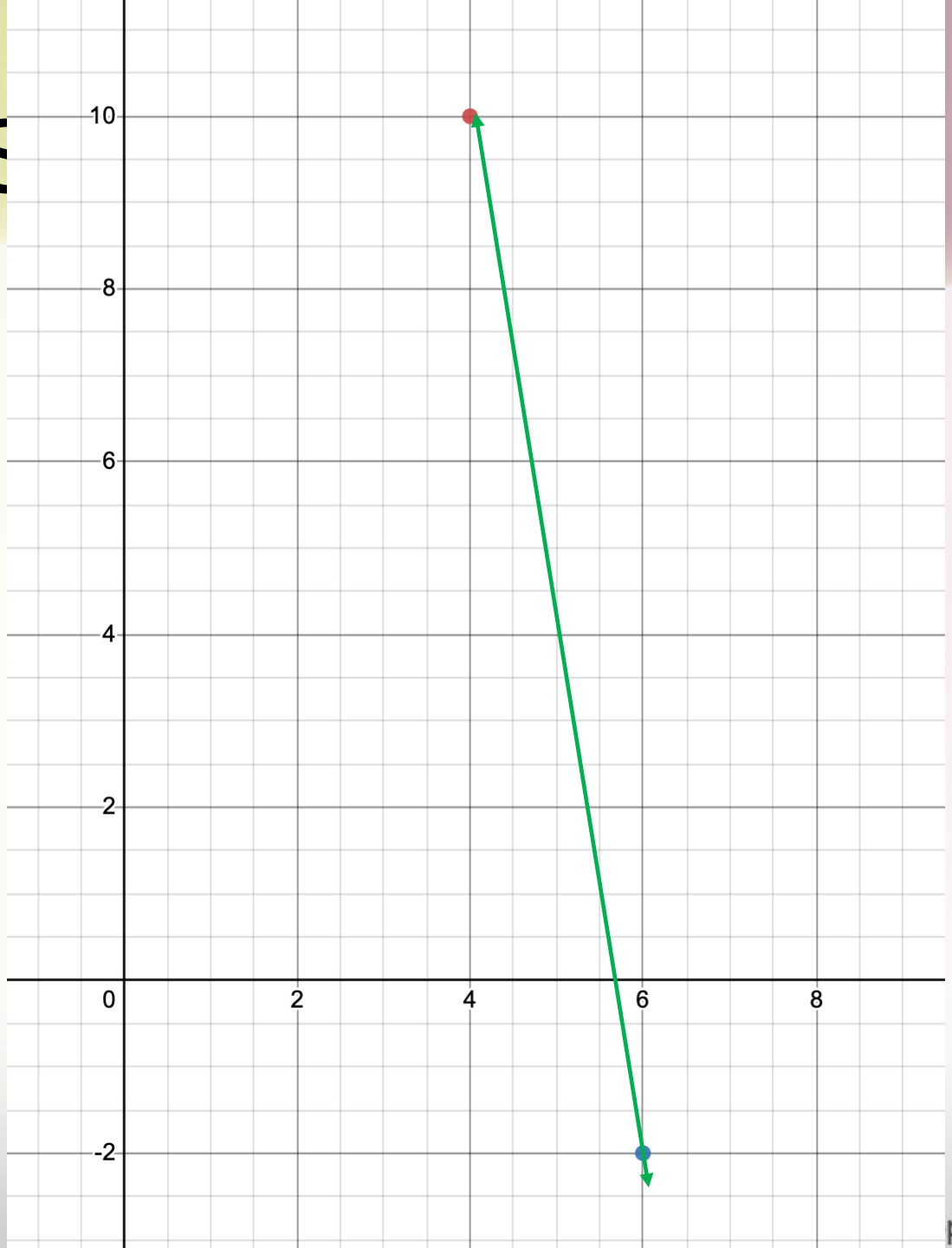
S

- (4, 10)

and (6, –2)?

Slope = –6

y=–6x +34



Why?

- Why is this empowering?
- If you only have rules, you have no power if you don’t remember the rules.
- If you have understanding, you can recreate ideas that you might have lost.
- This is about focusing on the “long haul” and not the “short haul”.

