CONTINUING OUR MATH CONVERSATION

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LET'S DO A LITTLE MATH

- The solution to an equation is $x = -12$.
- What might the equation be if there are some operations appearing in the equation?

Maybe

- $2x + 24 = 0$
- $5x + 8 = -x - 64$
- $x + 12 = 6x - 6x$

OR IN A DIFFERENT DIRECTION

- The parents made 400 chocolate chip cookies to sell.
- About how many cups of flour did they probably use?

agenda

- How has asking open questions or using parallel tasks been going?
- Teaching and assessing with intention

What has happened?

- Where did you have success?
- Struggles?

Teaching with intention

- It means looking at a curriculum standard and saying to yourself- What ideas do I want to emerge for kids? How would I make it happen?

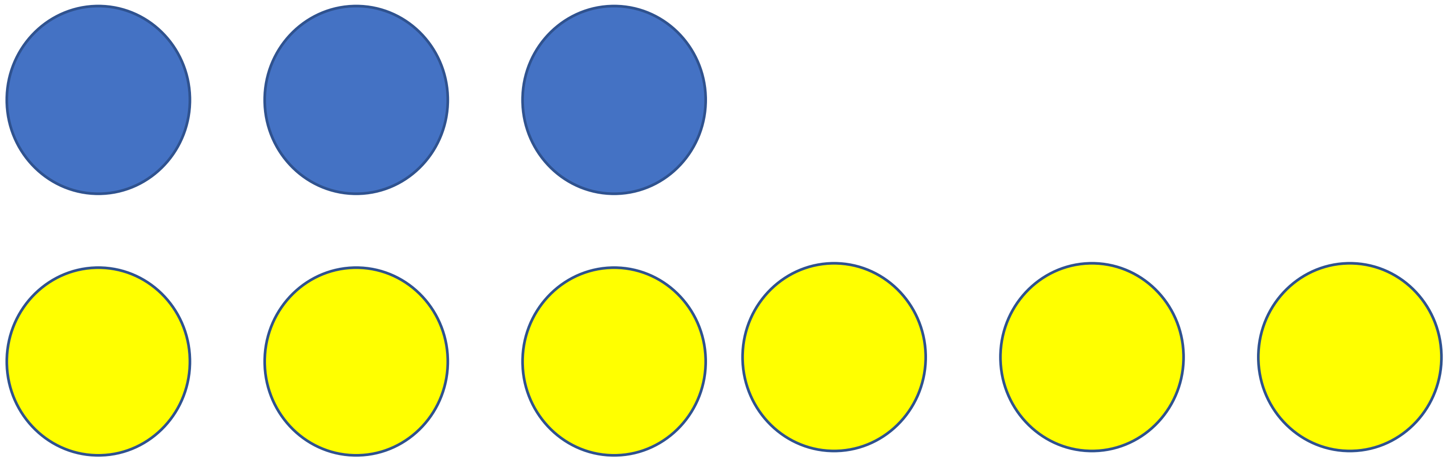
For example- Grade 6

- Understand the concept of a ratio including the distinctions between part:part and part:whole and the value of a ratio: part/part and part/whole. Use ratio language to describe a ratio relationship between two quantities.

The ideas YOU might include

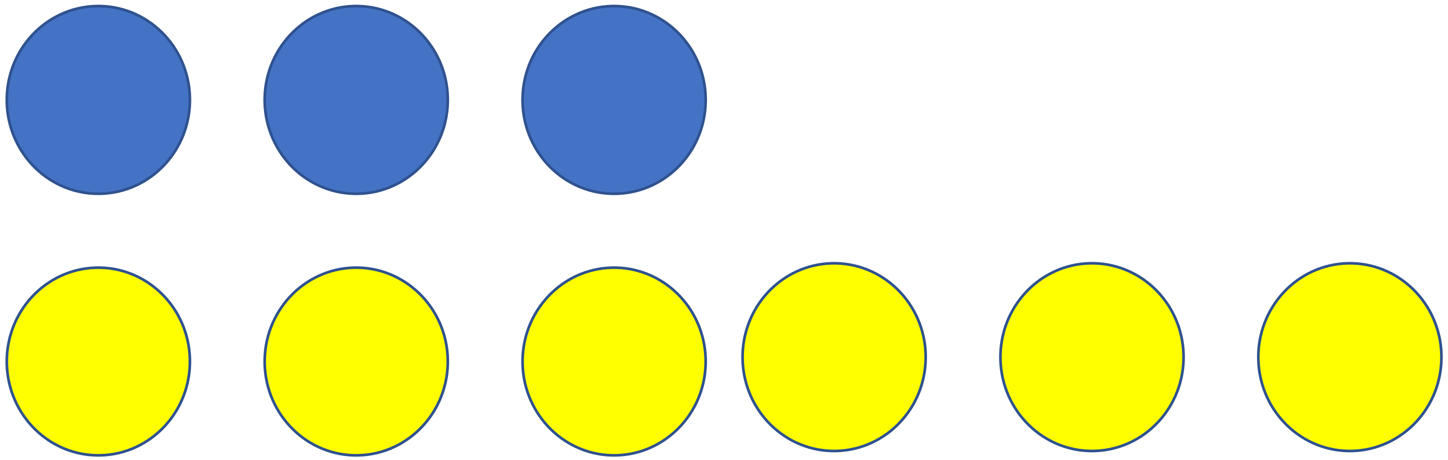
- Given a ratio describing a ratio situation, there are ALWAYS lots of other ratios and fractions describing that situation.

