UNDERSTANDING AND THINKING: THE FUNDAMENTALS

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We hear a lot from quite a varied set of folks about what aspects of math are important.

Not all of those viewpoints are consistent.

I have the opportunity today to share my perspective on what I think matters the most in teaching children mathematics and why.

- These ideas are not unique to me.
- In many ways, they derive from what our U.S. colleagues call “standards for mathematical practice” and what we call “processes”.

- Both value making sense of what is being learned.

1. Make sense of problems and persevere in solving them.

3. Construct viable arguments and critique the reasoning of others.

- reasoning and proving
- reflecting

- connecting

People like things that make sense.

- David Sousa and Carol Ann Tomlinson in “Differentiation and the brain: How neuroscience supports the learner-friendly classroom” suggest:
- Nothing goes into long-term memory unless it makes sense and has meaning.

- I am going to define, for today, understanding as making sense of information you've learned and being able to justify (an aspect, of course, of communication).

In Ontario

- There is an achievement chart category called “knowledge and understanding”.
- But what’s the difference between the two?

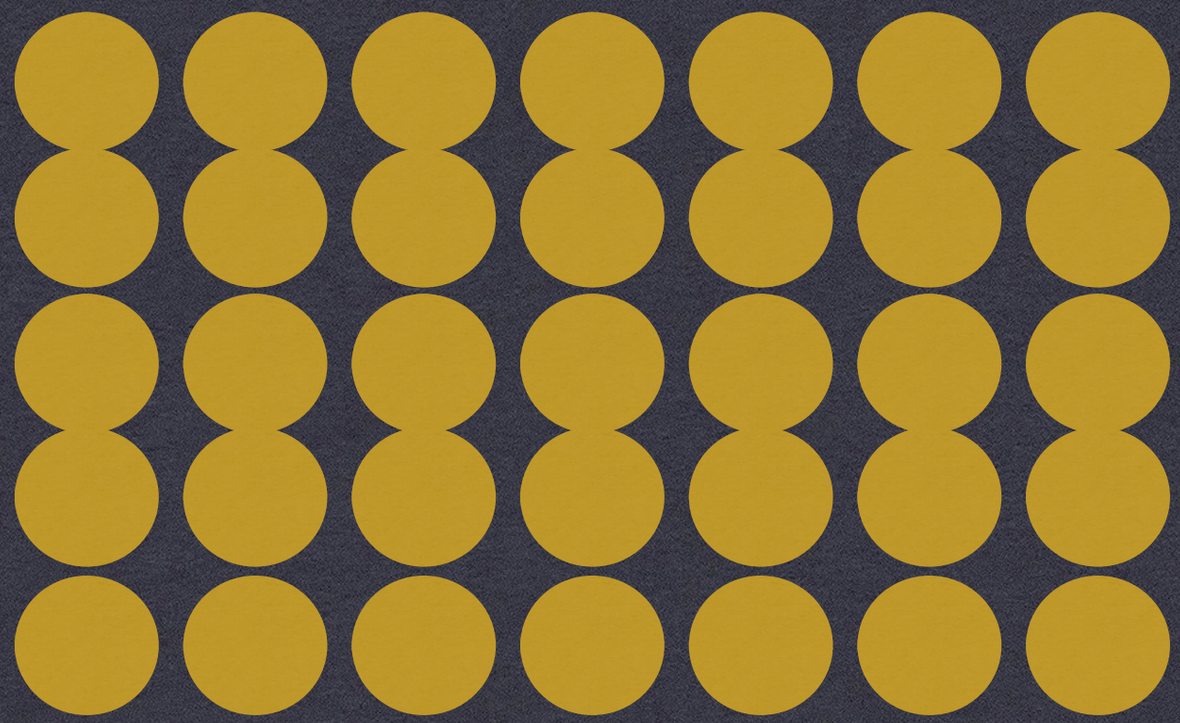
Categories	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding *Subject-specific content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding)*				
	The student:			
Knowledge of content (e.g., facts, terms, procedural skills, use of tools)	– demonstrates limited knowledge of content	– demonstrates some knowledge of content	– demonstrates considerable knowledge of content	– demonstrates thorough knowledge of content
Understanding of mathematical concepts	– demonstrates limited understanding of concepts	– demonstrates some understanding of concepts	– demonstrates considerable understanding of concepts	– demonstrates thorough understanding of concepts

