

Teaching Remotely with Focus

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May, 2020



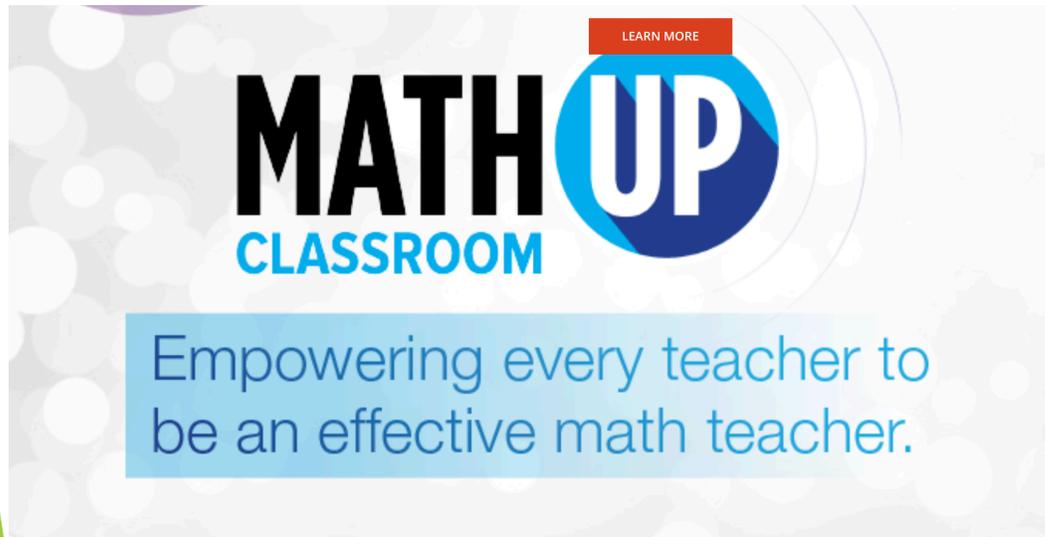
Ru**bi**con
celebrating 30 years

Sometimes

- ▶ What matters most now is “big stuff”, not details.
- ▶ What would that be for me now?

Sources

- ▶ Many of the provided screen shots come from my Open Question book series and my new resource MathUp.



Kindergarten

- ▶ Playing with numbers
- ▶ Counting practice

Playing with numbers

- ▶ Which is more? Which is less?
- ▶ Results of increasing and decreasing amounts
- ▶ Representing amounts

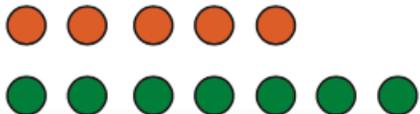
Which is more? Which is less?

Q One number is a lot more than another. What might the two numbers be? Explain your answer.

Which is more? Which is less?



Which of the three arrangements make it easy for you to tell whether there are more green or more orange counters?

1. 

2. 

3. 

Results of increasing and decreasing

- ▶ You start with 4 and make it a little bigger.
- ▶ What could it be now? Show me.

Results of increasing and decreasing

- ▶ You made a number a little smaller.
- ▶ Now it's 8.
- ▶ What was it?

Representing amounts

- ▶ Choose two numbers.
- ▶ Show each one in two different ways.

Representing amounts



Choose a number that is between 1 and 10. Then, draw a picture that has the same number of dots or lines as the number you chose. Tell a story about your picture.

Counting Practice

- ▶ Rote counting
- ▶ Cardinal counting

Counting Practice

- ▶ Count as high as you can
- ▶ Count a small collection of craft sticks
- ▶ Count a big collection of Lego bricks

Counting Practice

- ▶ How many forks do you think you should have in your house?
- ▶ How many do you have?

Kindergarten

- ▶ Although maybe not as mathematically critical, work with patterns and sorting suits the home environment.

Continuing patterns

- ▶ A pattern of emojis starts like this:
- ▶ How could you continue it?



Sorting

- ▶ Grab a handful of pebbles. Find a way to sort them.

Grade 1

- ▶ Adding and subtracting
- ▶ Representing Numbers
- ▶ Counting practice

Adding and subtracting

- ▶ When do you add? Subtract?
- ▶ What strategies do you use?



• Decide how many children are playing in a sandbox. Choose a number less than 10.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

• Decide how many more children join them.

