

Differentiating Math Instruction Grades 5 - 9

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Today

How and why we use open questions and parallel tasks



Our next 4 sessions



A focus on number



A focus on on patterns and relations



A focus on shape and space



A focus on statistics and probability



Sessions 1 to 4

- You will be asked to try out a few of the provided questions or your own variations.



Sessions 2 to 5

- We will begin with sharing what happened with the questions you tried.



Need for differentiation

- Every student has the right to benefit from instruction.
- We have an obligation to make sure it happens.



This
happens
by..

➡ Teaching to the zone of proximal development.



This means

- Pre-assessment data to have a sense of where kids are
- Believing in and embracing alternative approaches and choice
- Believing in a focus on bigger ideas and not always details



Let's talk
about
open tasks
first



What are open questions?

- Question to which a variety of responses are appropriate
- Contrast for example:
 - What are the factors of 12?
And
 - A number has more than 5 factors. What could it be?



Benefits

- A richer conversation
- Building confidence
- Building participation
- Changing perceptions of the nature of math



How they Work

- Think of them as a starting point and then realize we need to ask more questions as students answer to build a conversation



How they Work

- Think of some idea you hope students will deal with to help you phrase your original question



Let's see
some
examples.

Then we will
talk about
how you
create
them.

➡ We will have examples in
Grades 5 to 9.



Examples in Grade 5

- You multiply two numbers greater than 50 and the product has a tens digit of 2. What could the numbers be? Explain your answer.



Examples in Grade 5

- A fraction is just a little closer to $3\frac{1}{2}$ than $3\frac{1}{4}$.
- What could it be?



Examples in Grade 5

- ➡ It takes 18 base ten blocks to model a 2-digit by 2-digit multiplication.
- ➡ What might you be multiplying?



Examples in Grade 5

- A rectangle has an area of 100 cm^2 and is pretty long and thin.
- What could the length and width be?



Examples in Grade 5

- ▶ Create three increasing number patterns.
- ▶ One pattern will increase by 7 from one term to the next, and each of the two remaining patterns will increase by a different number.
- ▶ Include the number 120 in all of the patterns.
- ▶ Describe each pattern by using a pattern rule
- ▶ Which term has a value of 120 in each pattern?



Examples in Grade 5

Q Choose values for each of the geometric symbols in the equations below based on the following conditions:

- \triangle represents an even number.
- \square represents a number that is half the value of \triangle .
- \bigcirc represents a number that is the sum of $\triangle + \square$.

$$\square + T = \triangle \quad \bigcirc - R = \square$$
$$S = 2\triangle \quad \triangle \div W = 2$$

Then, solve for each letter variable.



Examples in Grade 5

- A certain 3-D object is a prism. What do you know for sure about the object?
- What are you not so sure about?



Examples in 7

- Investigate how percents are used in a sport you are interested in.
- Describe as many situations as you can, and tell how percents are used and calculated.



Examples in 7

- The difference between two fractions is $\frac{3}{8}$.
- What could the two fractions be?
- Think of three or more pairs of fractions.
- How can you easily even more possibilities?



Examples in 7

- Four numbers that are not too close together have the factors 3, 6, and 15 in common.
- What might the four numbers be?



Examples in 7

- Fill in the blanks to make this expression true.
- $0.3 < 0.[]5 < 0. []2 < [].4$
- Repeat with different digits.



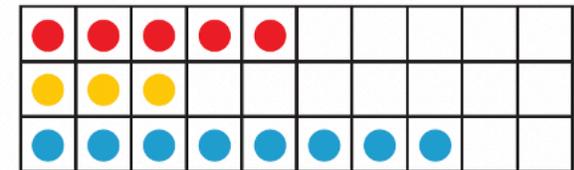
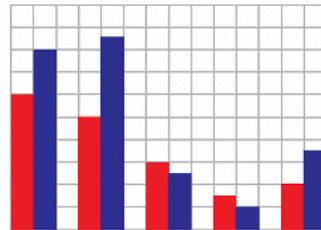
Examples in 7

- A linear increasing pattern is shown on a graph.
- The point that represents its 10th term has a value of 32.
- In a table of values, write a possible first 10 terms.
- Then, show what the graph might look like.

