What does it look like?

Fostering algebraic reasoning

Marian Small October 2018

What is it?

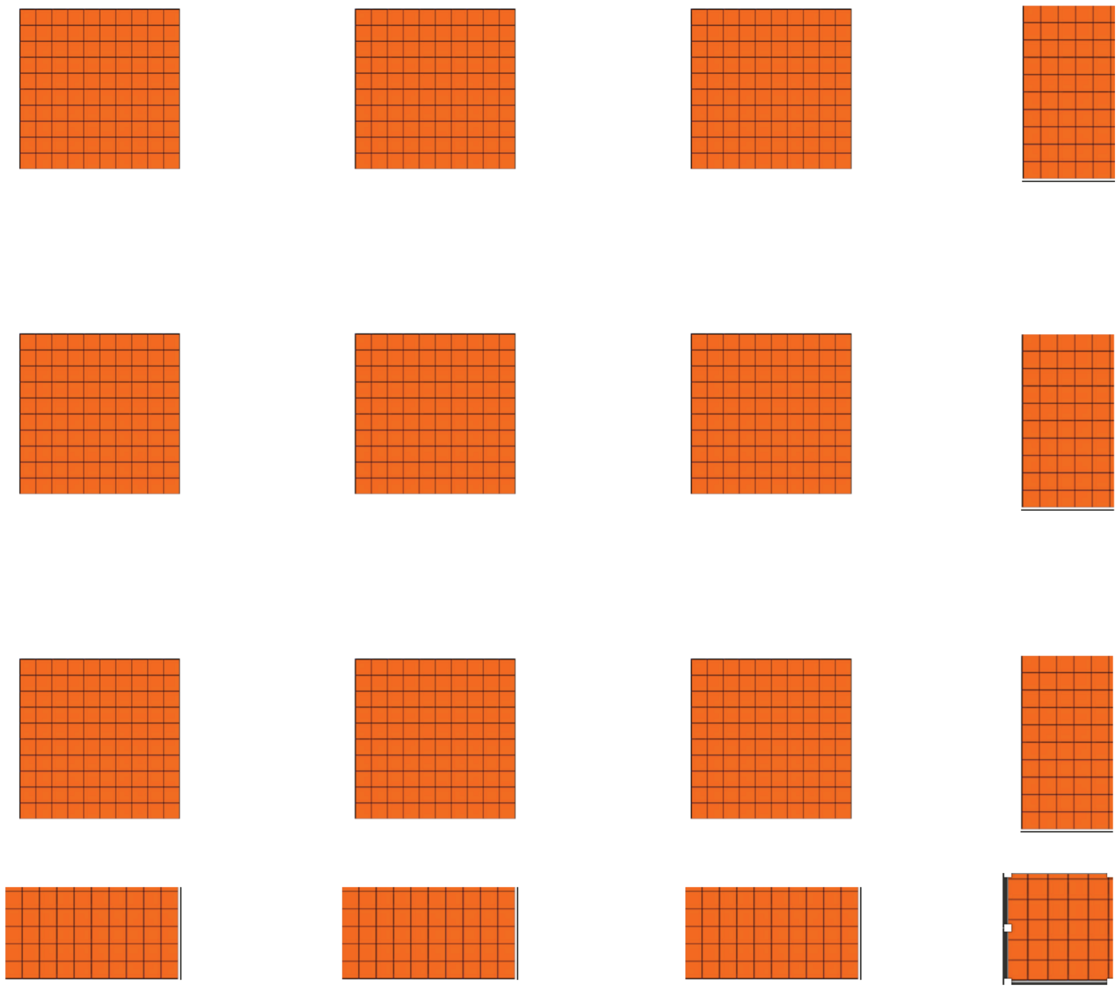
Algebraic Reasoning

Algebraic reasoning is the process students use when they generalize numerical situations, when they model situations using equations and variables, and when they study how quantities are related.