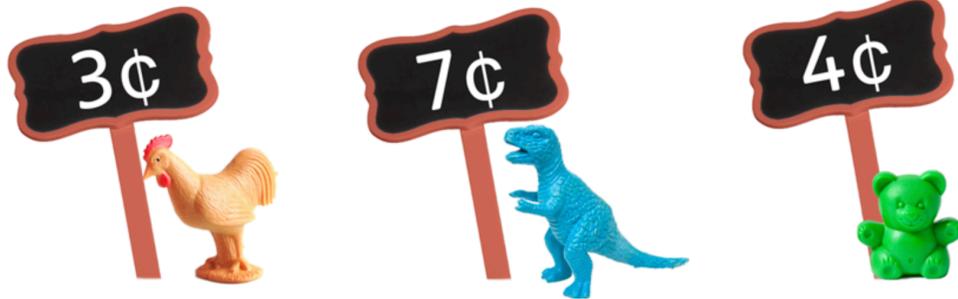
• How many children are there now?

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
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Adding and subtracting

You have 10¢.

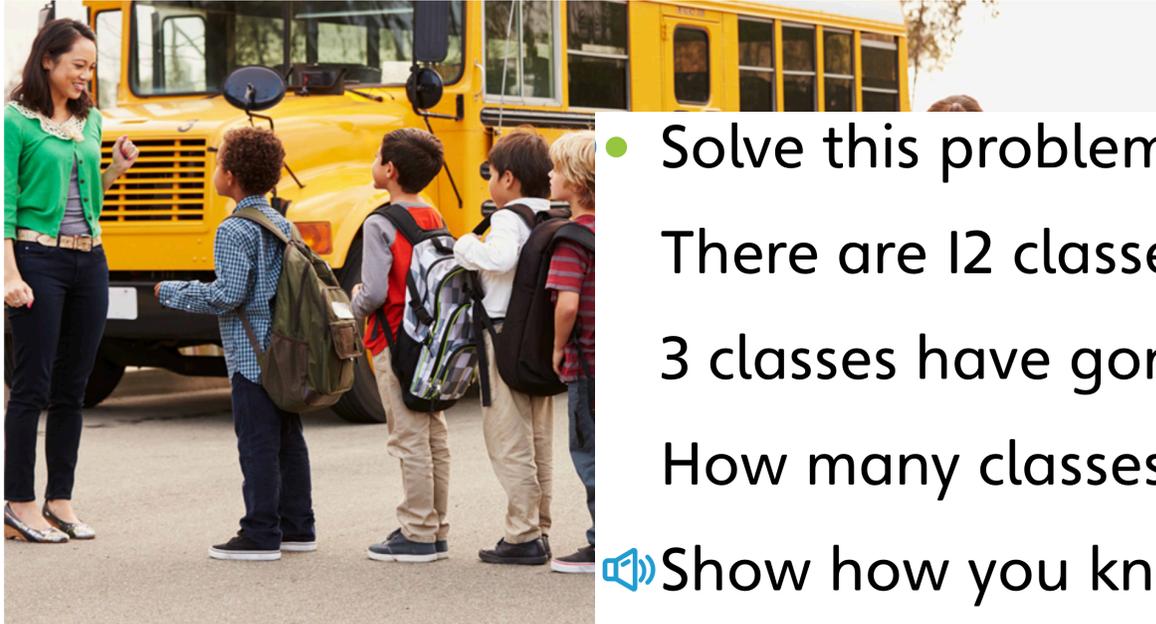
You buy one of these things.



How many cents will you have left?

How do you know?

Adding and subtracting



- Solve this problem:

There are 12 classes in Andrea's school.
3 classes have gone away to see a play.
How many classes are still in school?

-  Show how you know.

- Make up 2 problems that you can solve the same way. Do not make up problems about a school.

Adding and subtracting

- ▶ Make up a story where you would add $5 + 6$.
- ▶ Make up a story where you would take away 7 from 9.
- ▶ Make up a story where you would figure out how much more 12 is than 5.

Adding and subtracting



Lia and Ethan are playing a game.
Lia was 2 spaces ahead of Ethan.
Lia took a turn and then Ethan took a turn.
Now, Ethan is 2 spaces ahead of Lia.

- What square might they have been on in the game?
- What numbers might they have rolled?
- How did you figure it out?

Give more than one answer.