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Examples in 7

Which of these graphs is the least like the others?





Examples in 7

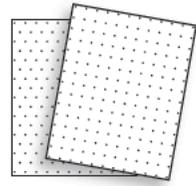
- The circumference of a circle is about equal to the perimeter of a rectangle.
- What could the dimensions of each shape be? Explain your thinking.



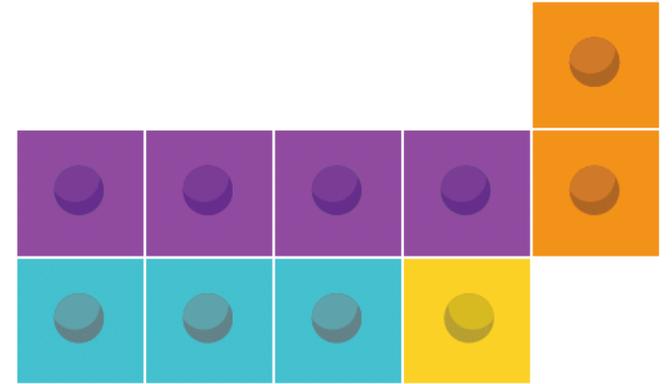
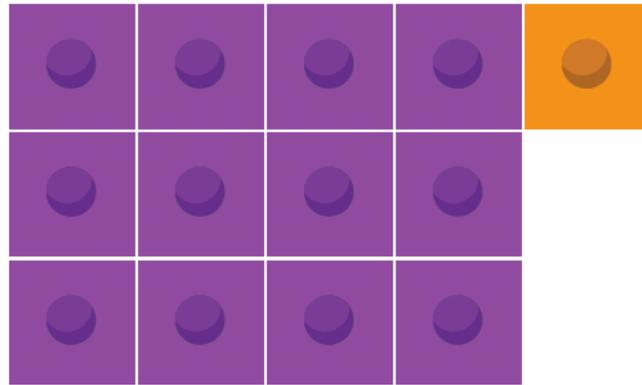
Examples in Grade 9

- A polygon is reflected on a Cartesian plane to create a symmetrical design.
- One vertex of the original polygon is $(1, \underline{\quad})$. One vertex of the reflected polygon is $(\underline{\quad}, 4)$.
- Fill in the blanks with numbers that are not 1 or 4. Then, draw the original polygon and the reflected polygon on a Cartesian plane, and include the line of reflection.

Examples in Grade 9



You can see the following two views of a composite 3-D object:



On square dot paper, draw what the structure could look like from the top, the bottom, the front, the back, the left side, and the right side. Then, calculate the surface area of the structure.



Examples in Grade 9

- Use 10 integer tiles to model an equation. What equation might you be solving? What is the solution?



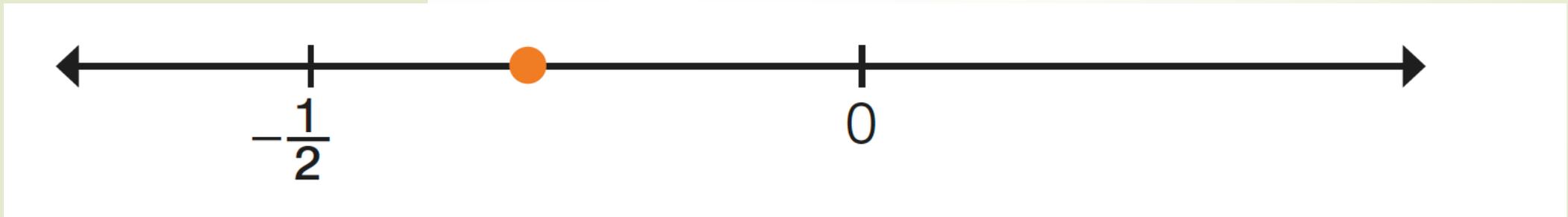
Examples in Grade 9

- A set of data has more than 12 values, and its mode might give people the wrong impression of the data if the mode were the only information they had.
- What might the values be?
- Give the mean, median, and mode of the data.
- Explain why the mode might be misleading.



Examples in Grade 9

- What is a reasonable value for the number at the dot? Explain your thinking.





