Teaching Math through Big Ideas & Using Open Questions

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October 2018

Using Open Questions with Intention

- Let’s look at some examples.

- The mean of a set of data is a LOT more than the median. What could the values be?

- Choose a car speed.
- Write that speed using as many equivalent rates as you can.

- Write a proportion by choosing values for the blanks:
- []/[] = []/x.

- Write a story for that proportion.

- The lines of two equations intersect at (4, –8).
- What could the lines be if they are not horizontal or vertical?

- A line goes through (4,2) and slants up and to the right.
- Name at least one other thing that you are sure is NOT true about that line.

- An exponential function is a LOT like $y = 2^x$.

- What might it be and how are the graphs similar and different?

Maybe you might try!

Think of a topic you teach and turn that more closed question into a more open one.

Teaching with Bigger Ideas in Mind

- I will make some suggestions about big ideas we should focus middle school/high school instruction on and how focusing on those ideas impacts instruction.

The reason this makes sense to me..

- The concepts should be the heart of your program, with the skills woven in but not as the most prominent feature.
- Thinking in big ideas allows for this.

How this works

The focus today will be on the more "thoughtful" questions we should be asking students, questions much more likely to engage them than repetitive questions.

Consider measurement

- A big idea is:
- Some measurements of an object are independent of other measurements, but some are not.

It comes up in

- areas/circumferences of circles
- area/perimeter optimization
- Pythagorean theorem
- areas and perimeters of composite figures

It comes up in

- formulas for volumes
- surface area formulas
- trig ratios
- similar triangles

SO

- instead of exercises that practise skills, you might ask:

Circle measures

- You know that one measure of a circle is 20 cm.
- What other measures of the circle do you know?
- Are there any you don’t know?

Area/perimeter optimization

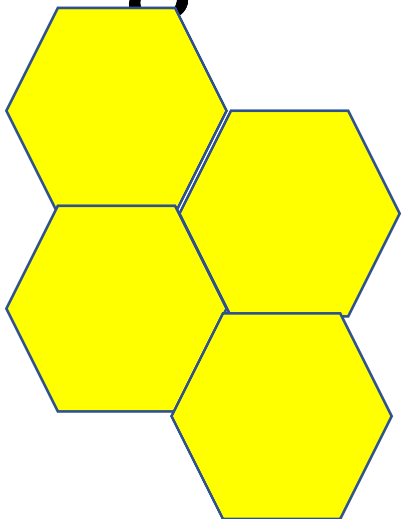
- You know the area of a rectangle.
- How sure are you about its perimeter?

Pythagorean theorem

- You know that the hypotenuse of a right triangle is 50 cm.
- What else do you know about its measurements?
- What if it’s an isosceles right triangle?

Composite Figures

- You know that the perimeter of this shape formed of regular hexagons is 56 cm.
- What measurements of the hexagon are you sure of?



Formulas for volumes

- You know that the volume of a cylinder is 200 cm^3.
- What one other measurement could I tell you that would tell you its surface area? (Or is there one?)

Surface area

- You know the surface area of a triangular prism and the perimeter of its base.
- Are there other measures that you are sure of?

Trig

- The sine of an angle in a right triangle is half the value of the cosine.
- What other measurements are you sure of?

So during instruction

- Instead of focusing on the actual values, you focus on what information automatically tells you other information and/or
- what you automatically know if you know some measurements.

In terms of assessment

- You might ask something like:
- Why do you know the surface area of a sphere if you know its volume, but not the surface area of a rectangular prism if you know its volume?

Or maybe

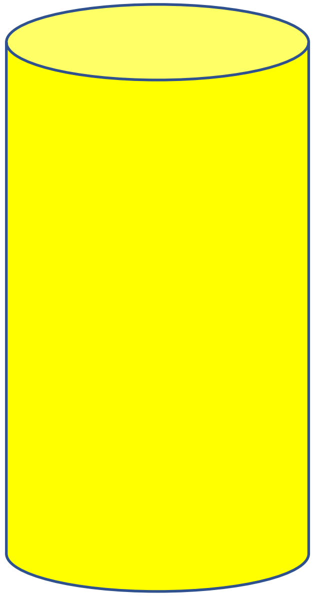
- You might ask something like:
- Describe four different examples from what you’ve learned where knowing one measurement of a shape or figure tells you another one.

Here is a different big idea

- Knowing the measurements of one shape or figure can sometimes give you information about the measures of another shape or figure.