Step 2

- Students are empowered by less focus on answers and more focus on process, strategies and thinking.

Step 2

- Even if teachers use problem solving tasks, the focus is often on the answer and how various students got the answer.
- That is still a focus on answers.

Step 2

- How do you take the focus away from “What is the answer? How did you get the answer?”

Step 2

- It could involve activities like:
- WITHOUT MULTIPLYING, tell me something you know about 28 x 93.
- Without talking about the answer, a student might suggest:

Step 2

28 x 93

- The product is even.
- The product is close to 2700.
- You could model this with a rectangle with length 93 and width 28.
- It is 28 more than 28 x 92.
- It is 93 more than 27 x 93.

Step 2

- It could involve activities like:
- WITHOUT ADDING, tell me what you know about $\frac{2}{3} + \frac{1}{4}$.

Step 2

$$\frac{2}{3} + \frac{1}{4}$$

- The sum is less than 1, but not a lot less.
- The sum is more than $\frac{2}{3}$.
- If you wrote the sum as a decimal, it would be a repeating decimal.

Step 2

- OR
- WITHOUT ADDING, tell me something you know about 37 + 48.
- It is less than 48 + 48.
- It is more than 37 + 37.
- It is a 2-digit number.
- The tens digit will be 8.

Step 2

- OR
- WITHOUT GRAPHING OR DOING ALGEBRA, tell me something you know about the common solution to $x + y = 9$ and $2x - y = 3$.

Step 2

$x + y = 9$ and $2x - y = 3$.

- The x and y cannot both be more than 9.
- They are both true only for one value of x and y.
- y cannot be negative since if it were, x would be very small for second equation, but big for the first one.

Step 2

- Let’s talk about what this looks like in terms of students working on a task.

Possible Task

Action Task

1. Ryan and Kyle served 249 meals at the community kitchen. Ryan served twice as many meals as Kyle. How many meals did each person serve? Show all your work.

Step 2

- Even if students have not gotten to answers, I could ask:
- How do you know that 100 and 200 are not the amounts?
- Will the numbers of meals each served be more or less than 100 and 200?
- Why might you divide?
- Why wouldn’t you just divide by 2?

Step 2

- Another task.

1. Describe a situation that fits each equation. Describe more than one situation for each equation, if possible.

a) $4x - 1 = 19$

b) $2x - 4 = 10$

c) $2x + 2y = 48$

d) $5x + 2y = 90$

2. Choose one of the situations above, and use a different equation to describe the same situation.

Step 2

- Even without students having completed the full task, I can ask:
- When you see a multiplication in an equation, how does that help you come up with a context?
- What about a subtraction?

Step 2

- Why can equations always describe more than one situation?
- Can situations usually be described by more than one equation?

Step 2

- This empowers students who are slower.

Step 2

- A strategy a teacher might use to avoid a focus on answers is to give students every answer and indicate that the students' only task is to figure out how you arrived at that solution.

Step 3

- Differentiation during instruction, and not only differentiation after instruction, empowers students.

Step 3

- This might be as simple as allowing students to choose values with which to work.
- Or allowing students to choose which of two or three tasks to pursue or which direction to take on an open-ended task.

Step 3

- For example, instead of a primary task where we ask students to represent the number 83 in three different ways, we could say:
- Choose a number greater than 10. Represent it in a bunch of ways.

Step 3

- For example, instead of a junior task where we ask students to determine the sum of $\frac{2}{3} + \frac{4}{5}$ in whatever way they wish, we could say:
- Choose two fractions between $\frac{1}{2}$ and 1 so that the sum is more than 1 $\frac{1}{2}$.

Step 3

- For example, instead of an intermediate task where we ask students to dilate a particular shape using a particular factor, we could ask:
- Choose a shape that sits in two quadrants of a Cartesian grid. Dilate it. Does it stay in the same two quadrants?

Step 3

- For example, instead of a secondary task where we ask students to determine the line of best fit for a set of data, we could ask:
- Create a set of data where it would be easy for you to estimate the line of best fit without using technology.
- Create a set where it would be a lot more challenging without technology.

Step 4

- Differentiation of assessment is another way to empower students.