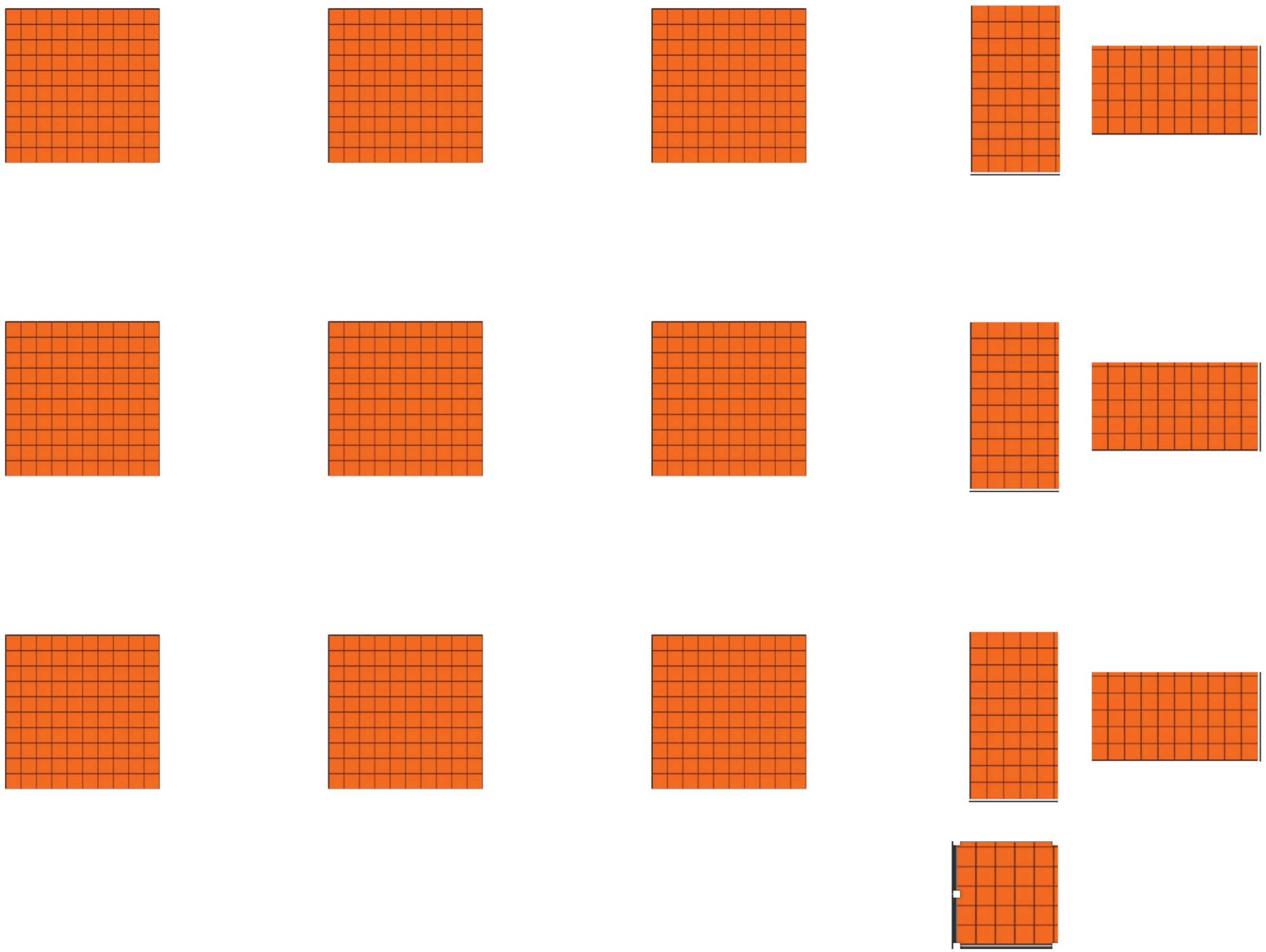
What might algebraic reasoning look like?

- It could be a numerical generalization:
- To square a number of the form []5, you multiply [] by [] +1 and stick a 25 at the end.
- For example, $35^2 = 1225$.
- How come?