For example- what ratios are here?



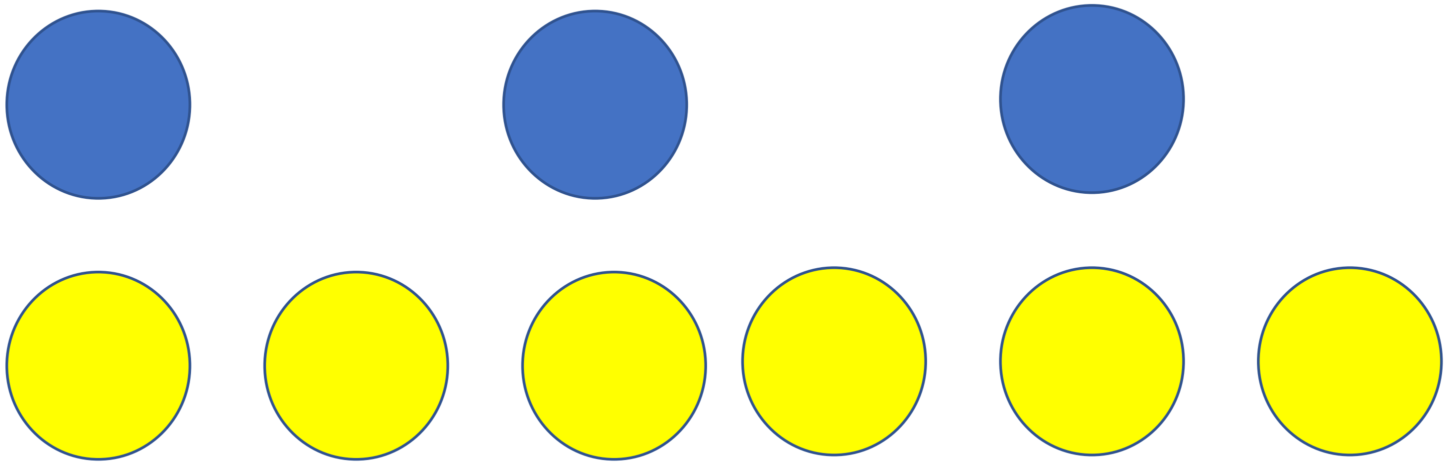
3:6 6:3 3:9 6:9 9:3 9:6

what fractions are here?



3/6 6/3 3/9 9/3 6/9 9/6

What other ones?



1:2 2:1 1:3 2:3 3:2 3:1

So

- I teach a lesson where kids focus on all the ratios they see in a situation (and all the fractions.)

I might ask

- Use counters and create a situation that shows, at the same time, the ratios 3:5 and 2:3.
- What other ratios are visible? What fractions?