Adding and subtracting

- ▶ Draw a picture to show why $9+8$ has to have the same answer as $10 + 7$.
- ▶ Draw a picture to show why $12 - 9$ has to have the same answer as $13 - 10$?

Representing Numbers

- ▶ What do numbers to 20 mean and look like?

Representing Numbers

Choose 2 numbers
between **10** and **20**.

10 ? ? **20**

How are the 2 numbers the same?
How are they different?

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Representing Numbers Greater Than 10



Representing Numbers

 What numbers under 20 can you show in the shape of a triangle? How does your picture show how your number can be broken up into parts?

Representing Numbers

- ▶ What number might be $15 + \text{a little}$?
- ▶ What number fills more than 1 ten frames?

Counting Practice

- ▶ Some skip counting, but mostly counting collections

Counting Practice

- ▶ You are counting and say 12. What could come next?

Counting Practice

There are a lot of balls
on the gym floor.

How could you place them
so that they are easy to count?

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Counting in Groups



Counting Practice

- ▶ How high can you count?
- ▶ How did you make sure you counted all the Lego blocks?

Grade 2

- ▶ Adding and subtracting
- ▶ Representing Numbers
- ▶ Growing Patterns

Adding and Subtracting

- ▶ When do you add? Subtract?
- ▶ What strategies?

 1. The problem starts:

Erica has collected the
garbage from 3 rooms.

 Finish the problem. Make it a problem you could solve by adding. Write an equation for the problem. Explain why your problem works.

 2. The problem starts:

Erica has collected the garbage from 3 rooms.

 Finish the problem. Make it a problem you could solve by subtracting. Write an equation for the problem. Explain why your problem works.

 4. The problem starts:

There were 10 dogs
in the park.

 Finish the problem. Make it a problem that you could solve either by adding or by subtracting. Write 2 equations for the problem. Explain why your problem works.

Adding and Subtracting

- ▶ Make up a story where you would add $25 + 16$.
- ▶ Make up a story where you would take away 12 from 21.
- ▶ Make up a story where you would figure out how much more 31 is than 22.

Adding and Subtracting

4. Suppose you know $8 + 5 = 13$. What subtraction equations can you figure out now?

3. What other fact could you use to help you figure out these facts? How does it help?

- $9 + 8$

Adding and Subtracting

- ▶ Draw a picture to show why $18 + 12$ has to have the same answer as $20 + 10$.
- ▶ Draw a picture to show why $42 - 19$ has to have the same answer as $43 - 20$?

Representing Numbers

- ▶ What do numbers to 100 mean and look like?

Representing Numbers



8



5

1. Make some towers of 5 counters. Make some towers of 8 counters.
2. Put a few towers into a group. How many counters are in the group?
3. Repeat the task 2 more times. Make different numbers.
4. What numbers can you compose using 5s and 8s?

Representing Numbers

- ▶ Draw a picture to show how 65 and 56 look different.
- ▶ Draw a picture that shows that 122 is a little more than 12 tens.
- ▶ How would you explain how much 314 is?

Growing Patterns

- ▶ Predicting how patterns continue

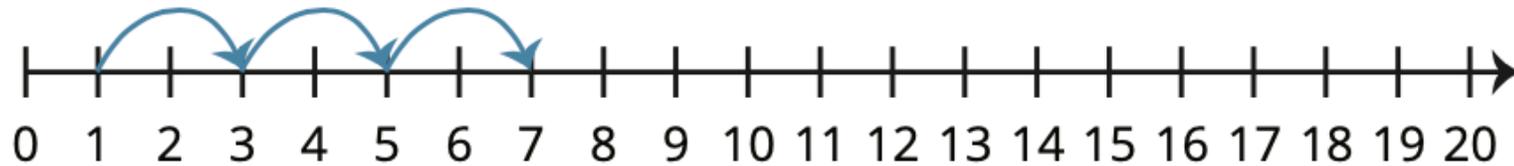
Growing Patterns



1. Create a growing pattern using triangles. Start with 4 triangles.
2. Represent the same pattern in different ways.
 - Use a number line.
 - Use actions.

Growing Patterns

→→→→→ **Q** If you continued jumping on the number line with this pattern, what are some numbers greater than 20 you would never land on? Why wouldn't you?



Growing Patterns

- ▶ How long does it take this pattern to get to 100?
- ▶ 5, 8, 11, 14,....