Step 4

- A teacher might allow students these sorts of choices.
- Students might have choices on a test or quiz of which 5 or 10 questions they choose to answer.
- They might get to decide whether they respond to tasks in writing or orally.

Step 4

- We might allow students to choose which of a number of pieces of work they wish you to assess rather than using tests or quizzes.
- They might get to choose a test vs a presentation.

Step 5

- Less direction instruction and more learning through problem solving empowers students.
- By definition, the instructor has the power, not the student, in direct instruction.

Step 5

- When teaching through problem solving, students would have their own personal connections to draw on when they hear what we want them to learn. This empowers them.
- Students feel empowered when you trust them to figure things out. This happens when teaching through problem solving.

Step 5

- Empowering students honours their differences. Direct instruction makes that more difficult.

Step 5

- Direct instruction is often quicker in the short term, but the question is whether the learning lasts.

Step 6

Some of the most successful math students notice a lot of things that not all students notice.

We empower other students by helping them notice, too.

Step 6

We need to set tasks that make it possible for students to notice what is the important learning.

Step 6

For example, suppose we want students to notice that when you subtract a number from another one, the result could be more, less or the same as what you subtracted.

Step 6

I might ask students to complete these subtractions:

84 – 4 84 – 78 84 – 43

Then I'd ask: What do you notice about how the difference compares to what you subtracted?

Step 6

For example, suppose we want students to notice how the distributive property works.

I might ask students what they notice here.

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Step 6

Suppose I want students to notice that because an equation is a balance, you can also predict information about the solution before solving.

Step 6

For example, how could you predict that the solution to
$35x + 19 = 23x + 8$ could NOT be a positive value, but
$35x + 19 = 23x + 88$ could have a positive solution?

Step 6

For example:
The length of a rectangle is double the width.
What do you notice about the perimeters?

Step 6

Length	Width	Perimeter
8	4	24
6	3	18
10	5	30
20	10	60

Step 6

For example

The sum of two numbers is double the difference.
What could the numbers be?

Step 6

Numbers	Sum	Difference
3, 1	4	2
30, 10	40	20
15, 5	20	10
24, 8	32	16

Step 6

When you notice patterns and relationships, you are empowered.

Some students do this instinctively, but we can empower other students by helping them notice.

Step 7

- Conversation can empower students if we handle it well.

Step 7

- We need to provide opportunities for student –student talk, student- teacher talk.
- But we need to show students we are listening to them in our responses to them.

Step 7

- Let's try this.
- I will ask a question and try to respond to what you say.
- How could you predict that 11 x 12 is more than 10 x 13 without referring to the answers?

Step 7

- Some students might be reluctant to talk.
- We might want to provide anonymous ways for students to ask questions.

Step 7

- Some students are cautious about speaking again once they have made an error and it was noticed.

Step 7

- We might want to find a way to turn a wrong answer into a correct one (without misleading students).
- For example, I pose a problem:
- I have a rectangle with whole number side lengths.
- The length is 4 times the width.
- I calculate the perimeter.

Step 7

- What are some possible values for the perimeter?
- The student says 24 cm.
- What do I do?
- Oh, right, you decided the perimeter was 4 times the width instead of 4 times the length. Got it!

Step 7

- Another way to look at this is taking the position– ok, let's look at your problem instead of mine.

Step 8

- Students are empowered by learning to self-assess and not depend only on our confirmation.

Step 8

- We need to provide meaningful success criteria.
- We can encourage students to get feedback from peers on a regular basis.

Step 8

- We might insist that students tell us if they are correct, not the other way around.

- We might ask, in response to a query about whether they are right, what do you think the best part of what you did was?

Step 8

- I watched a masterful teacher two weeks ago who never reacted to wrong or correct responses, but just recorded them and darned if the students didn't figure out themselves what was right or wrong.

Step 9

- Students are empowered when they are free to ask for what they need and when they are provided, without fanfare, the tools they need.

Step 9

- Students need to be provided with the tools that might empower them without feeling negatively judged for using them.
- We don’t’ say “You can use … IF YOU NEED THEM.”
- We don’t force them to get up in front of all their peers to get the materials they need.