MAYBE

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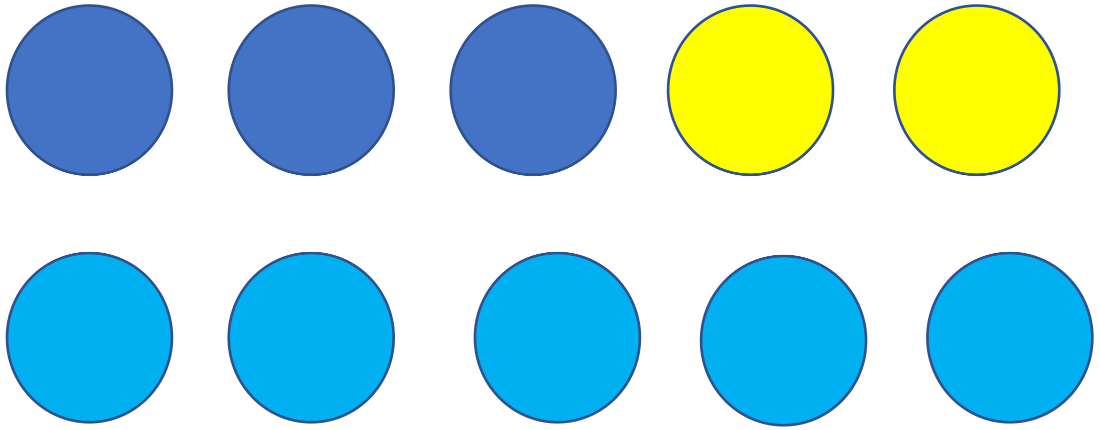
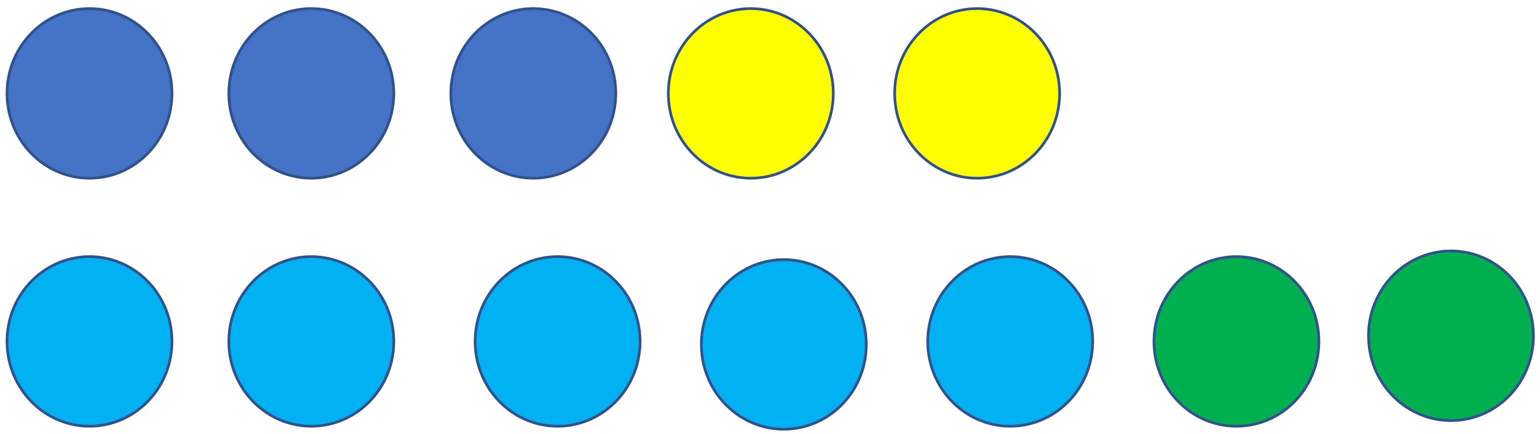


MAYBE



maybe

- When you multiply or divide both terms of a ratio by a positive whole number, the ratio describes the same relationship.
- When you add to or subtract from both terms …, the ratio usually does not describe the same relationship.

I might ask

- 4:12 describes the ratio of adults to kids at a family gathering.
- Do you know, for sure, how many people were at the gathering?

4 adults: 12 kids

- What is the fewest number of kids there could be?
- If there were more than 50 kids, how many adults might there have been?

Or you might ask

- 4:[]   = 6:{}
- How far apart could [] and {} be?

FOR EXAMPLE- GRADE 7

- Apply and extend previous understandings of addition and subtraction to add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

WHAT IDEAS REALLY MATTER?

- The sum of two negatives is the opposite of the sum of the opposites.
- For example, $-3 + (-4) = -(3 + 4)$

SO I MIGHT ASK

- I added two negative numbers that were one apart.
- What could the sum be?
- What could it not be?

WHAT IDEAS REALLY MATTER?

- Knowing that $n + (-n) = 0$ allows you to add any positive to any negative.
- For example, $5 + (-2) = 3 + 2 + (-2)$, so it's 3.

I might ask

- Chris said that thinking about 14 as 8 + 6 helps you figure out – 8 + 14.
- Do you agree or not? Why?

OR

- I had some positive integer tiles and added them to four times as many negative tiles. What integer might I have been representing?
- What could I NOT have been representing?

WHAT IDEAS REALLY MATTER?

- The difference of two integers, $a - b$ is what you add to b to get a result of a.
- It also happens to be the same as $a + (-b)$.

I might ask

- You jumped forward from a negative integer to a positive integer on a number line and you moved 12 steps.
- Write an addition equation to describe what happened. Then write a subtraction equation.

e.g.

- from –4 to 8
- –4 + 12 = 8 OR
- 8 – 12 = –4 OR 8 – (–4) = 12
- Why does it make sense that it's the same as   8 + 4?

WHAT IDEAS REALLY MATTER?

- If one pair of integers has the same sum as another, it's because one in the second pair is the same amount less as the other is greater than the original numbers.
- E.g. 7 + (–3) = 5 + (–1) since 5 is 2 less than 7 and –1 is 2 more than –3.