Grade 3

- ▶ Multiplication and division
- ▶ Comparing numbers
- ▶ Fractions

Multiplication and division

- ▶ What do they mean?
- ▶ How do you model them?

Multiplication and division



This picture shows 6×2 since there are 6 equal groups of 2 wheels.

- Draw your own picture that shows multiplication.
- Tell why your picture shows multiplication.
- What multiplication expression does your picture show?
- What addition expression does your picture show?

Multiplication and division

Follow these steps to land on an odd number on a number line.

- Start at 0.
- Make a jump.
- Make more jumps of the same size.
- You must end on an odd number.

Multiplication and division



A pile of books is shared among 4 students.
Each student gets the same number of books.
No books are left over.

- How many books could be in the pile?
- How many books could **not** be in the pile?
- How do you know?

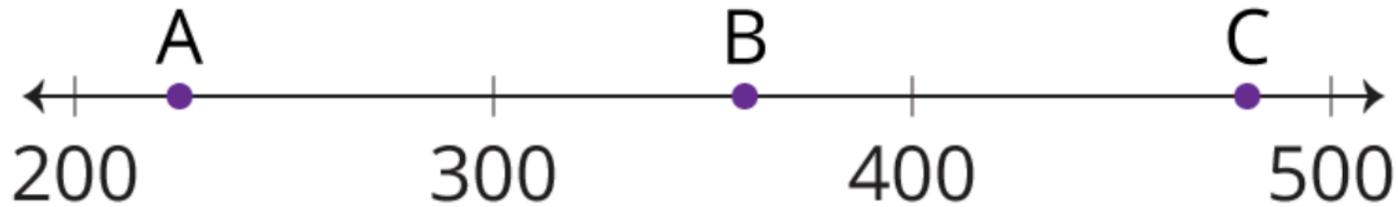
Multiplication and division

- ▶ Make up a story that is about 4×5 .
- ▶ Make up a story that is about $24 \div 4$.
- ▶ Draw a picture that shows how to figure out 5×7 .
- ▶ Draw a picture that shows how to figure out $15 \div 3$.

Comparing Numbers

- ▶ Which is bigger? Why?

Comparing Numbers



- What number might each dot represent?
Why do you think that?

Comparing Numbers

Arizona's class is comparing the numbers of students at different schools.



School A: 48
School B: 21
School C: 5 4
School D: 3
School E: 3 8

1. Choose digits to complete the school numbers above.
Use different numbers to make each statement below true.
Show how you know your digits work.
- a) School A has more students than School C.
 - b) School A has fewer students than School C and School E.
 - c) School D and School E have close to the same number of students.

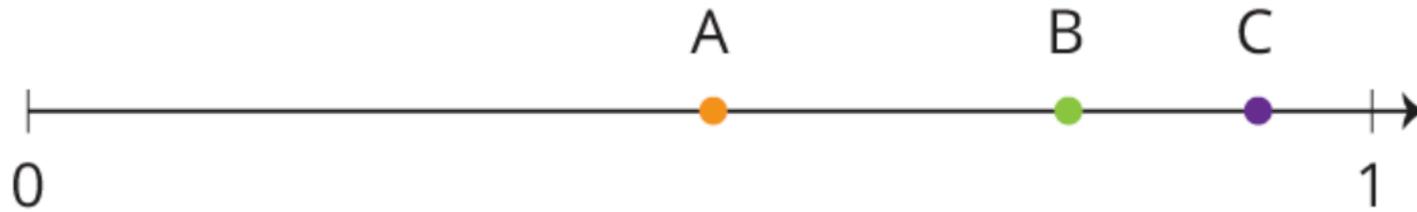
Comparing Numbers

- ▶ How could the first number be bigger?
 - ▶ []42 and []91
- ▶ How could the second one be bigger?
 - ▶ []42 and []91