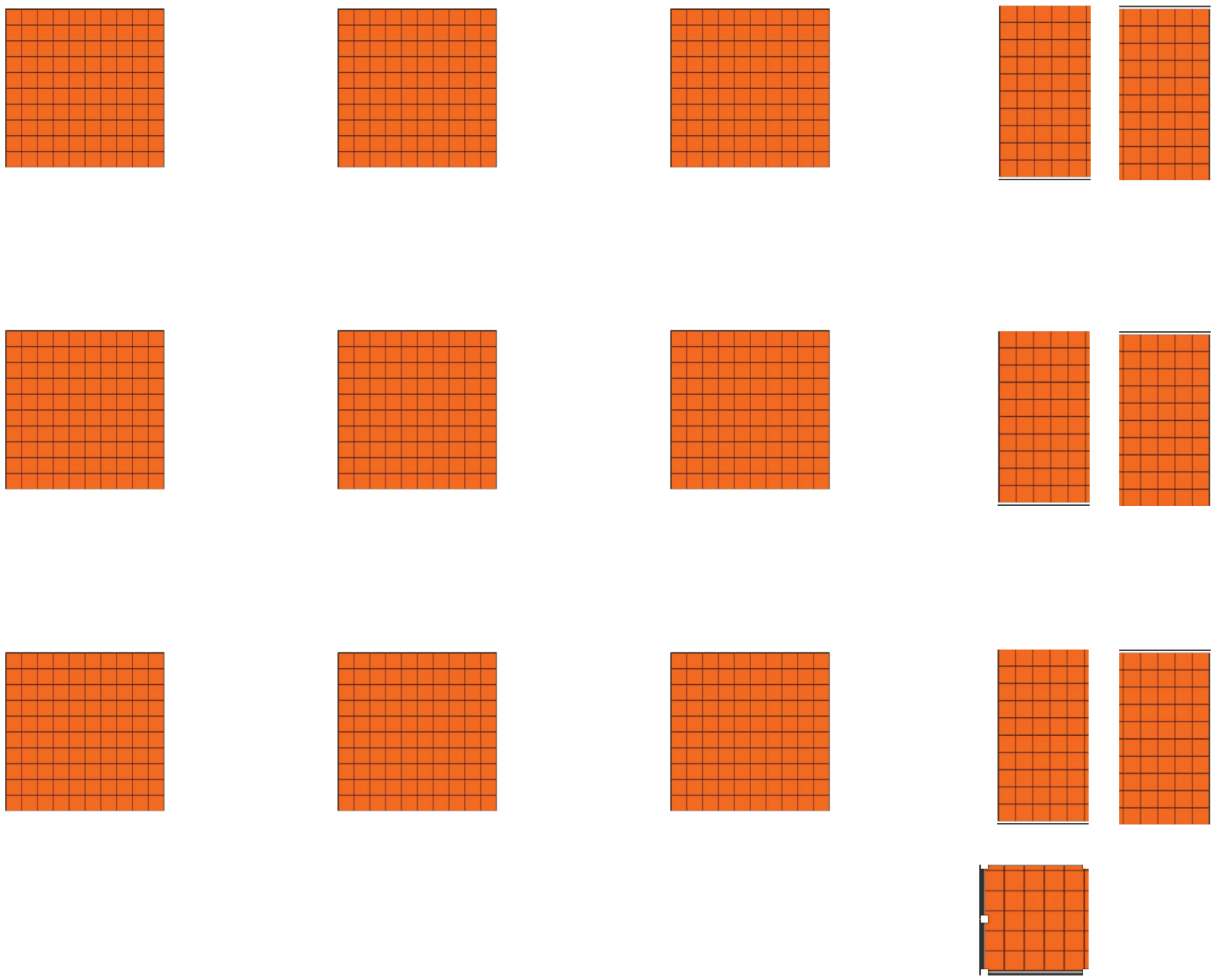
A visual (but still algebraic) approach



A visual (but still algebraic) approach



A visual (but still algebraic) approach



Using variables

- $(10a + 5)(10a + 5) = 100a^2 + 50a + 50a + 25$
- $= 100a(a + 1) + 25$

What might algebraic reasoning look like?