I MIGHT ASK

- You have a pile of negative counters and a somewhat smaller pile of positive counters.
- Decide on values and write an addition to tell how many altogether.

now

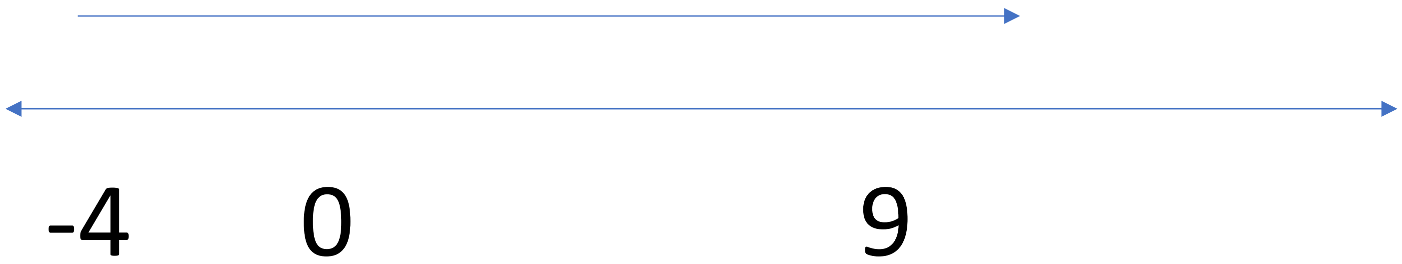
- Move some counters from one pile to the other.
- Write the new addition sentence.
- How has the old sentence changed?

WHAT IDEAS REALLY MATTER?

- If one pair of integers has the same difference as another, it's because each in the second pair is the same amount more or less than its match in the first pair.
- E.g. 7 – (–3) = 10 – 0 since each is increased by 3.

SO I MIGHT ASK

- Figure out the distance from -4 to 9.
- What subtraction sentence asks for that distance?



THEN

- If you had started at 0 instead of -4 and went the same distance, where would you end?
- How does this help explain why $9 – (–4) =\quad 9 + 4$?

FOR EXAMPLE- GRADE 8

- Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. For rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number

What ideas really matter?

- It is hard to tell if a number is rational or irrational just by looking at what the calculator says.

So I might ask

- My calculator display is showing this:
- 1.7320508075688772
- My friend's shows: 0.333333334
- Is each number irrational or rational?

What ideas really matter?

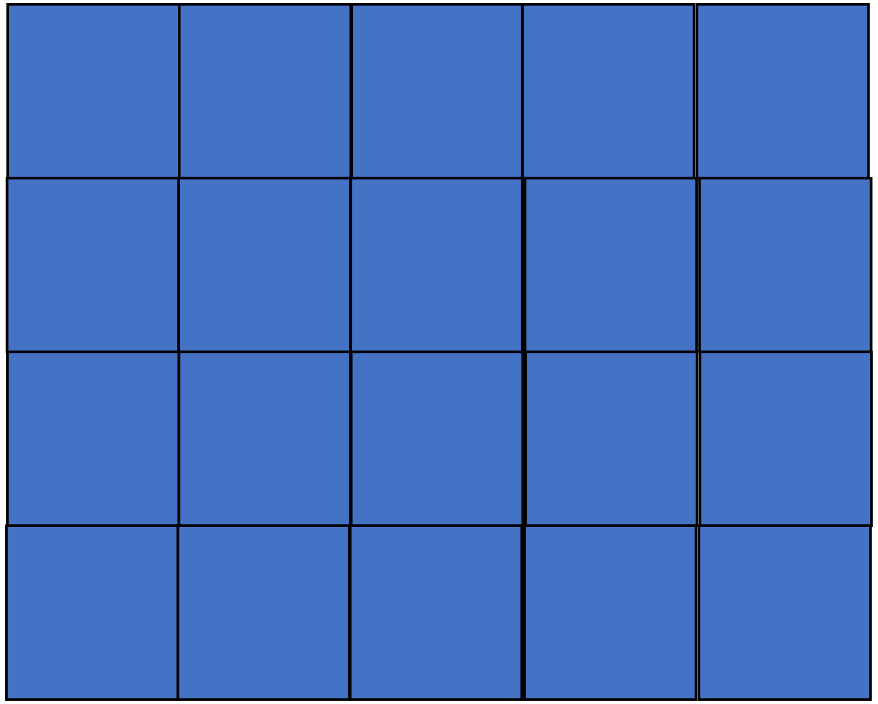
- Every square root of an integer that is not a perfect square is irrational.

SO I MIGHT ASK

- What is a perfect square?
- Choose a number that is NOT a perfect square.
- Draw a picture that shows that its square root is not an integer.