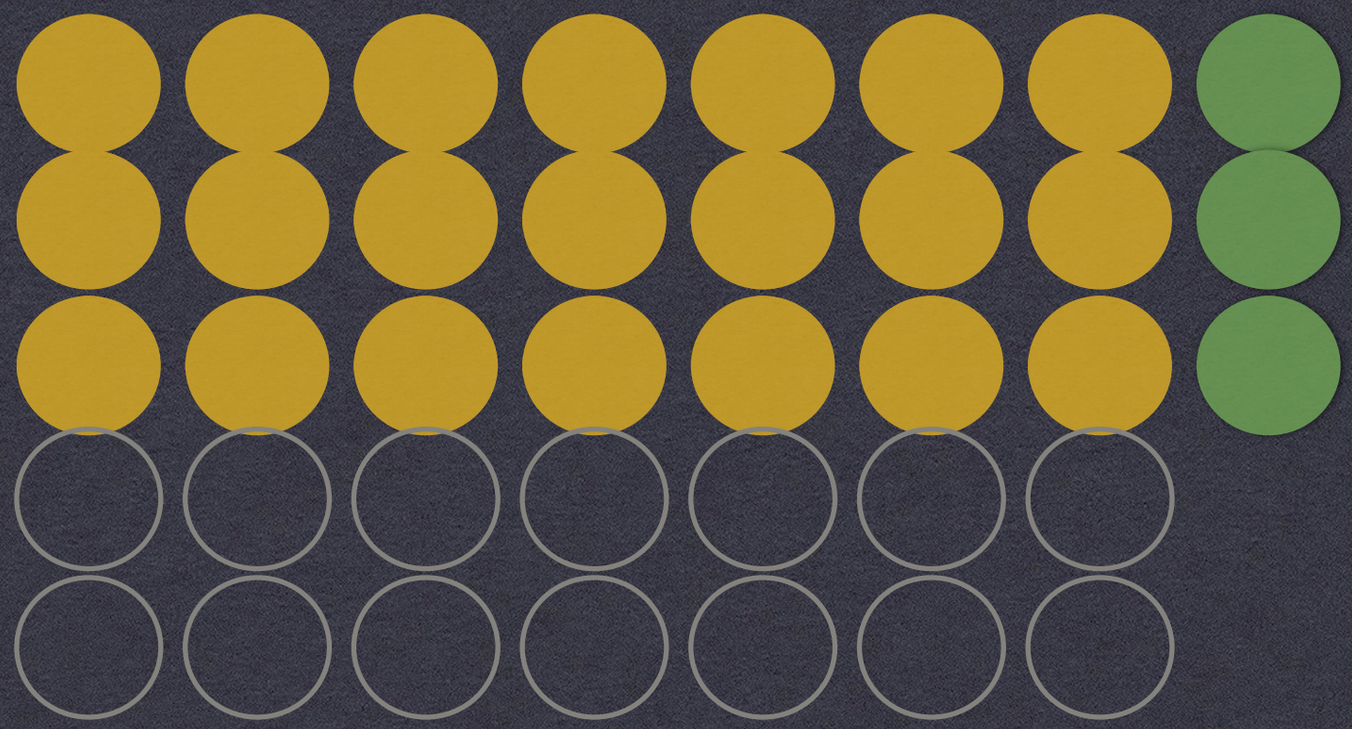
KNOWLEDGE AND UNDERSTANDING

- The document doesn’t tell us.
- But what is it?
- Look at these examples

Grade 4

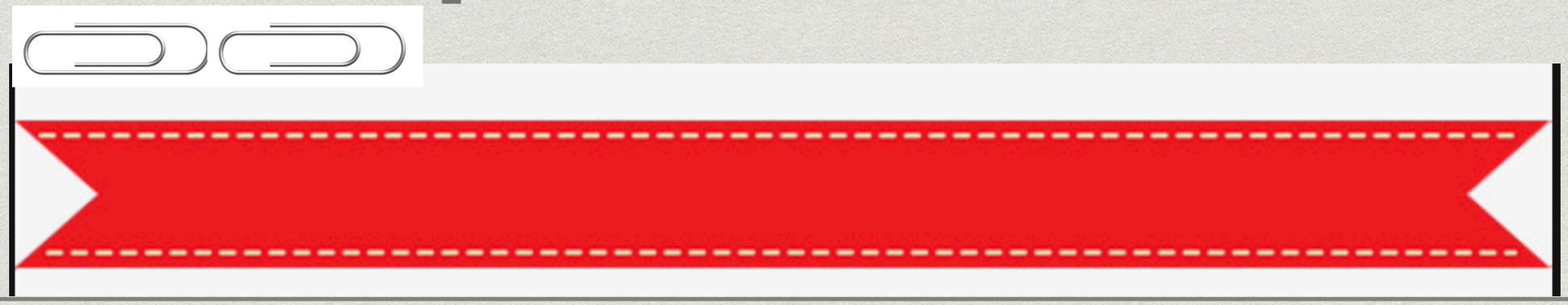
- **Knowledge:** What is 4 x 8?
- **Understanding:** How do you know that 5 x 7 > 3 x 8 without calculating the products?