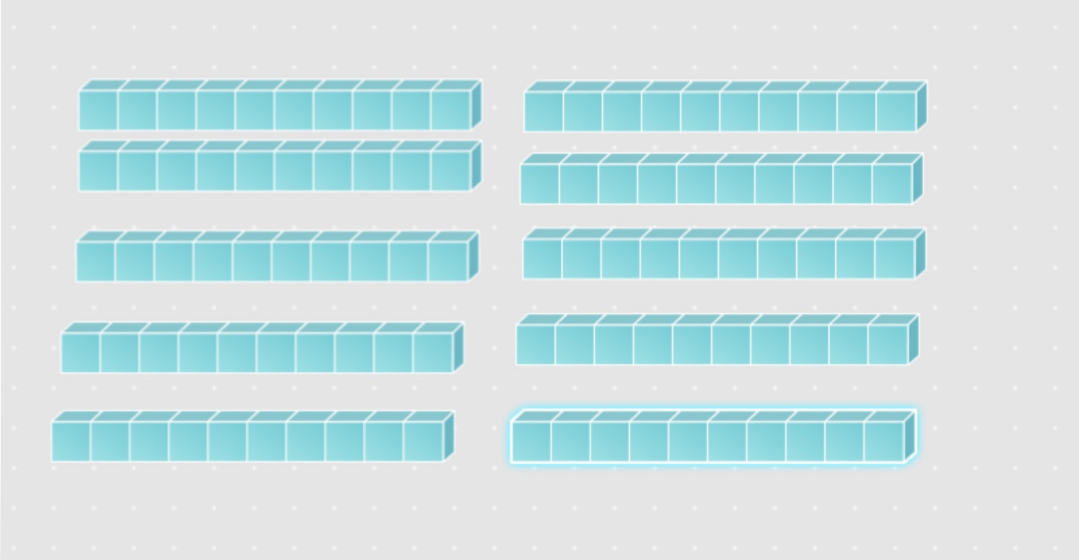
Step 9

- It is valuable to ask questions based on manipulative use to signal that you think manipulatives are a great idea to use.

- For example, I might ask young ones to tell me what number I might be thinking of if I filled four ten frames and part of another one.