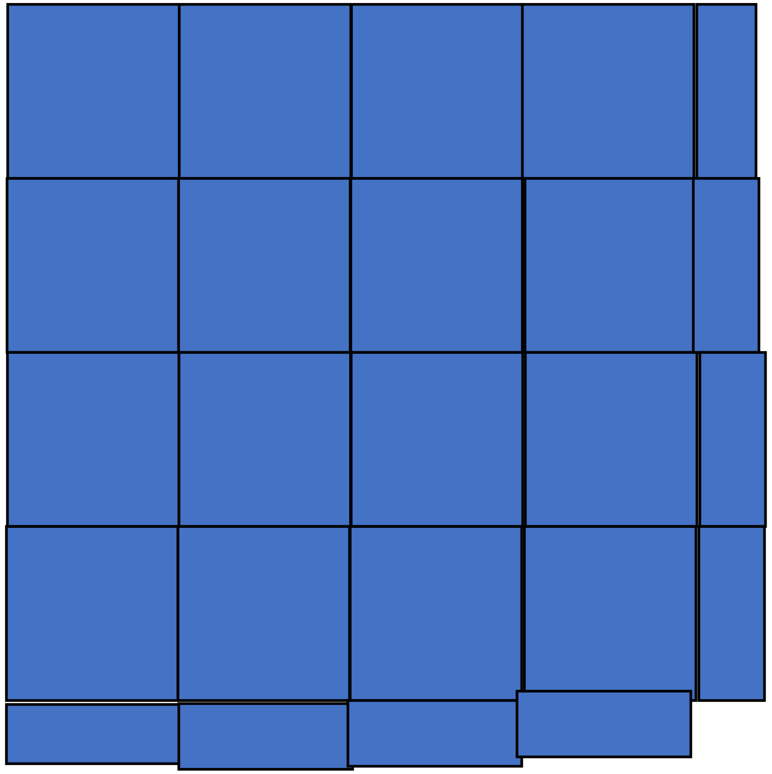
e.g.

- $\sqrt{20}$



e.g.

- √20



What ideas really matter?

- Every expression which, when simplified, involves π, is irrational.

What ideas really matter?

- If you add or subtract irrationals, the result is usually irrational.

What ideas really matter?

- If you multiply or subtract irrationals, the result is sometimes rational and sometimes irrational.

So I might ask

- Which of these is rational? Irrational?
- $4 + \pi$
- $\pi - \pi$
- $\sqrt{3} \times \sqrt{4}$
- $\sqrt{2} \times \sqrt{2}$
- $\sqrt{5} - \sqrt{2}$

What ideas really matter?

- The decimal representation of a rational either terminates or repeats.

What Do the decimals look like?

- Choose one of these numerators.

  1 2 3

- Choose one of these denominators.

  5 8 12 20

What Do the decimals look like?

- Create a fraction and determine the decimal equivalent.
- Which ones “terminate” and which ones don’t? What did you notice?

What Do the decimals look like?

- What do you think will happen with these fractions’ decimal representations?
- 3/40 2/45 9/45 3/80

Now You try

- Choose one of these topics and think about some ideas that matter for your topic and what sort of task might bring this out?

Now You try

- Gr 6 – solving equations OR statistics
- Gr 7- percent OR using equations
- Gr 8- systems of equations OR exponent issues

Focusing students on what matters

- Note taking
- Focusing on the important and not everything

Imagine a lesson on dividing fractions

- Write a problem that would be solved by dividing 2/3 by ¼.
- Tell why you might write 8/12 ÷ 3/12 and say the answer is 8/3.OR
- Tell why the answer HAS TO BE 2/3 x 4.

Assessing with intention

- In the same way we’ve been discussing, your assessment should have lots of attention to the ideas that matter.
- We will consider assessment for learning and assessment of learning.

Let’s consider some topics

- Consider the topic of common factors and multiples.