- It could be something simple, e.g.

What might algebraic reasoning look like?

- I might ask:
- You want Ax – B to be worth a lot when x is only around 10.
- What values would you use for A and B? Why?

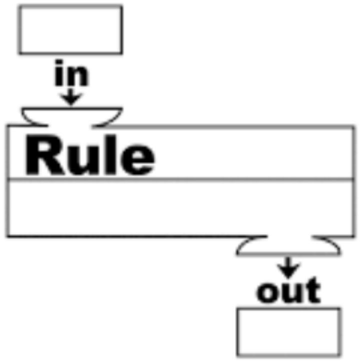
What might algebraic reasoning look like?

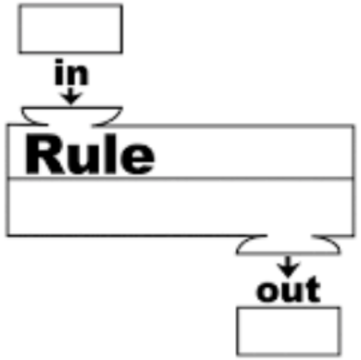
- I might choose a large positive value for A and a negative value for B

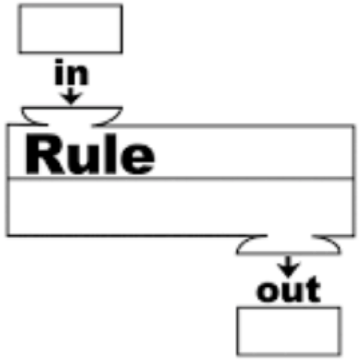
Or Play Guess my Rule

•

0 4 5





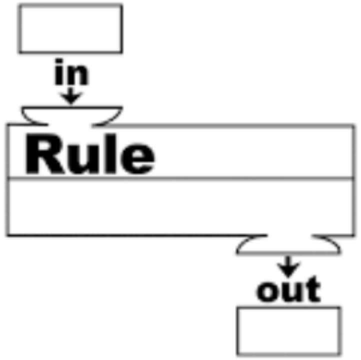


–8 8 17

Maybe $x^2 - 8$ OR $x^3 - 8x^2 + 20x - 8$

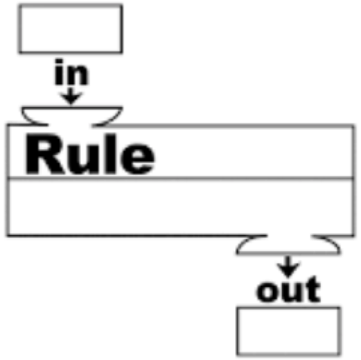
-

0



-8

4



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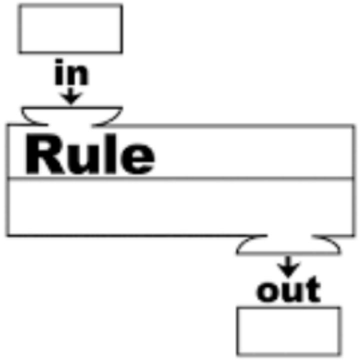