It could be volume

- What are some other shapes that have to have four times the volume of this one?



Or

- One pyramid has 12 times the volume of another.
- What could the two pyramids be?

It could be the Pythagorean theorm

- You know that the legs of a right triangle are a and b.
- How does the hypotenuse of that triangle compare to the one with a triangle with legs of $\frac{a}{2}$ and $\frac{b}{3}$?

In terms of instruction

- You don’t necessarily do one shape at a time, but you focus on the relationship idea.
- For example, what can I do to each shape to double the volume or surface area?

In terms of assessment

- Give an example where knowing the surface area of one figure helps you figure out the surface area of another.

Other measurement big ideas

- The unit chosen for a measurement affects the numerical value for that measurement in a predictable way.

Other measurement big ideas

- Measurement formulas allow us to use measurements that are simpler to access in order to calculate measurements that are more difficult to access.

Now

- Let’s look at algebra.

One big idea

- Interpreting equations or expressions involving unknowns is based on using properties of numbers and meanings of operations.

Curriculum topics

- Add and subtract polynomials
- Multiply a polynomial by a monomial
- Expand and simplify polynomial expressions

Curriculum topics

- Solve first-degree equations
- Rearrange formulas
- Substitute into and evaluate algebraic expressions involving exponents

Curriculum topics

- Expand and simplify second-degree polynomial expressions
- Factor polynomial expressions
- Solve quadratic equations that have real roots

Curriculum topics

- Determine the value of a variable in the first degree using a formula
- Express the equation of a line in one form given the other form

Curriculum topics

- Solve systems of linear equations

Curriculum topics

- Simplify polynomial expressions by adding, subtracting, and multiplying
- Simplify rational expressions
- Determine if two algebraic expressions are equivalent

Curriculum topics

- Simplify algebraic expressions containing integer and rational expressions

So…

- instead of pages of exercises, I might ask:

Add and subtract polynomials

- How does knowing that to figure out 60 – 14 you figure out what to add to 14 to get to 60 help you figure out what $(-4x - 12) - (3x^2 - 2x)$ is?

Expand and simplify polynomial expressions

- What picture can you draw to show 5 x 3?
- How could you use a similar picture to show 3n (n+4)?

Substitute into algebraic expressions involving exponents

- How much more is 5×4^3 than 4×4^3?
- So how much more is $5n^3$ than $4n^3$?

Solve quadratic equations

- You know the product of two numbers and have to figure out what at least one of them is.
- Is it easier if the product is 24 or 0? Why?

Express equation of a line in one form in another form

- How else can you write: 4 +8 +[] = 24?
- How else can you write Ax + By + C = 0?
- How are these processes similar?

Solve systems of equations

- It has to be true that $3x + 2y + 2 = 36$ AND $2x – y = 18$.
- How does knowing that 18 is half of 36 help you figure this out?

In terms of instruction

- The focus is on how what you know about the operations and numbers helped you figure these things out.

In terms of assessment

- You might ask
- What do you know about multiplication that would help you interpret why there is no positive solution to $4x + 3 = 8x + 10$?
-

Another big idea:

- Variables can be used to efficiently describe relationships.

You might ask

- You choose two numbers.
- When you add them, the answer is double what you get when you subtract them.
- How could writing this algebraically help you figure it out the numbers?