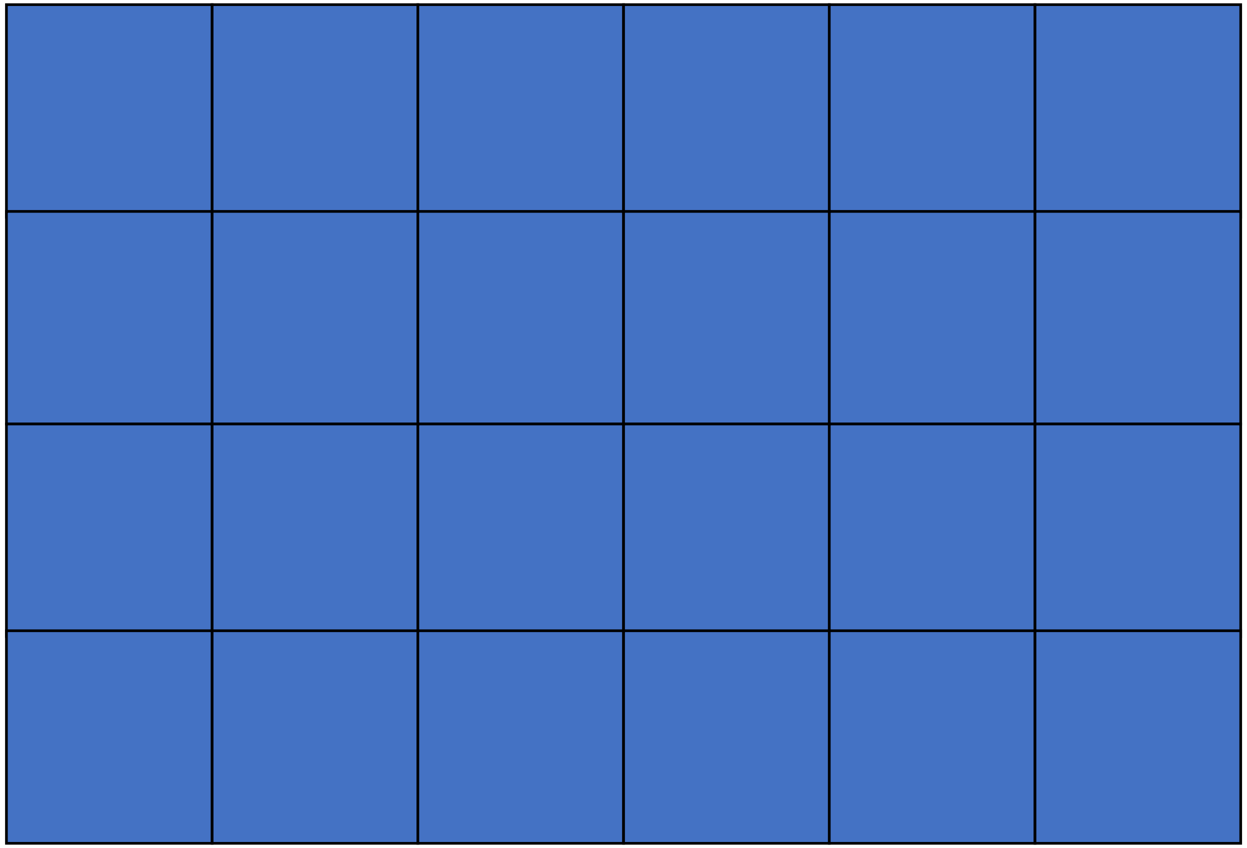
Assessment for learning

- Observations:
- Do students show they know that once a number is factored into primes, no more factoring is possible?

Assessment for learning

- Do students have a physical understanding of what common factors and multiples mean?

Looking at multiples and factors



Looking at multiples and factors

Assessment for learning

- Do students have multiple strategies for determining common factors or multiples?

skills

- Factor…. into primes.
- List common factors for…
- List common multiples for numbers in standard and prime factored form.

concepts

- Jane and Lisa are both factoring 320 into primes.
- Jane began: 320 = 32 x 10
- Lisa began: 320 = 2 x 160

- Will they end up with the same prime factorization?.

concepts

- A number is factored as a x b x c.
- How do you know whether it can be factored even more?

concepts

- If you were asked to figure out one common factor of 10 784 and 5792 other than 1, what answer might come to someone very quickly?

concepts

- If you were asked to quickly come up with a multiple of 117 and 5, what might be a way to do that?

concepts

- Suppose you know that 110 is a common factor of two numbers. Name two other common factors.
- Suppose you know that 110 is a common multiple of two numbers. Name two other common multiples.

concepts

- Why is it useful to write two numbers in prime factored form to figure out a common factor or common multiple?

concepts

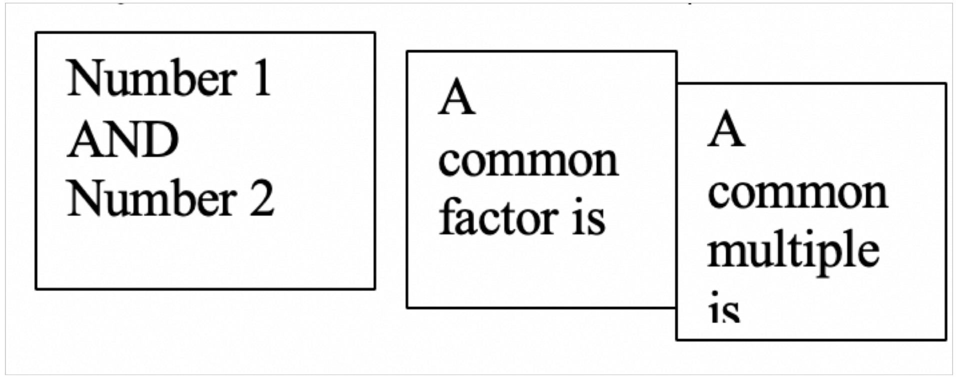
- Suppose you know that two numbers have a least common multiple of 2^3 x 3. What else do you know about the two numbers?

concepts

- Suppose you know that two numbers have a greatest common factor of 2^3 x 3. What else do you know about the two numbers?

Performance task

- Create a concentration game using 20 cards where there are ten pairs of matching cards.
- Every card looks like one of these:



Performance task

- Cards match if the card with two numbers matches a card with a right common factor OR a right common multiple.
- On some of the number cards, write the numbers in prime factored form.
- Play the game with a partner.