Fractions

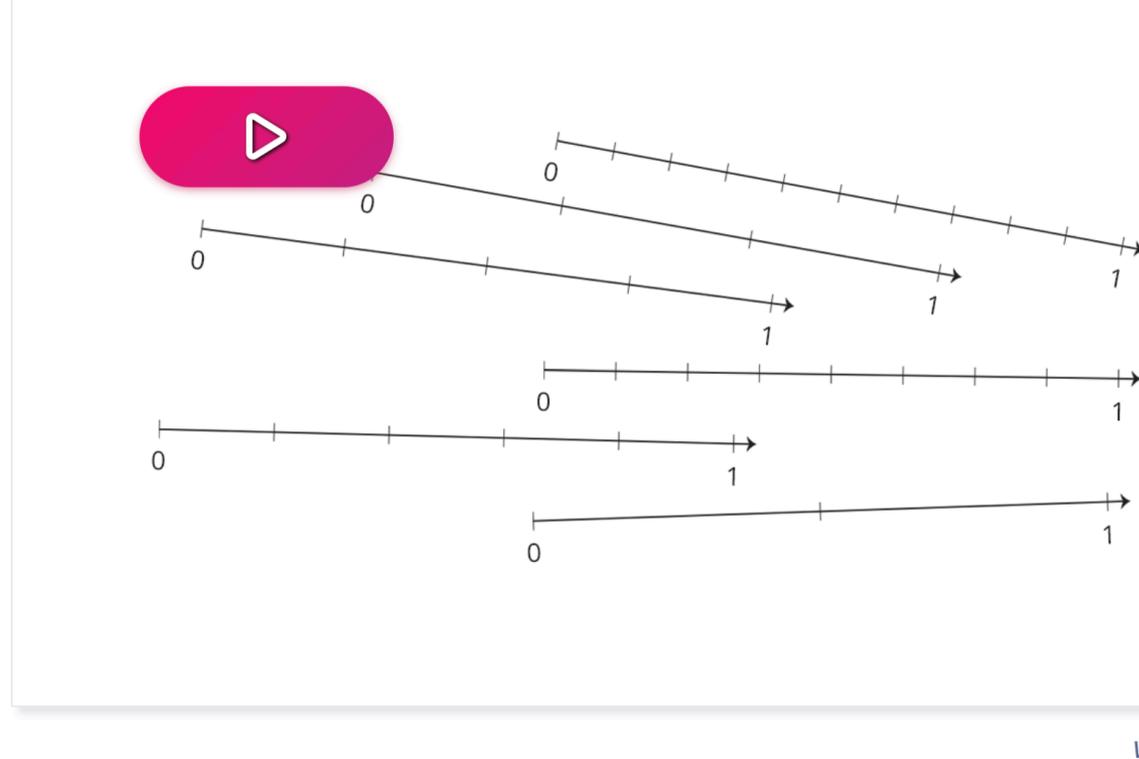
- ▶ What do they mean?
- ▶ How are different models related?

Fractions



- Which letter do you think shows three-fourths on this number line? Why?

Fractions



1. Use number lines to make fractions.

a) Choose one of these numbers:
one, two, three, four, five, six, eight, ten

Choose one of these words:
halves, thirds, fourths, fifths, eighths, tenths

Make a fraction with your 2 choices. Your fraction must be less than 1 whole.

Fractions

- ▶ Show $\frac{4}{5}$ on a number line.
Show it with a group of stickers.
- ▶ This is $\frac{1}{3}$. Show me what $\frac{2}{3}$ looks like.
- ▶ Show what the whole looks like.



Measurement

- ▶ Not critical, but... What can you measure?

Measurement

- ▶ How could you figure out how long your bed is?
- ▶ How could you figure out how big your pillow is?
- ▶ What else could you measure that might be in your bedroom?

Grade 4

- ▶ Multiplication and division
- ▶ Comparing fractions
- ▶ Representing Numbers

Multiplication and Division

- ▶ What do they mean?
- ▶ What strategies do students have?
- ▶ Working on the facts

Multiplication and Division



Julia has four times as many coins as Ben.

- How many coins might Ben and Julia have?
- Suppose Ben has three coins.
Why does 4×3 tell the number of coins Julia has?
- Who do you think has more money? Can you be certain?

Multiplication and Division

Q Which one of these phrases do you think doesn't belong?

- 3 times as many as 8
- 4 times as many as 6
- 6 times as many as 3
- 2 times as many as 12

Multiplication and Division



Create three multiplication problems that use these numbers:

23, 45, 3, 10, 6, 15

Each problem should use different numbers. Then, solve the problems.

Multiplication and Division

- ▶ Make up a story where you would multiply 9×9 .
- ▶ Make up a story where you would divide 54 by 6.
- ▶ Make up a story where you would multiply 15 by 4.

Multiplication and Division

- ▶ How does knowing 3×7 help you figure out 9×7 ?
- ▶ Why do $24 \div 6$ and $12 \div 3$ HAVE to have the same answer?