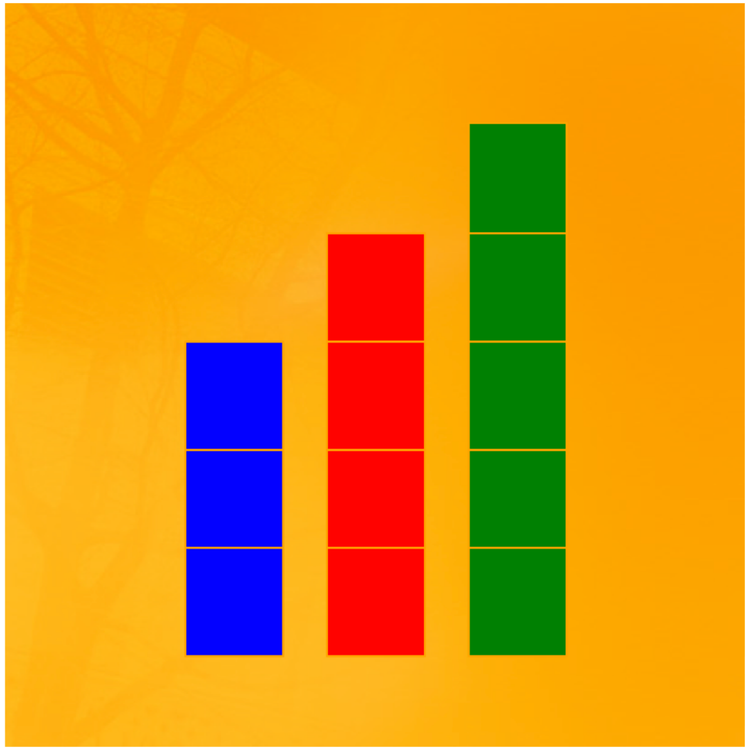
You might ask

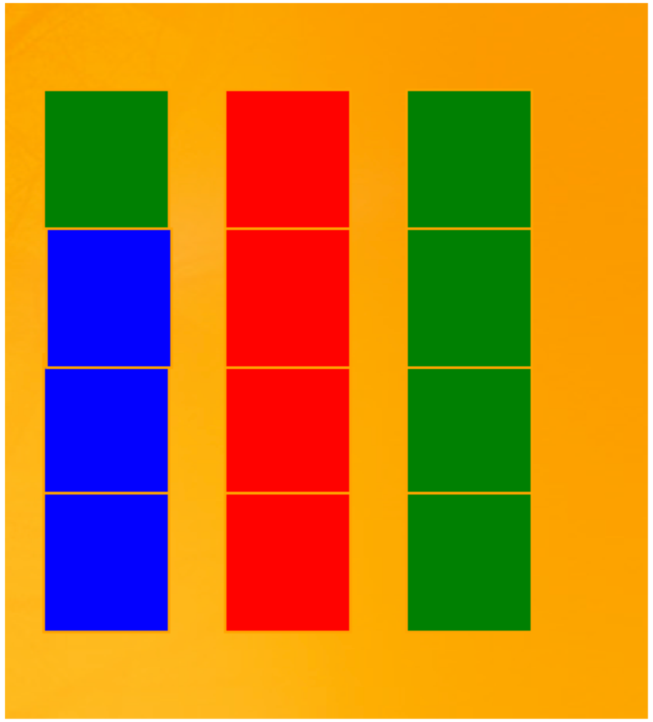
- Choose three consecutive whole numbers. Add them.
- Repeat with three other consecutive whole numbers.
- What do you notice?
- How could you describe this algebraically?

Algebra:

- $n + (n+1) + (n+2) = 3(n+1)$ OR
- $(n - 1) + n + (n + 1) = 3n$

Visually

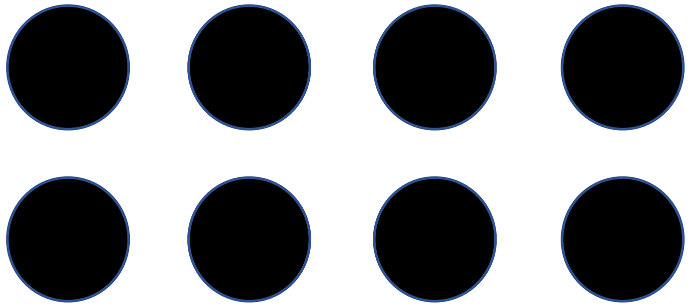




You might ask

- Choose a number.
- Multiply it by two more.
- Compare the result to the squre of the number between the two.
- What do you notice?

Visually



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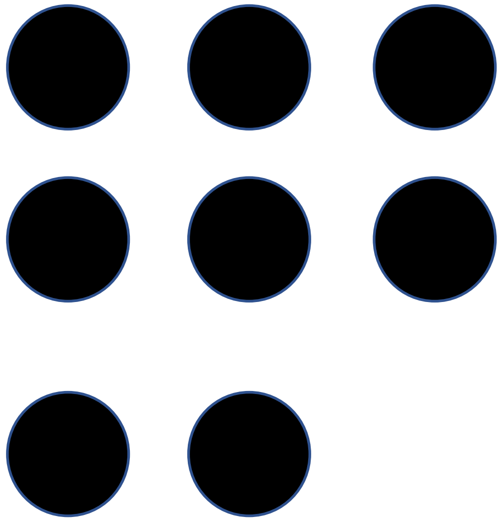