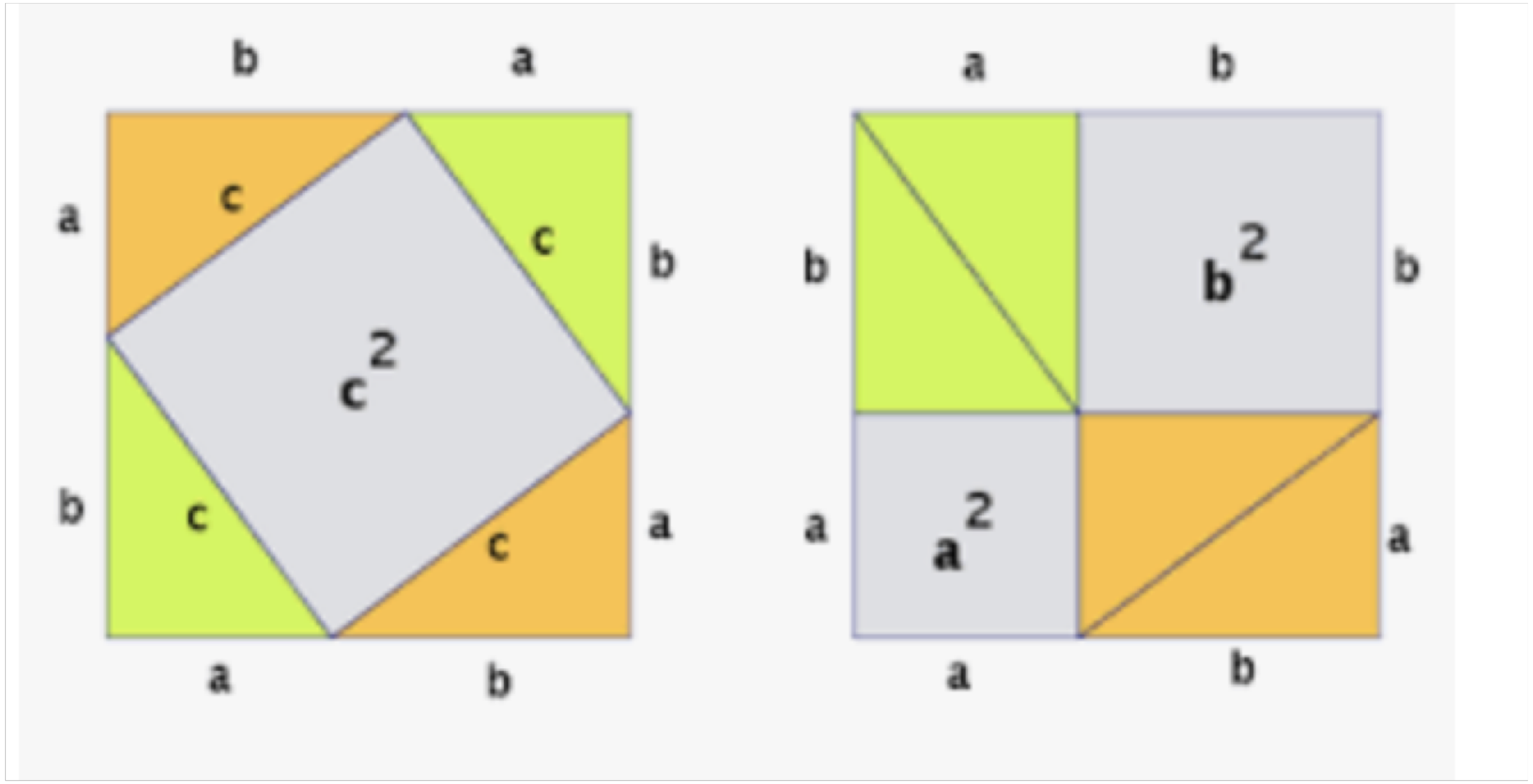
Let’s consider a different topic

- The Pythagorean theorem

What might you observe?

- Can they interpret a decomposition approach to the Pythagorean theorem?
- Can they make a connection between the geometric and numerical interpretations?

Decomposition to show Pythagorean theorem



What might you observe?

- Use the theorem backwards as easily as forwards?
- Know when to apply the theorem
- Relate Pythagorean triples

What might you observe?

- Recognize when solving a problem might benefit from using the theorem
- Explain why whole number multiples of Pythagorean triples are also such triples

skills

- Determine a missing side length given both legs
- Determine a missing side length given a hypotenuse and one leg

skills

- Are these side lengths of a right triangle or not?

4 – 7 – 10

6 – 10 – 14

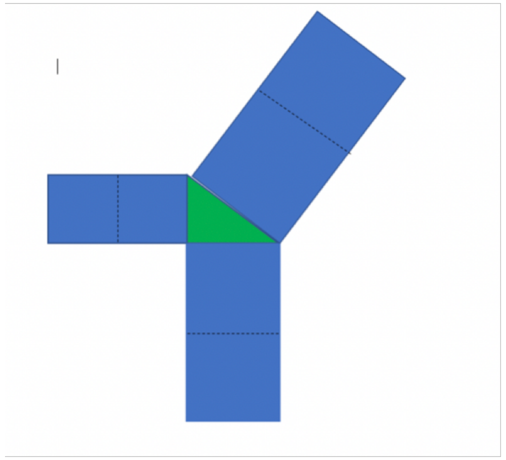
36 – 48 – 60

CONCEPTS

- How could you show that 9-12-15 is a right triangle using a diagram?