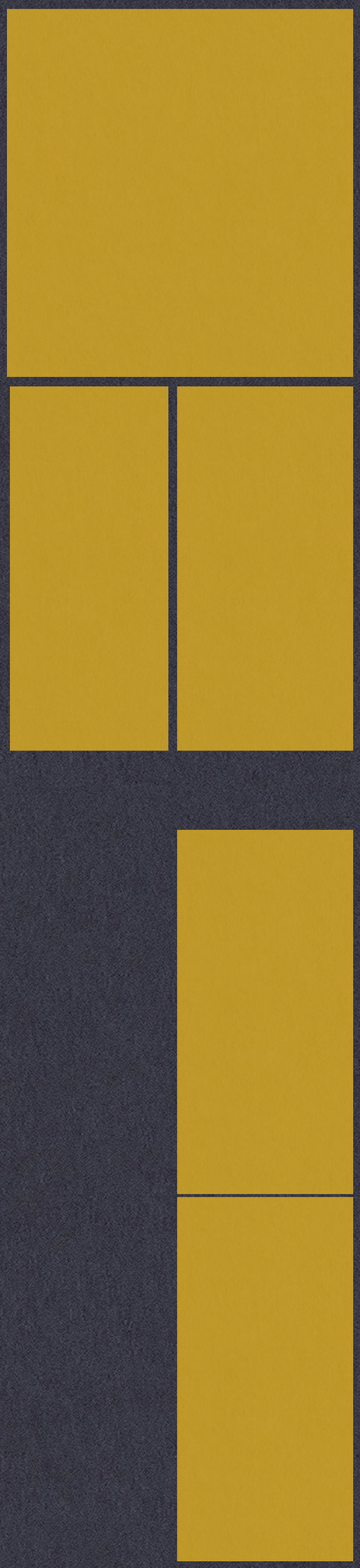
Grade 1

- **Knowledge:** How many paper clips long is this ribbon?[You have lots.]
- **Understanding:** How do you know this ribbon is not 5 of these paper clips long? [You only get these two to use.]



Grade 5

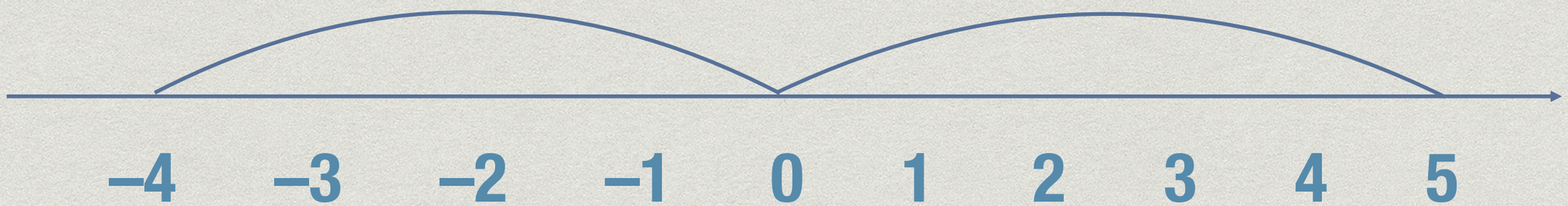
- **Knowledge:** What is the area of this rectangle?
- **Understanding:** Two rectangles have the same area. One has twice the length of the other. What do you know about the widths?



Grade 7

- **Knowledge:** What is 5 – (–4)?
- **Understanding:** Use a model to show why 5 – (–4) has to have the same answer as 5 + 4.