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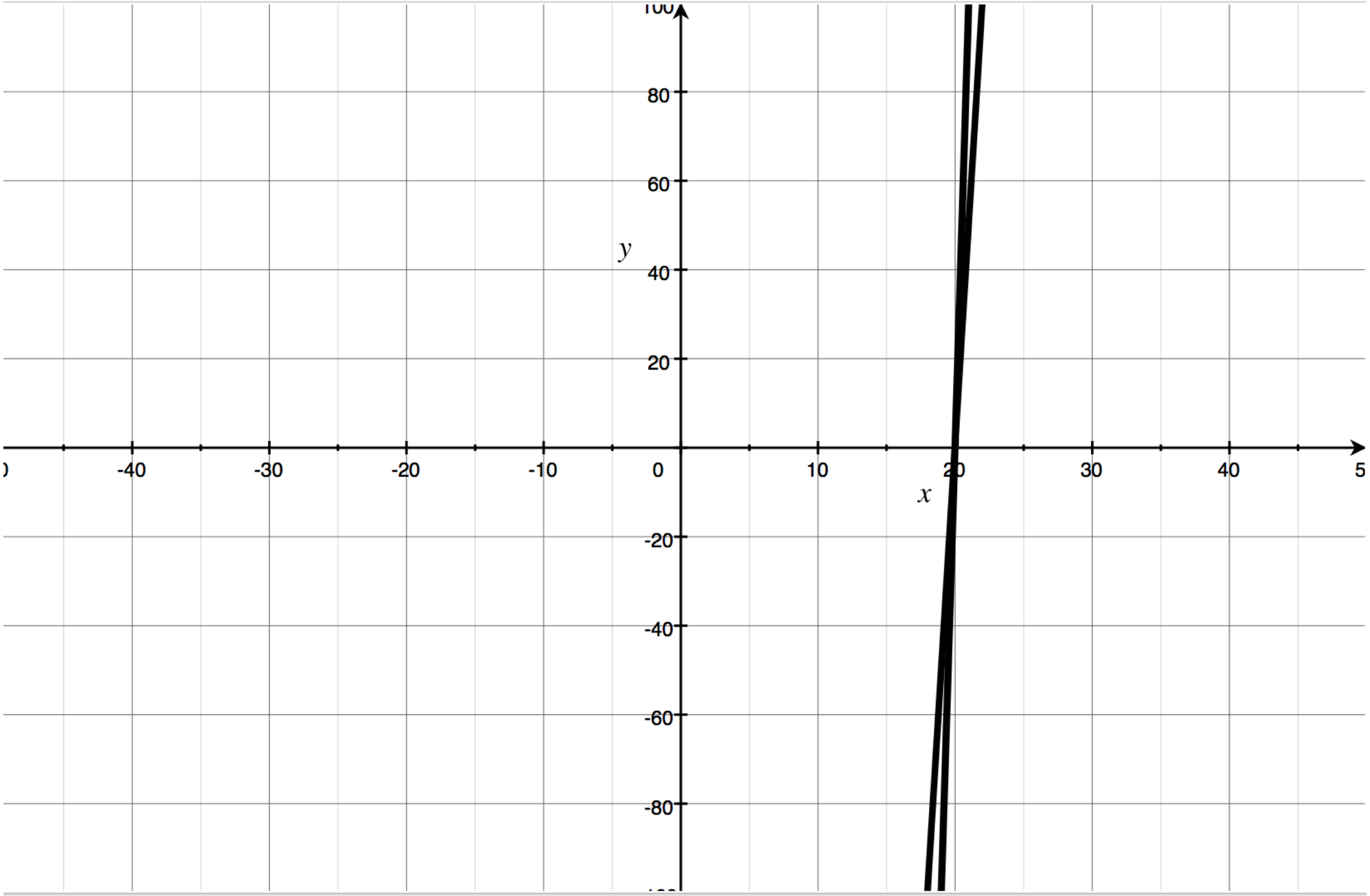
Or I might ask something a little more complex

- Two lines intersect at (20,1).
- Could both slopes be steep?

Or a little more complex

- If they are, what else do you know about those lines?
- If not, why not?

No Equation Selected



Or even more complex, but cool

- Let’s look at Desmos Racing Dots.

- https://teacher.desmos.com/activitybuilder/custom/56d139907e51c4ed1014b51f

So…

- What is the difference between focusing on algebraic reasoning vs. algebraic skills?

You will see reasoning in

- Exploring generalizations
- Exploring equality

You will see reasoning in

- Creating and testing conjectures
- Justifying and proving

MAYBE

- A) Solve $3x - 80 = 2x - 40$
- VS

MAYBE

- B) **WITHOUT SOLVING**, predict which two solutions will be closer and why.
- #1: $3x – 80 = 2x – 40$
- #2: $4x – 70 = 3x – 40$
- #3: $10x – 80 = 2x – 40$

OR

- A)Graph $y = 3(x - 3)^2 + 8$
- VS

OR

- B) Without drawing, predict how these graphs are alike and different?
- $y = 2(x - 3)^2 + 8$
- $y = 0.5(x - 8)^2 + 3$