Examples in 9

- Choose values for a , b , c and d so that $a/b < -c/d$.
- Create four sets of values that you are sure also work.



Examples in Grade 6

- ▶ Choose values from 1 to 10 for A and B in the following pattern rules:
- ▶ Pattern 1: Start at A, and add B for each new term.
- ▶ Pattern 2: Start at B, and add A for each new term.
- ▶ Create a table of values for the first 5 terms of each pattern
- ▶ Compare the graphed patterns. What do you notice?
- ▶ Repeat for two different values of A and B. What do you notice?
- ▶ Do you think your observations would be the same for any values of A and B?



Examples in Grade 6

- You reflect a triangle on a coordinate grid and one of the original vertices $(4,2)$ lands at $(2,4)$.
- Show what might have happened.



Examples in Grade 6

- Write a number of the form $0.[][][]$ for each statement.
- a) It's greater than 0.4162.
- b) It's less than 0.1.
- c) It's between 0.398 and 0.4.
- d) It's closer to 0.47 than 0.393.
- Represent the numbers on a number line.



Examples in Grade 6

➡ How might you sort the following numbers into two groups?

4 9 16 26 36 49 59 64 100



Examples in Grade 6

- A composite number has 12 factors.
- What might the number be?
- Think of three or more possibilities.
- Use tiles to show how these factors form the dimensions of rectangles.



Examples in Grade 6

 Choose four improper fractions to place on the number line. Choose fractions so that there is one in each coloured section.
Describe how you know your fractions belong where you put them.





Examples in Grade 8

- The surface area of a cylinder is close to 100 cm^2 .
- What could the dimensions of the cylinder be?



Examples in Grade 8

- ▶ Kelly performed a probability experiment that had two independent events.
- ▶ There were more outcomes for the second event than there were outcomes for the first event.
- ▶ What might the experiment have been?
- ▶ Use a model to show the theoretical probability of each outcome.
- ▶ Then, perform the experiment, and compare the theoretical and experimental probabilities. What do you notice?



Examples in Grade 8

- Draw a graph of a linear increasing pattern that goes through $(2, 8)$.
- What is an equation that could represent the linear relationship between points in your graph?
- Show two or more possible patterns.



Examples in Grade 8

- Suppose you saved $\square\%$ on an item, which led to a savings of a little more than \$5.
- What do you think the item might have cost?



Examples in Grade 8

- Choose a rate to describe a realistic scale for a map of a real place.
- Then, decide how far apart five pairs of places would be on the map.
- Explain your thinking.



Examples in Grade 8

- The quotient of two fractions is almost 2..
- What might the fractions be?
- Draw pictures to show why your answer makes sense.



Examples in Grade 8

- You multiply two integers and the answer is about 50 less than one of them. What might they be?



How do we
create
open
questions?

- Turn a question around
- Replace a number with a blank
- Use “soft” words
- Generalize the details of a closed question



Turn it
around

- Instead of...
What is a common multiple of 20 and 35?
- Ask: A common multiple of two numbers is 140.
What could the numbers be?



Replace a
number
with a
blank

- Instead of...
What is a common
multiple of 20 and 35?
- Ask: Choose two
numbers to fill the blanks.
Then answer the
question. What is a
common multiple of ____
and ____ ?



Use a soft
word

- Instead of...
What is a common
multiple of 20 and 35?
- Ask: A number is almost a
common multiple of 20
and 35. What could it
be?



Generalize
the details

- Instead of...
What is a common multiple of 20 and 35?
- Ask: Choose a number and 5 less than its double. What is their common multiple?



Generalize the details

- Instead of...
What is a common multiple of 20 and 35?
- Ask: Choose two numbers 15 apart and figure out a common multiple?



Generalize the details

- Instead of...
What is a common multiple of 20 and 35?
- Ask: Choose two numbers between 10 and 40. Figure out a common multiple.



What are
parallel
tasks?

➤ Sets of tasks designed to meet the needs of students at different developmental levels that get at the same bigger idea and are usually similar in context.