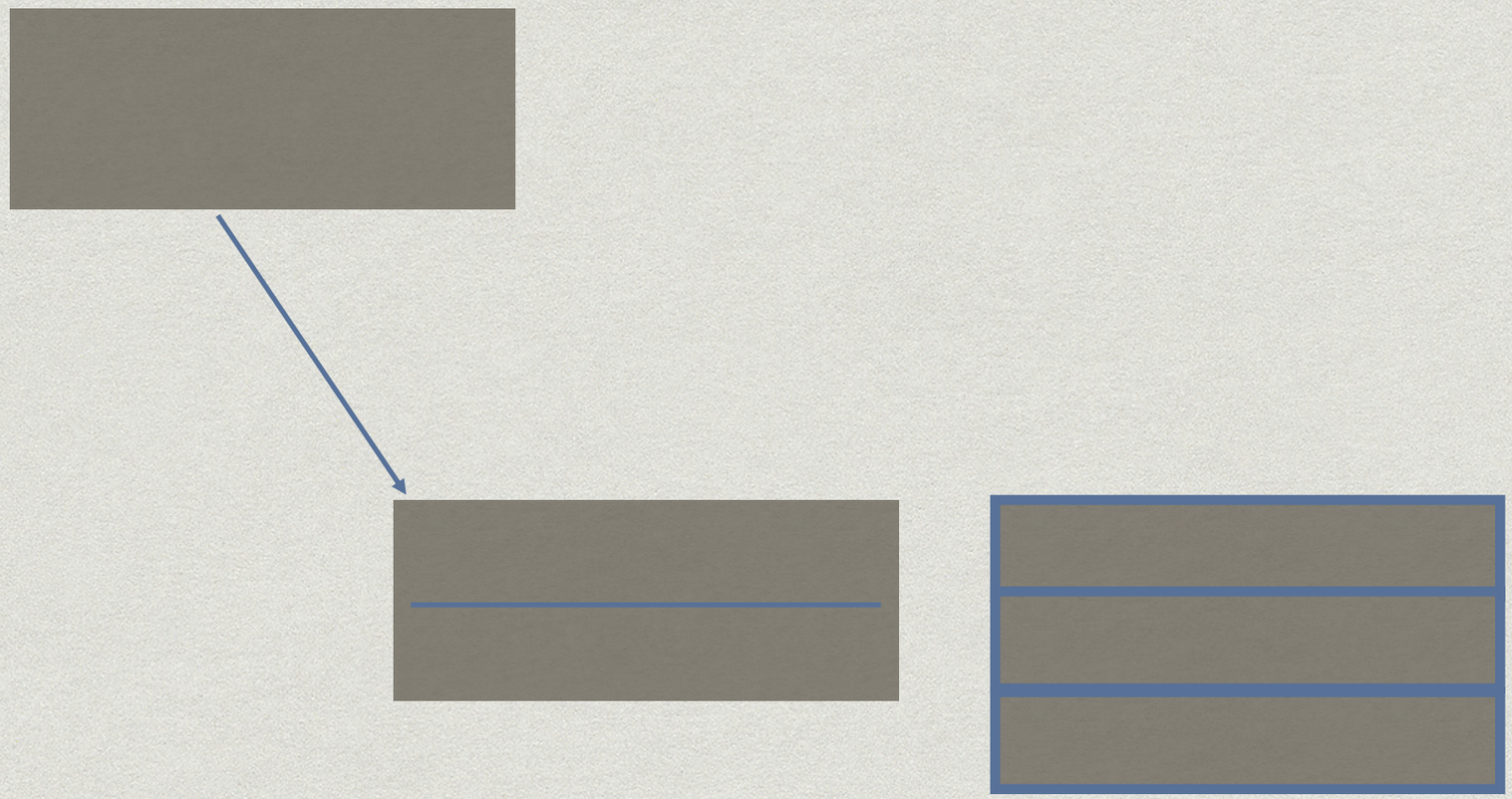
Grade 7



Grade 3

- **Knowledge:** Shade 2/3.

- **Understanding:** This is 2/3 of something. What could the something be?



Grade 8

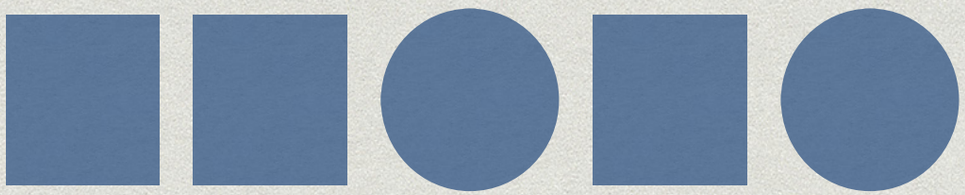
- **Knowledge:** What is 5/8 x 7/8?
- **Understanding:** I want to multiply two fractions so that the product is more than half of either of them. What could I multiply?