Comparing fractions

- ▶ How can you compare mentally (rather than procedurally)?

Comparing fractions

A large group of students from Grades 1 to 6 needs to be organized into five smaller groups for sports day.

Create five groups of students. Call them Groups A, B, C, D, and E.

- Group A must be exactly $\frac{1}{3}$ students from Grades 1 to 3.
- Group B must have a greater fraction of students from Grades 1 to 3 than Group C.
- Group D must be almost all students from Grades 1 to 3.
- Group E must be about $\frac{1}{2}$ students from Grades 1 to 3.

The groups do **not** have to be equal in size.

There could be 10, 12, 15, or 20 students in any of the groups.

Show all your work.

Comparing fractions

Q Think of some fractions where the numerator of a fraction is three less than the denominator. Which of these could be true?

- The fraction is less than $\frac{1}{2}$.
- The fraction is greater than $\frac{1}{2}$.
- The fraction is greater than $\frac{3}{4}$.
- The fraction is greater than $\frac{9}{10}$.
- The fraction is less than $\frac{1}{8}$.

Explain your thinking.

Comparing fractions

- ▶ How do you know that $\frac{2}{3} < \frac{4}{5}$ by drawing a picture?
- ▶ How do you know without a picture?
- ▶ How can you be sure that $\frac{2}{15} < \frac{8}{9}$ without doing any work?

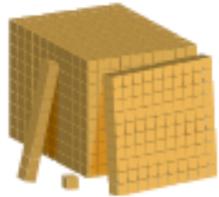
Representing Numbers

- ▶ What do you know about numbers in the thousands?

Representing Numbers

 You read a whole number and you say exactly five words. What could the number be?

Representing Numbers



List three numbers that you can represent with 28 base ten blocks. Describe another way you could represent each number using base ten blocks.

Representing Numbers

- ▶ How do you know it would take more \$10 bills to show \$220 than \$20 bills?

Grade 5

- ▶ Multiplication and Division
- ▶ Decimals
- ▶ The Rectangle

Multiplication and division

- ▶ Strategies for working with larger numbers

Multiplication and division



To model 31×14 , Sarah used 20 base ten blocks. What might her model have looked like? What other two-digit multiplications might be modelled with 20 base ten blocks?

Multiplication and division

 Choose a remainder of 2, 3, or 4. Try to divide lots of three-digit numbers by one-digit numbers to get that remainder. Describe your strategy.

Multiplication and division

\$ **Q** Choose prices so that the cost of one T-shirt is slightly more than the cost of one pair of shorts. Tell how you solved the problem.

- 3 T-shirts cost \$ _____.
- 5 pairs of shorts cost \$ _____.

Multiplication and division

- ▶ Make up a story where you would multiply 22×25 .
- ▶ Make up a story where you would divide 150 by 3.
- ▶ Make up a story where you would multiply 135×4 .

Multiplication and division

- ▶ Draw a picture to show what 15×34 looks like without drawing a ton of things.
- ▶ How does knowing 20×40 help you figure out 20×45 ?
- ▶ Why do $124 \div 16$ and $62 \div 8$ HAVE to have the same answer?
- ▶ Why is the answer to $513 \div 7$ less than 100?

Decimals

- ▶ What are they?
- ▶ How do you operate with them like you do with whole numbers?

Decimals



A decimal number with thousandths is closer to 4.12 than to 4.7.

- What might the number be? Think of several possible numbers. Explain your choices.
- What could the number **not** be?

Decimals



1. Create two problems that could be solved by adding 1.935 and 2.45. Make the problems sound quite different.
Solve the problems.

Decimals

- ▶ What whole numbers could you subtract to help you figure out $4.5 - 2.14$?
- ▶ What would you have to add to 3.8 to get to 5.13 ? How could you write that as an equation?
- ▶ Draw a picture to show why $0.12 < 0.4$.

The Rectangle

- ▶ Measurement formulas - perimeter and area

The Rectangle

- 1. Choose two numbers in a row greater than 20. What rectangles can you make with those numbers of square units as the area? Each rectangle should be made up of full squares.
 - Build as many rectangles as you can for each area.
 - For each rectangle, write an equation that relates the area to the number of squares in the rectangle.
-
- 2. a) Predict whether there will be more rectangles for the next number.
 - b) Test to see if you are right.