So how can you…

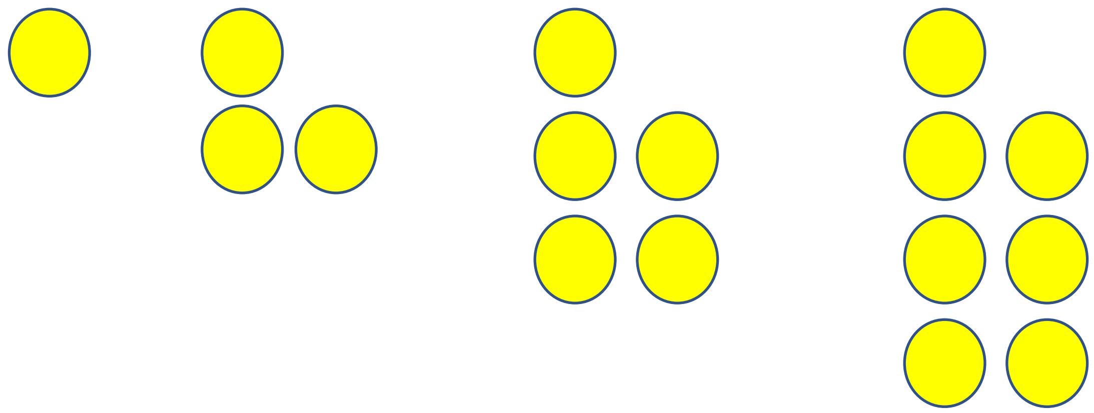
- Switch to include MORE algebraic reasoning in which to embed work on algebraic skills.

Start with an outcome

- Grade 8:
- Discrete linear relations

A skill question

- I might ask students to predict the 20th term of the pattern below. They might or might not use reasoning.



What kind of more reasoning question could you ask?

- A linear growing pattern is made using square tiles.
- The first term uses more than 2 tiles.

What kind of more reasoning question could you ask?

- One of its terms uses 25 tiles.
- Another of its terms uses 49 tiles.
- What other numbers of square tiles could be used in other terms? Why?

S/he thinks

- If one term has 25 tiles and one has 49 tiles, the difference is 24 tiles.

S/he thinks

- I must go up by a factor of 24, like 8.
- My pattern could be 9, 17, 25, 33, 41, 49,….

Grade 8

- Using an expression to describe a relationship

A skill question

- Write the phrase *One more than three times a number* algebraically.

A more reasoning question

- Describe a series of two computations you might use to get from 10 to 35.
- Use variables to describe that relationship so it could be applied to other numbers as well.

Grade 8

- Evaluate algebraic expressions

A skill question

- Which value is greater when you substitute $x = -8$:

- $-3x - 4$
- $-10x + 36$
- $-2x + 6$

Or I might ask

- Create an algebraic expression which has a value close to –100 when x is close to 19.

- You think:–100 is about subtracting 5 sets of 20, so I would use $1 – 5x$.

Grade 8

- Two-variable discrete linear relations

A skill question

- Suppose $2y + 3x = 5$.
- Write y in terms of x.

Or I might ask

- Suppose $5x + 2y = 125$ and x and y are integers.
- How do you know there are lots of solutions?
- How do you know x is odd?
- How far apart can the x values and y values of different solutions be?

Grade 9

- Solving and verifying linear equations

A skill question

- What is the solution to $3x - 18 = -2x + 32$?
- Explain your strategy.

But I might ask

- How do you know that the solution has to be negative WITHOUT SOLVING or even ALMOST SOLVING?

- $5x + 80 = 3x + 10$

Grade 9

- Linear functions: slope and equations of lines

So I might ask

- The line $Ax + By + C = 0$ has a negative slope.
- How many and which of A, B, and C can be negative to make this happen?