Kindergarten

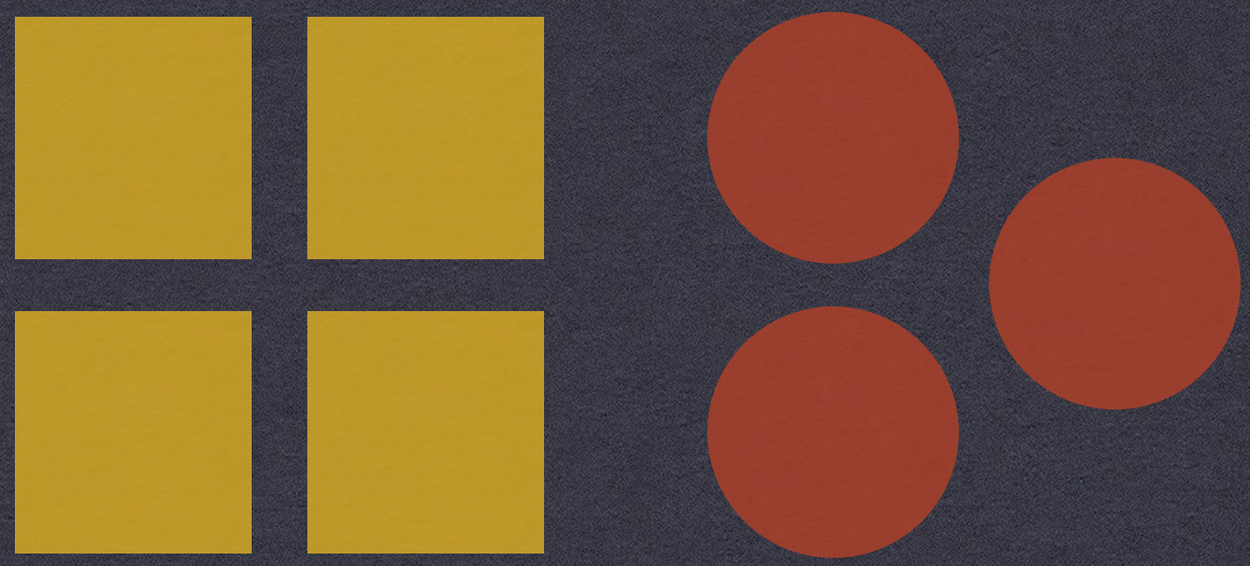
- **Knowledge:** Sarah has 5 stickers. Lia has 2. Who has more?
- **Understanding:** Lia has fewer stickers than Sarah. Lia has more stickers than Kevin. Who has the most stickers?

Grade 6

- **Knowledge:** What is the ratio of squares to circles?



- **Understanding:** Draw a picture that shows all of these ratios at the same time: 4:3, 4:7 and 7:3