The Rectangle

Q The area of Rectangle A is twice the area of Rectangle B. The perimeter of Rectangle A is 20 units greater than the perimeter of Rectangle B. What could the dimensions of the two rectangles be? Construct your rectangles to show you are right.

The Rectangle

- ▶ A rectangle has side lengths between 10 and 20.
- ▶ Pick side lengths.
- ▶ What is the rectangle's perimeter and area?

- ▶ Could a rectangle have a big number for its perimeter and a little number for its area?

Grade 6

- ▶ Ratio and percent
- ▶ Cartesian grid
- ▶ Using variables
- ▶ Area formulas

Ratio and Percent

- ▶ What are they?
- ▶ What does it mean to have an equivalent ratio?

Ratio and Percent

Kate has only nickels and dimes in her coin purse.
She has two times as many dimes as nickels.

- How much money might she have? How do you know?
- Write a ratio to compare the number of dimes and nickels that Kate has.

Ratio and Percent

VISIBIE 

1. Read this list of ratio and percent statements.
Some will make sense to you, but others will not.



Statement A:

85% of all children are toddlers.



Statement B:

The ratio of teachers to students in most classes is 9:1.



Statement C:

About 2% of the students in a school take the bus.



Statement D:

The ratio of flour to sugar in a cake recipe is often about 3:1.



Statement E:

When you open a new carton of milk, it's usually about 95% full.



Statement F:

A newborn's length is about 25% of what its adult height will be.



Statement G:

There are many families with a ratio of adults to children of 1:1.



Statement H:

The ratio of time you're awake to the time you're asleep is about 6:6.

- a) Choose two statements that make sense.
Explain why they make sense.
- b) Choose two statements that don't make sense.
Explain why they don't make sense.

Ratio and Percent

- ▶ Use at least 50 circles to show a ratio of 4:5.
- ▶ 25% of a number is 15. What is 75% of that number?

Decimals

- ▶ What are they?
- ▶ How do you add, subtract, multiply and divide with them?

Decimals

$$\begin{array}{ccc} \text{A} & \text{B} & \text{C} \\ 0.0 \square 2 4 \square \square & 0. \square \square 3 4 \square \square & 0.02 \square 4 \square \end{array}$$

1. Choose values for the missing digits so that each statement below is true. Explain how you know your answer to part a) is correct.

a) $A < B < C$

b) $A < C < B$

c) $C < A < B$

2. Choose digits for the blanks to make this statement true:

$$0. \square 4 \square 1 5 2 < 0. \square 5 \square \square 1 < 0. \square \square 5 2 < 0.8 \square \square 4 \square$$

Decimals

 You multiply a decimal number with a 7 in it by 100.
What do you know for sure about the product?

Decimals

- ▶ Draw a picture to show how little 0.0003 really is.
- ▶ Some people think that this is how you write seventeen ten thousandths: 0.1 710 000.
- ▶ Other people think it is 0.00017.
- ▶ What do you think? Why?

Cartesian grid

- ▶ How do you locate?

Cartesian grid

1. a) Draw a horizontal axis and a vertical axis on grid paper. Label the lines on both axes from 0 to 10.
 - b) Draw three different parallelograms on your coordinate grid. Two sides of each parallelogram should be either vertical or horizontal lines on the grid.
 - c) Label the vertices of each parallelogram.
2. What did you notice about the vertices of the parallelograms?

Cartesian grid

- ▶ Create a set of clues for another student to guess a point on your grid.
- ▶ Make sure your friend needs 3 clues to figure it out.

Cartesian grid

- ▶ Draw a picture of a hexagon on a Cartesian grid where some sides are short and some are really long. Tell the coordinates of the vertices.

Using Variables

- ▶ What different things does an equation mean?
- ▶ What is a variable?

Using Variables

Sort these equations into two groups.

What is the same about all the equations in each group?

Equation 1: $2 \times n + 7 = 11$

Equation 2: $2 \times n = a$

Equation 3: $2 \times n - 2 = 16$

Equation 4: $35 = 3 \times n - 1$

Equation 5: $A = b \times h$

Equation 6: $A = l \times w$

Equation 7: $4 \times n + 2 \times n = 18$

Equation 8: $4 \times n + 2 \times n = 6 \times n$

Using Variables

Choose a number to start with and a rule that leads to a result of 20. Don't write down the rule or tell anyone what number you started with.