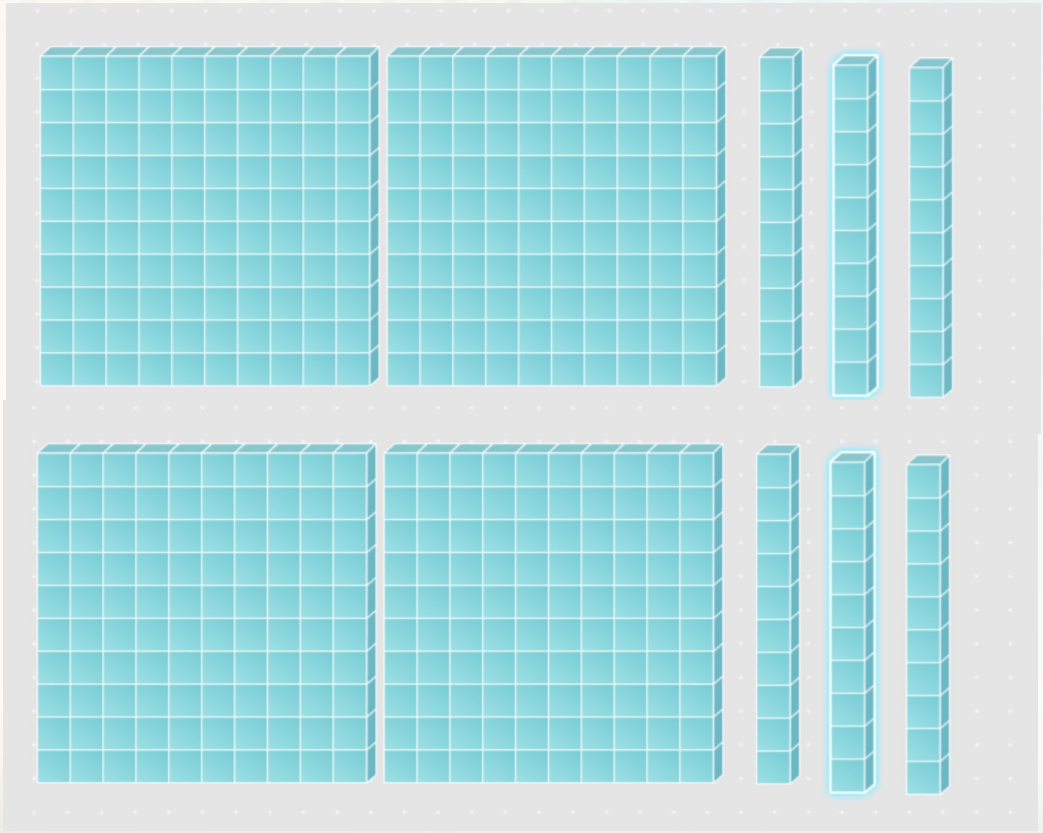
Step 9

- Or I might ask junior students what multiplications they could show me using 10 base ten blocks.
- E.g.



Step 9

- or



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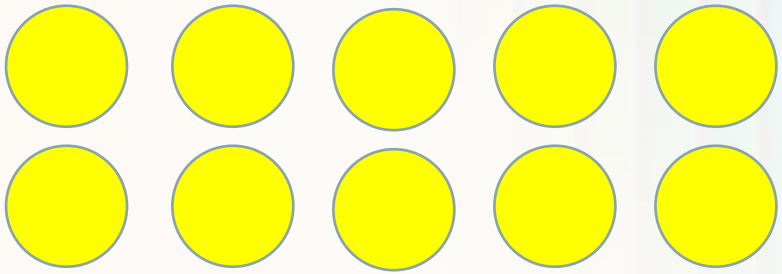
- Or older students what integers they can show using exactly 10 integer chips.

- E.g. 

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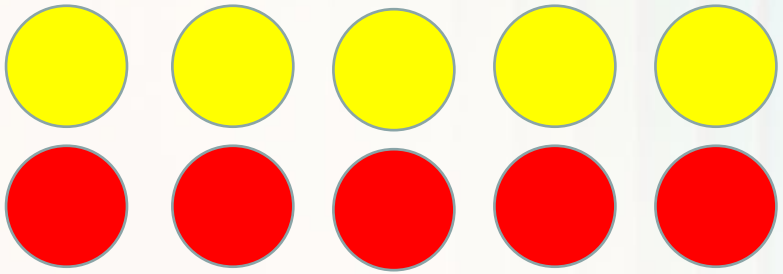

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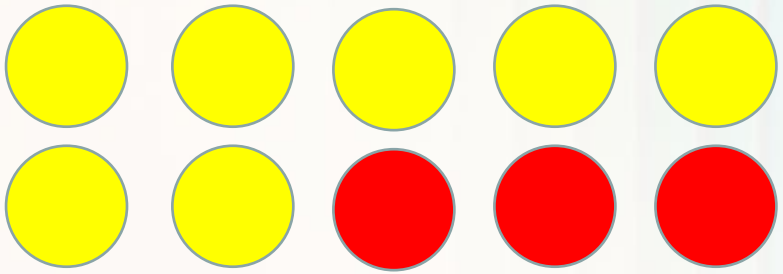

Step 9

- Students need to have an opportunity to select tools and strategies.
- That means that we provide an assortment of tools on the table.
- That means they get to decide what an efficient strategy for them is.

10

- Empowered students need empowered teachers- teachers who reflect regularly on their practice and make professional judgments that are thought through.

Step 10

- So many teachers do not give themselves permission to make their own decisions.
- They talk about preparing students for what the next teacher wants.
- They talk about what another teacher tells them they should do.

Step 10

- They choose resources to teach from where they just do what they are told.
- If a teacher does not feel empowered, it shows.

Step 10

- If it shows, the students see it and they are less comfortable believing they have choices.

Step 10

- I work hard at encouraging teachers to feel that they have the right and responsibility to make professional decisions, ideally having thought through various options.

In summary

- I am sure I could have stated the steps differently or I could have changed the order or the examples I used, but…

- These are the things that I have come to believe make the biggest difference.

Any questions?