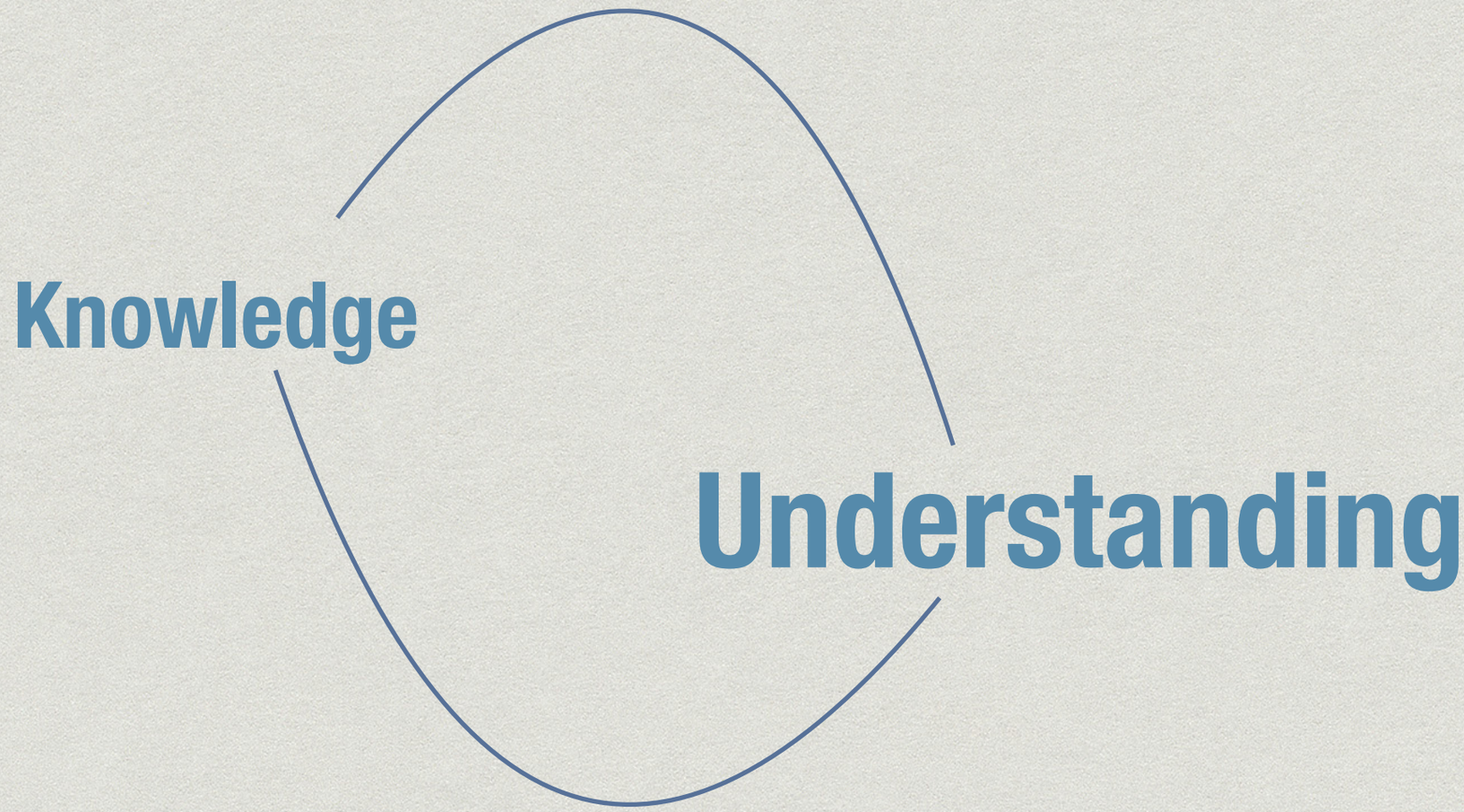
Grade 2

- **Knowledge:** Decompose 42 into tens and ones.
- **Understanding:** Are there more ways to decompose 42 into two equal groups or into one big group and one little one? Why?

Is knowledge addressed while working on understanding?

Is understanding addressed while working on knowledge?

Does one have to precede the other?


Knowledge
Understanding

Now let’s talk about thinking

- I am going to use the notion of thinking as dealing with a somewhat complex new situation and making sense of it.

For example

For example

Use each number once.

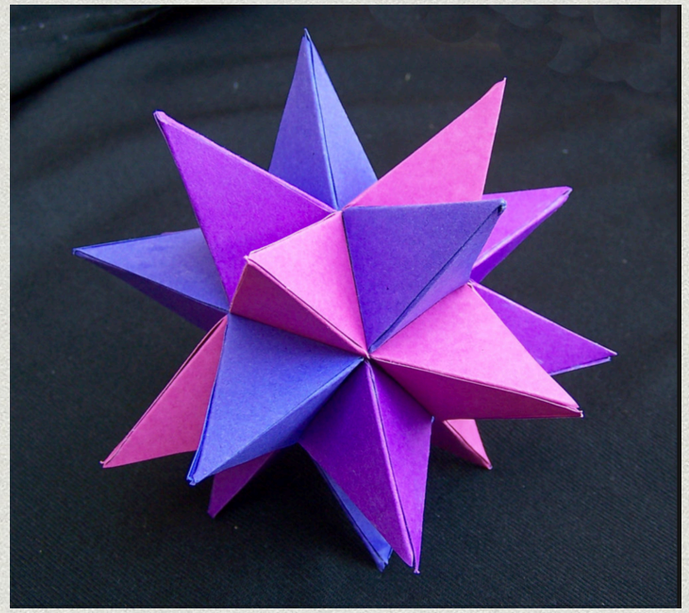
1 2 3 4 5 6 7 8 9

1. Write 3 equalities.

Use one of the numbers in each blank.

- □ − □ = □
- □ + □ = □ + □
- □ − 1 = □

For example



- You can stellate a 3-D figure if you put a pyramid on each face.
- How might you make a stellated figure with 90 edges?

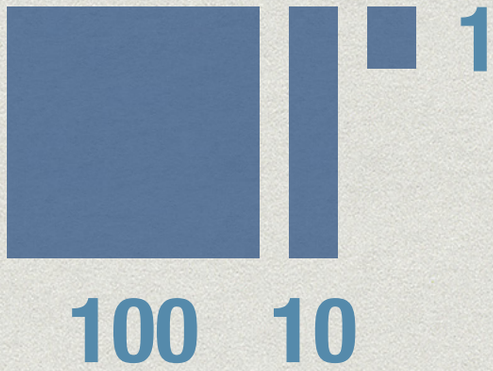
An aside

- To talk about thinking, I believe I need to talk briefly about why math might be attractive to many who pursue it.

What do people who love math love about it?

- Most likely that:
- It’s a lot like puzzle solving.

Let’s all try this puzzle Which is the lie?



- 68 can be represented with 32 base ten blocks.
- 148 can be represented with 43 base ten blocks.
- 502 can be represented with 142 base ten blocks.

- People love puzzles and mysteries!
- Check out a book called The Puzzle Instinct

- So let’s go back to thinking.

- But how does thinking differ from application?
- And how does thinking differ from understanding?

- Let’s look at how it differs from application first.