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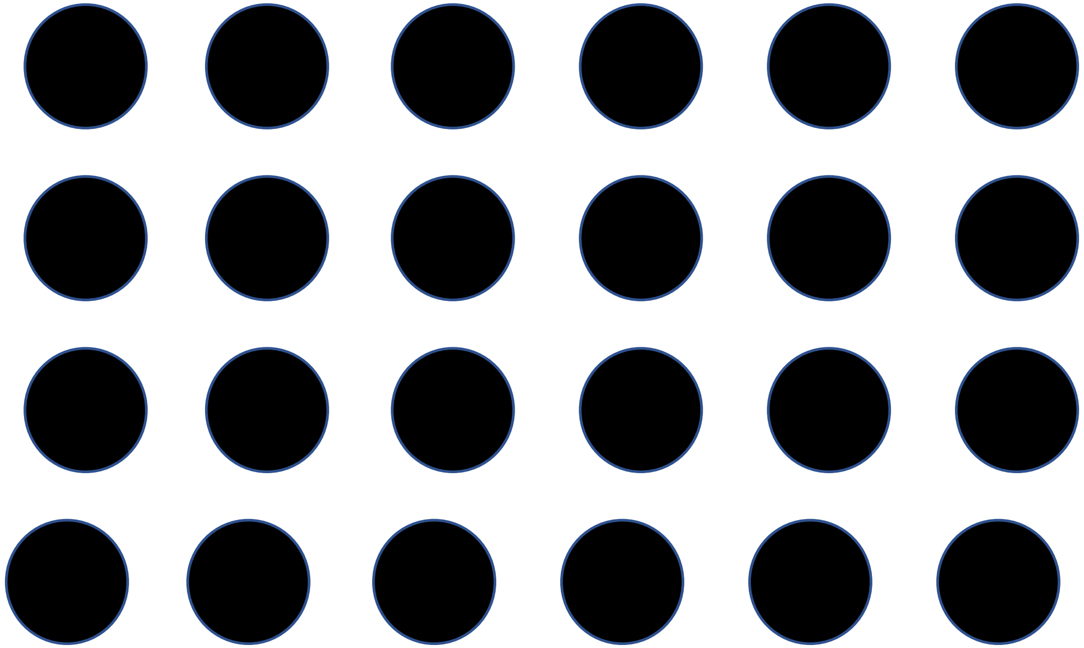

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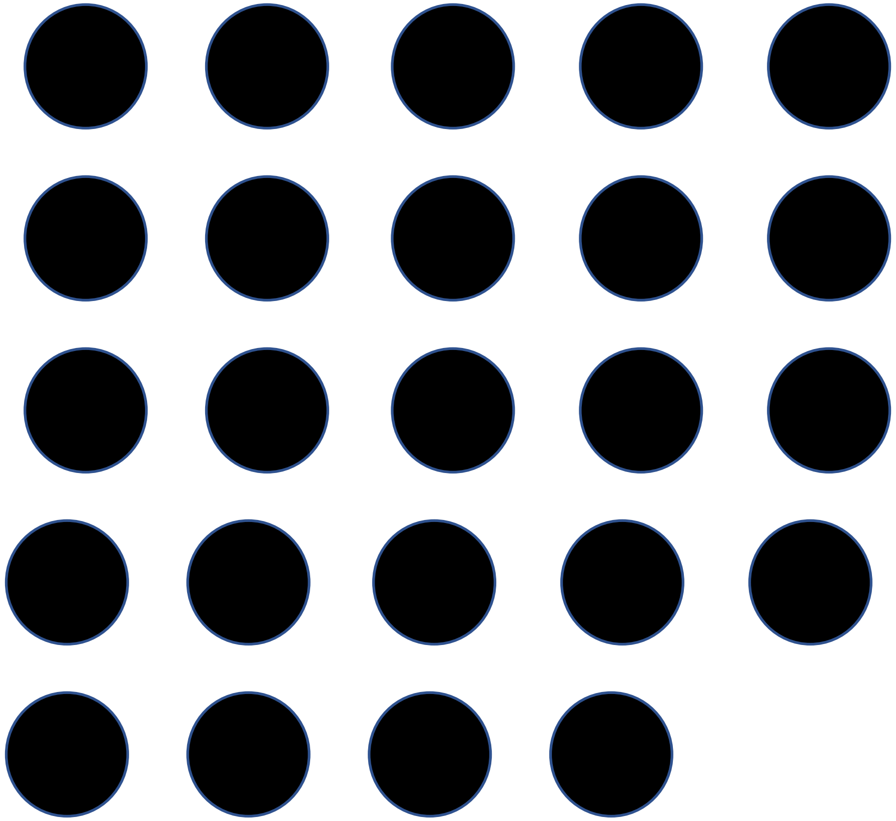


Algebraically

- $n(n+2) = n^2 + 2n = (n+1)^2 - 1$ OR
- $(n-1)(n+1) = n^2 - 1$

In terms of instruction

- You focus on how long it would take you to work out things or say things if you didn't use variables.