CONCEPTS

- What do you know about the areas of the rectangles on the sides of this right triangle? Why?



CONCEPTS

- TV screens are often measured by the lengths of their diagonals. If a giant TV is 48” tall and 72” wide, what would its diagonal length be? Show your thinking.

CONCEPTS

- What could be the lengths of the diagonals of a rectangle with an area of 30 in^2 and whole number side lengths?

CONCEPTS

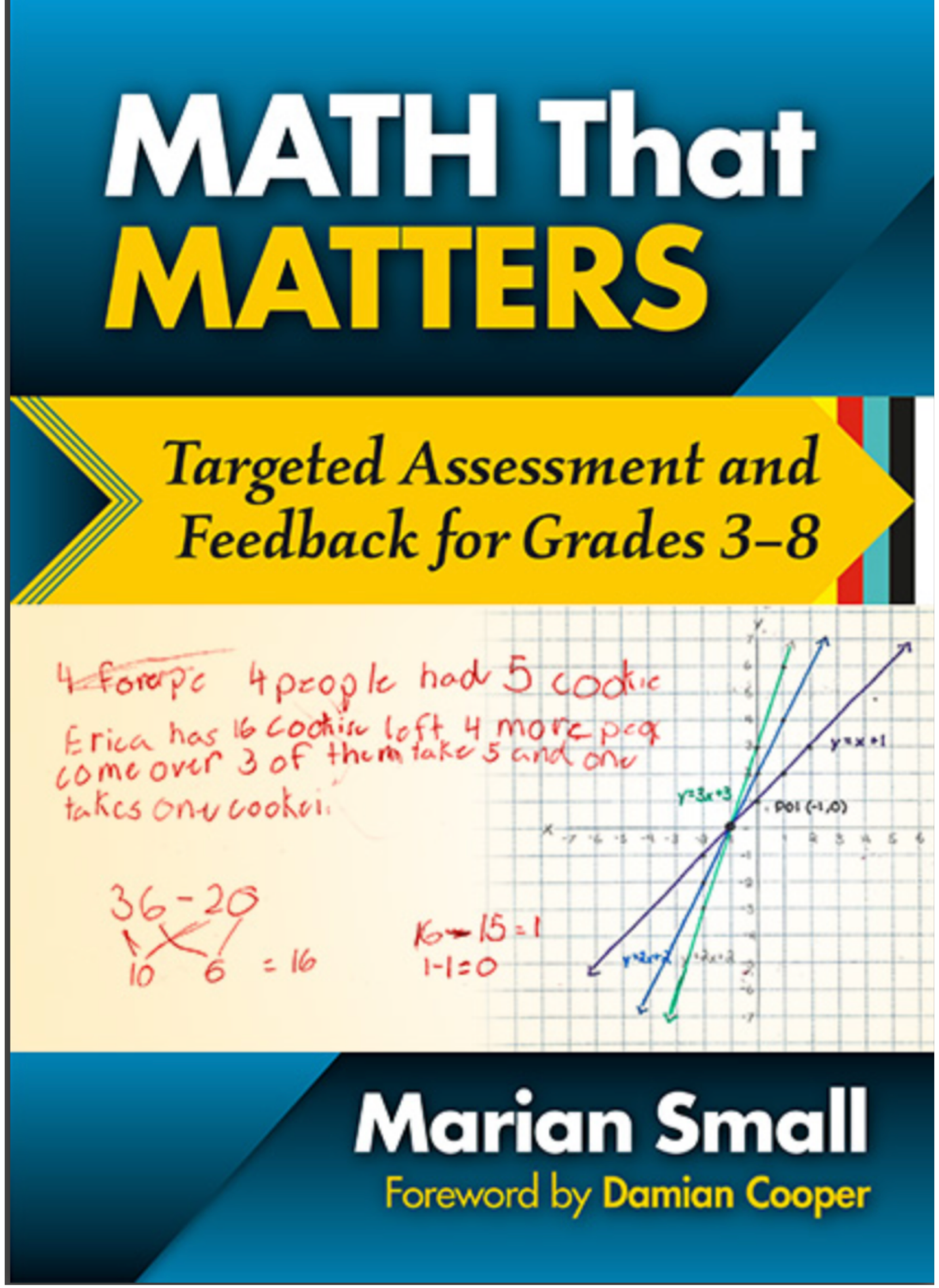
- An equilateral triangle has a side length of 4”. What is its area? Show your thinking.

CONCEPTS

- Describe a problem not like those in the previous questions where you might use the Pythagorean theorem to solve it.

YOUR TURN

- Think about an assessment you might be interested in creating.
- What observations would you make?
- What concept questions?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