For example, for Intermediate

- **Application:** A shirt costs $18.95. You get a 20% discount. What do you pay?
- **Thinking:** You pay the same for a shirt that is 20% off as for a jacket that is 40% off. How were the original prices related?

For example, for JI

- **Application:** What is the area of a basketball court that is 22.5m long and 12.8m wide?
- **Thinking:** A basketball court's length:width ratio is about 7:4. What might be reasonable dimensions? How many people would fit on it?

For example, for primary

- **Application:** There were 45 students in Grade 3 and 68 in Grade 4. How many were there in both grades?

For example, for primary

- **Thinking:** There were 48 kids on the bus. They were all in Grade 2 or Grade 3. But there exactly 4 more from Grade 2 than Grade 3.
- How many were in each grade?

For example, for primary

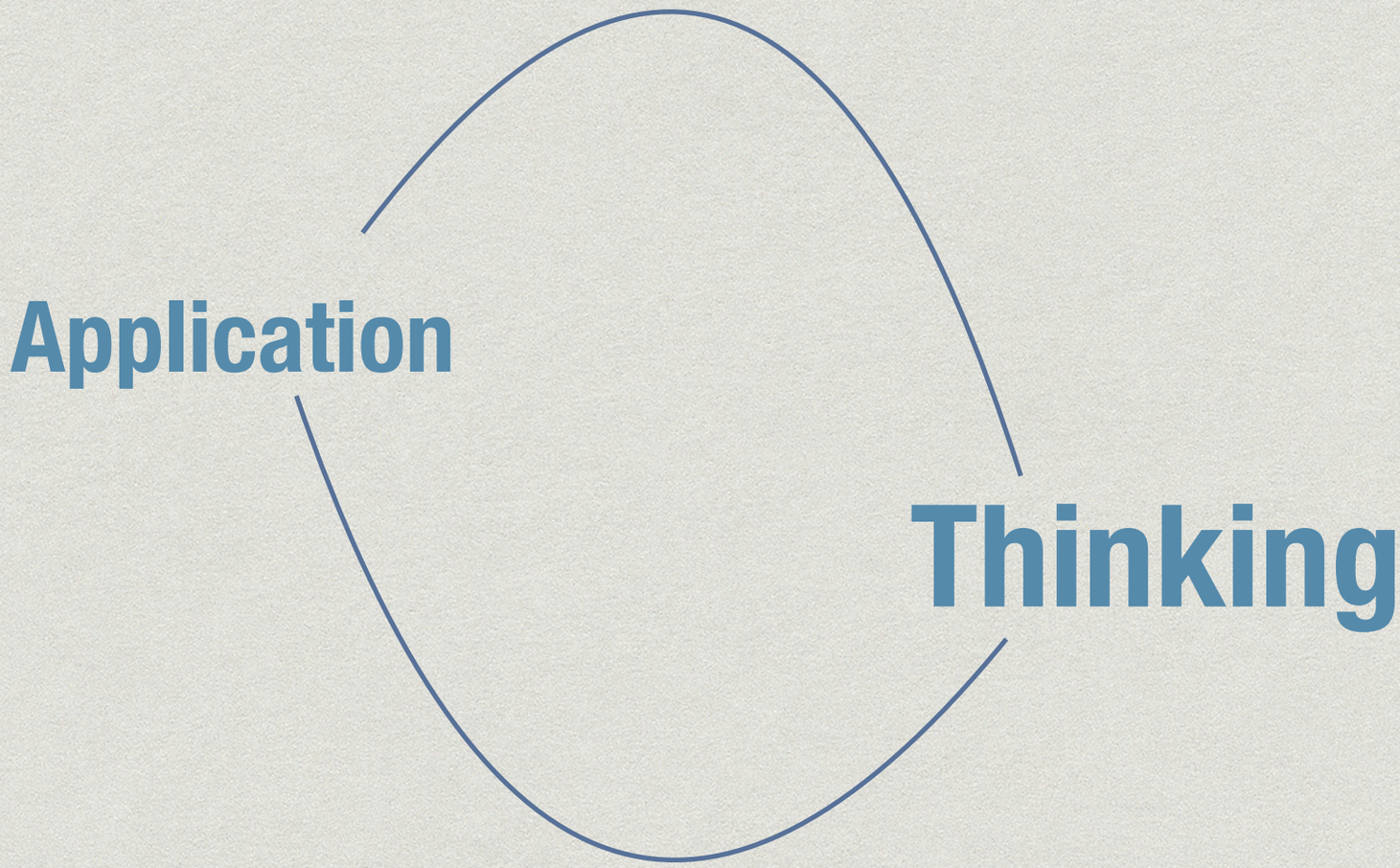
- **Application:** How many days are there in 3 weeks?
- **Thinking:** Your brother's birthday and yours are in the same month. Yours is on a Monday and his is on a Thursday. How many days apart might they be?