An example

- Task 1: Choose a denominator $[\]$ that is more than 4. Draw a picture that explains why $4/[\]$ is what $4 \div [\]$ means.
- Task 2: Choose a denominator $[\]$ that is less than 4. Draw a picture that explains why $4/[\]$ is what $4 \div [\]$ means.



Common questions

- What does dividing, for example, 10 by 5 mean?
- What does e.g. $5/10$ mean?
- What does $10/5$ mean?
- How did knowing what division means help you draw a picture?



Another example

- Task 1: A number between 20 and 30 is 80% of another number. What could the second number be?
- Task 2: A number between 20 and 30 is 150% of another number. What could the second number be?



Common questions

- Is the second number greater or less than the one between 20 and 30 that you chose?
- How did you decide whether it was greater or not?
- How far apart are possible values for the second number?



Another example

- Task 1: An equation with integer coefficients has a rational number solution. What could the equation be?
- Task 2: An equation with rational coefficients has an integer solution. What could the equation be?



Common questions

- ▶ How many variables will you use in your equation? Why?
- ▶ Is an integer a rational number?
- ▶ Can an equation with all positive coefficients have a negative solution or not?



Another example

Task 1: Jennifer is 5000 days old. Describe her age in a unit that makes more sense.

Task 2: Leah is 5 000 000 minutes old. Describe her age in a unit that makes more sense.



Common questions

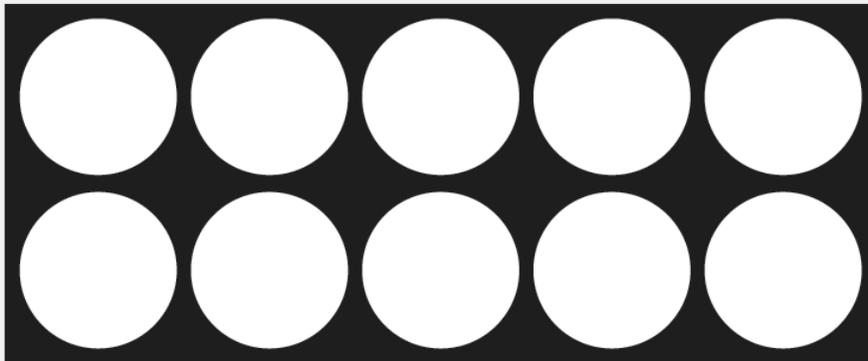
- Why is it hard to tell how old the girl is right away?
- Do you think she is a baby? Explain.
- Do you think the girl is older or younger than you are?
- What unit did you change to ? Why?
- How did you make the change work?



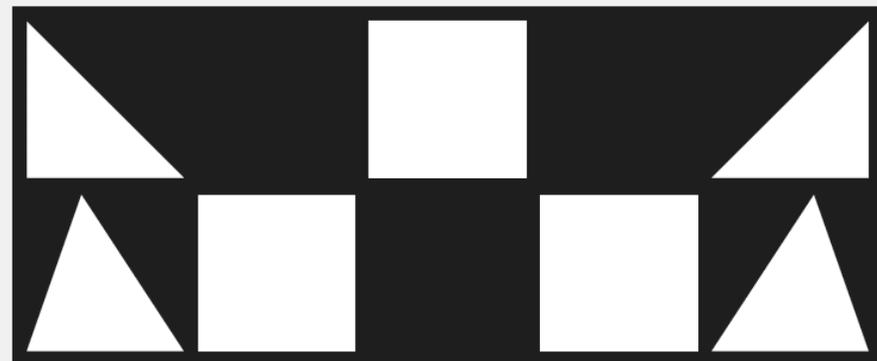
Another example

What fraction of the area of the rectangle is black?

Choice 1:



Choice 2:





Common questions

- Is the black area more or less than half the total area? Why do you think that?
- What formulas did you use to help you?
- Could you tell the fraction just by looking?



How do
you create
them?

- Start with a task.
- Think about where stumbling blocks might be.
- Change those details.
- Think of common questions that work for both tasks (often not the answers).



I am going
to ask you
to

- Try two tasks, either both open questions, both parallel tasks or one of each built on ideas we have discussed today.
- I will ask several of you to talk about what happened. Please bring some student samples for us to see or hear.



And now

➤ Are there questions you want to raise?