In terms of assessment

- Do these say the same thing or not? If not, why not?
- If yes, which description do you prefer? Why?

- Choose a number.
- Multiply it by 4 more.
- Compare the result to the square of the number halfway between them.
- OR $(n-2)(n+2) = n^2 - 4$.

Another big idea

- The same algebraic expression or equation can be related to different real-world istuations, and different algebraic expressions or equations can describe the same situation.

You might ask

- The equation $4x – 5 = 15$ describes two VERY DIFFERENT situations.
- What might they be?

Or

- Choose a quadratic function.
- Provide two different situations it might be describing.

In terms of instruction

- You focus on any situation and ask students to write more than one equation to describe it.

- You also regularly ask for different situations for any equation.

In terms of assessment

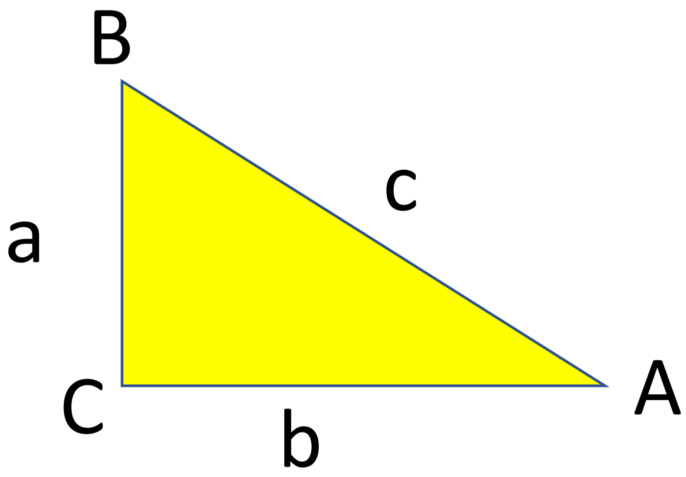
- Sarah says that ANY equation can be written in more than one way.
- Do you agree or not?
- Convince me your answer makes sense.

Another big idea

- Many equivalent representations can decribe the same situation or generalization. Each may give somewhat different insight into certain characteristics of the situation or generalization.

For example

- Use different equations to describe what you see here.



It might be

- Choose a linear relation.
- Show it as a graph, a table of values and an equation.
- Describe what each shows best about the relation.

It might be

- Consider the function $f(x) = (6-x)^2$.
- How could you represent it as:
- a sum, a difference, a product, and a quotient?

In terms of instruction

- You focus on what representations are convenient; help you see other relationships; etc.

In terms of assessment

- It could be:
- Consider vertex form, general form and factored form for a quadratic.
- Which is most useful when?

Let's think about algebra

- Comparing mathematical relationships either algebraically or graphically helps us see classes of relationships with common characteristics and help us describe each member of the class.

You could ask

- Create a relation that behaves a lot like $y = 3x$, but not exactly. How are they alike and different?

You could ask

- Are $y = 3(x-2)^2 + 5$ and $y = 8(x-2)^2 + 5$ more alike when x is between 0 and 5 or when x is closer to 100? Why?

It could be

- Which two of these are most alike and why?
- $y = 3x - 4$
- $y = 3x + 8$
- $y = -3x - 4$

It could be

- Describe four ways that a quadratic and linear relationship are different.

It could be

- A bunch of right triangles have a sine between 0.5 and 0.6.
- How else are they alike?

It could be

- Draw a graph of a parabola that grows quickly as x increases from 10 to 20 and the graph of a parabola that grows slowly in that domain. What are their equations?

In terms of instruction

- The focus is on noticing similarities and commonalities, as well as differences, in a variety of algebraic situations (or measurement or geometric ones).

In terms of assessment

- You might ask:
- Think of some ways that a bunch of quadratics could be alike- but different from other ones.
- How would their algebraic descriptions make that clear?

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