For example, for junior

- **Application:** You bought 15 small notebooks at $4 each. How much did they cost?
- **Thinking:** You spend close to $70 to buy some $4 items and some $5 items (and nothing else). What numbers of each type could you have bought?

For example, for JI

- **Application:** What is the missing number if the mean is 15?
- 30 2 8 14 ▯
- **Thinking:** The mean of 5 numbers is 2/3 of the median. What could the numbers be?


Application
Thinking

What about communication?

- Do you see how communication fits into both understanding and thinking? (probably more than into knowledge and application)

What about understanding vs thinking?

- I am not sure it is worth the effort making the distinction, but…
- For me it is about the breadth of the question- is it focused on one and only one idea, but still requires understanding, or is it broader?

For example, in Grade 8

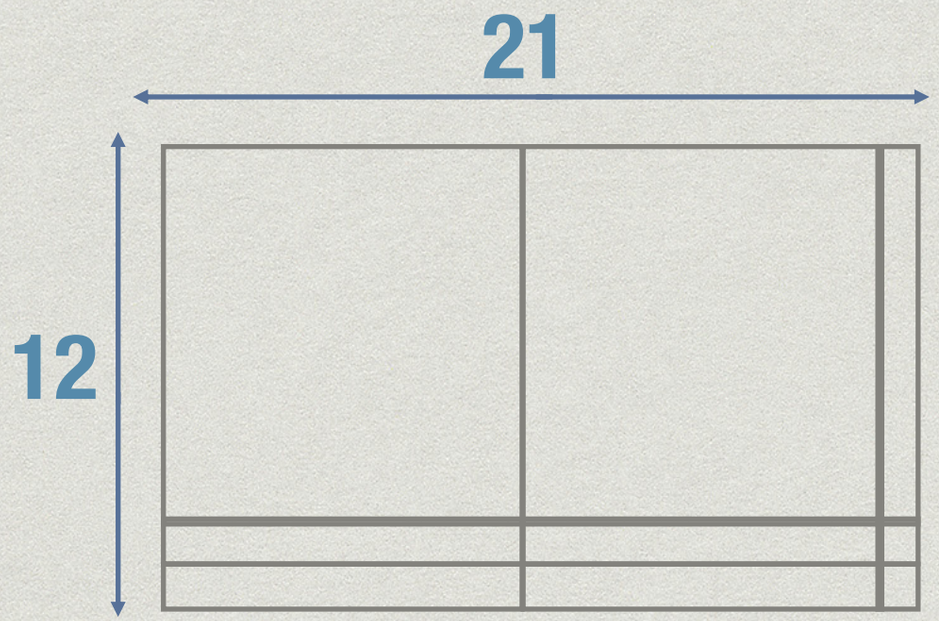
- **Understanding:** Why is the product of a positive and a negative negative?

For example, in Grade 8

- **Thinking:** The product of two integers is 50 less than one of the integers. What could the two integers have been? List all possibilities.

For example, in Grade 5

- **Understanding**: Use base ten blocks to model 21 x 12 and tell me how the model helps you see that the answer is more than 250, but not a lot more.


21
12

For example, in Grade 5

- **Thinking:** Model a multiplication involving 2-digit numbers using base ten blocks that requires you to use exactly 36 blocks in the model.

For example, in Grade 2

- **Understanding:** How could you add to solve this? How could you subtract?
- Kassie picked 43 berries.
- Maya picked 27 berries.
- How many more did Maya pick?

For example, in Grade 2

- **Thinking:** When you add two numbers, you get 10 more than when you subtract them. What could they be?

So, what is fundamental?

- I have tried to make the case that knowledge is not enough when we know that people seek to make sense.
- We need some knowledge to use to develop understanding, but we can’t wait too long to move to understanding.
- I see the need for a back and forth.

So, what is fundamental?

- Similarly, I have tried to make the case that application is not enough when we know that people seek puzzles.
- We need knowledge and perhaps application and/or understanding to use in thinking, but we can’t wait too long to move to thinking.
- I see the need for a back and forth.

In the end…

- It is always about our beliefs about what math is and what education is for that determine what we see as fundamental.

I believe

- that I want my grandkids to come out of school seeing that learning new stuff is exciting and seeking challenge.
- I want them to believe that what they learn in math makes sense.

- Of course I want them to know their times tables and be able to add, etc., but that is not even close to enough for me in a world looking for thinkers.
- Understanding and thinking are, for me, what is fundamental.