Write your rule and an equation that describes how your rule changes your number to get to 20.

Using Variables

Q Which equation do you think doesn't belong?

$$3 \cdot a \div 6 = 5$$

$$4 \cdot a + 52 = 92$$

$$3 + 18 \cdot a = 183$$

$$3 \cdot a - 12 = 11$$

Using Variables

- ▶ What do you mean when you write $3x = 18$?
- ▶ What do you mean when you write $3x = x + x + x$?

Area Formulas

- ▶ Triangles
- ▶ Parallelograms

Area Formulas



Choose four consecutive whole numbers. One number will be the area of a square, one will be the area of a rectangle, one will be the area of a triangle, and one will be the area of a parallelogram, all in square centimetres. Construct these shapes using centimetre grid paper or dynamic geometry software.

Area Formulas

 The area of a parallelogram is exactly 10 cm^2 greater than the area of a triangle. What could the dimensions of the shapes be?

Area Formulas

- ▶ A triangle and a parallelogram both have areas of 20 cm^2 . What might be their heights and bases?

Grade 7

- ▶ Integer operations
- ▶ Percent
- ▶ Algebra

Integer operations

- ▶ How is operating with integers like working with whole numbers? How is it different?

Integer operations

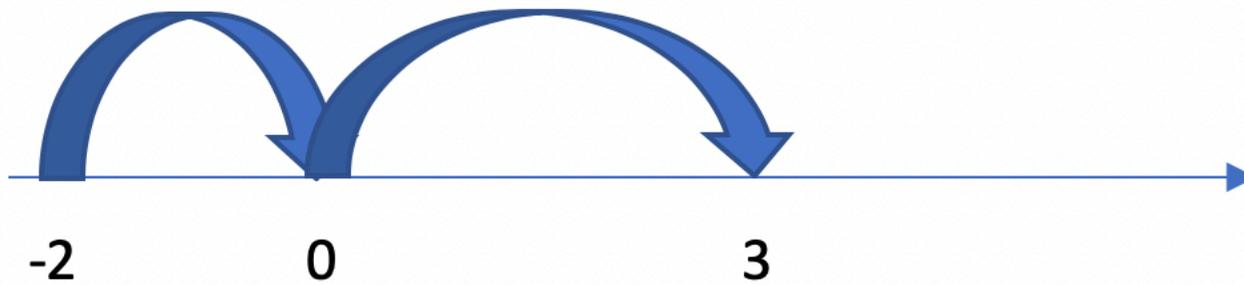
- ▶ You add a bunch of positive integers to three times as many negative integers.
- ▶ What totals could you get? Not get?

Integer operations

- ▶ You multiply two integers and the result is about 50 less than one of them.
- ▶ What could they be?

Integer operations

- ▶ To figure out $3 - (-2)$, Andrew drew this picture. Why?



Percent

- ▶ Problems/calculations involving percent

Percent

- ▶ Priya paid \$45 for a jacket on sale.
- ▶ Choose the sale discount and tell the original price of the jacket.

Percent

- ▶ You know that 30% of B is less than 20.
- ▶ What other percent statements could you make?

Percent

- ▶ You know that 30% of number B is less than 20.
- ▶ What other percent statements could you make?

Percent

 Investigate how percents are used in a sport you are interested in. Describe as many situations as you can, and tell how percents are used and calculated.

Percent

- ▶ Draw a picture to show why 35% of 80 is 28.
- ▶ You paid \$21 for a jacket that is 70% off.
- ▶ Draw a picture to show how to figure out the original price.

Algebra

- ▶ Interpreting meaning of expressions

Algebra



The function machine tells you what to do with the number that goes in to get the number that comes out.

- What do you think the rule for this machine is?

Algebra

A $4x = 28$

B $2x = x + x$

C $x + 12 = 3x$

D $41 = 3x + 20$

Algebra

- Create three Which Does Not Belong? activities using these three formats:
 - four algebraic expressions with their models
 - four algebraic expressions that require evaluation
 - four equations that can be solved using models

For each activity, make sure there are a couple of reasonable choices for which one does not belong.

- Explain why there are a couple of choices that could be selected in each case.

Algebra

- ▶ An algebraic expression involving $[\]x - [\]$ is worth more when $x = 2$ than when $x = 10$. What could it be?

Any questions?