Grade 10

- Systems of linear equations

So I might ask

- Why might someone say it's easier to solve the first pair of equations than the second?
- Pair 1: Pair 2
- $6x - 20\,y = 18$ $3x - 2y = 10$
- $3x + 20y = 24$ $4x + 5y = 20$

Grade 10

- Equations of parallel and perpendicular lines

I could ask

- Write the equation of a line parallel to $y = 3x - 4$ that passes through the origin.

Or I could ask

- A parallelogram is drawn on a coordinate grid.
- One of the sides is on the line $4x + 2y = 3$.
- What lines might the other sides sit on?

Grade 10

- Connecting ordered pair with a meaning of an algebraic solution

I might ask…

- How could you quickly create the equation of a line of the form $ax + by = c$ (not horizontal or vertical) that passes through $(4, -2)$?

Grade 10

- Simple factoring

I might ask…

- Explore this number pattern.
- Relate it to factoring algebraic expressions.

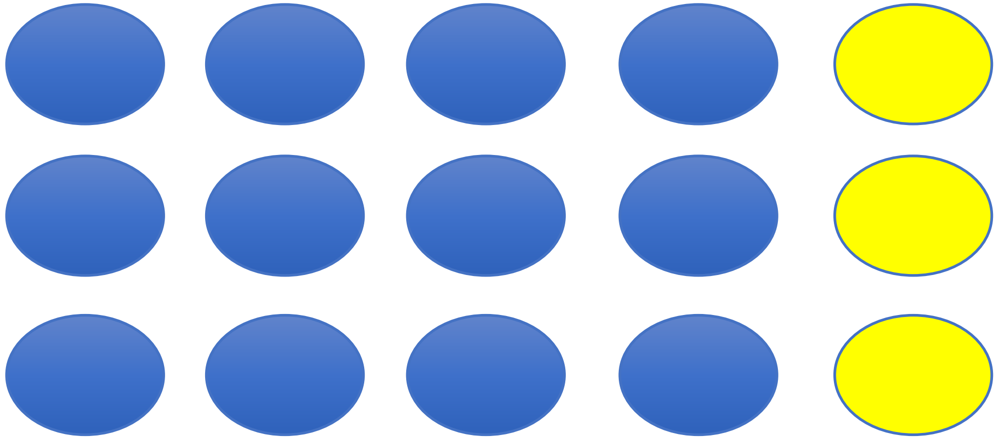
I might show…

- 3 x 5 = 15
- 4 x 6 = 24
- 5 x 7 = 35
- 6 x 8 = 49
- 7 x 9 = 63

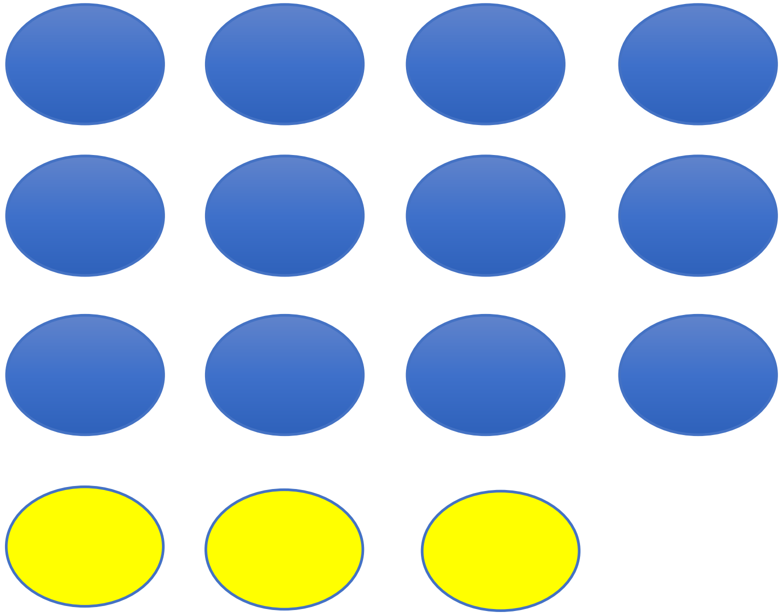
Or….

- 3 x 5 is one less than 4 x 4
- 4 x 6 is one less than 5 x 5
- 5 x 7 is one less than 6 x 6

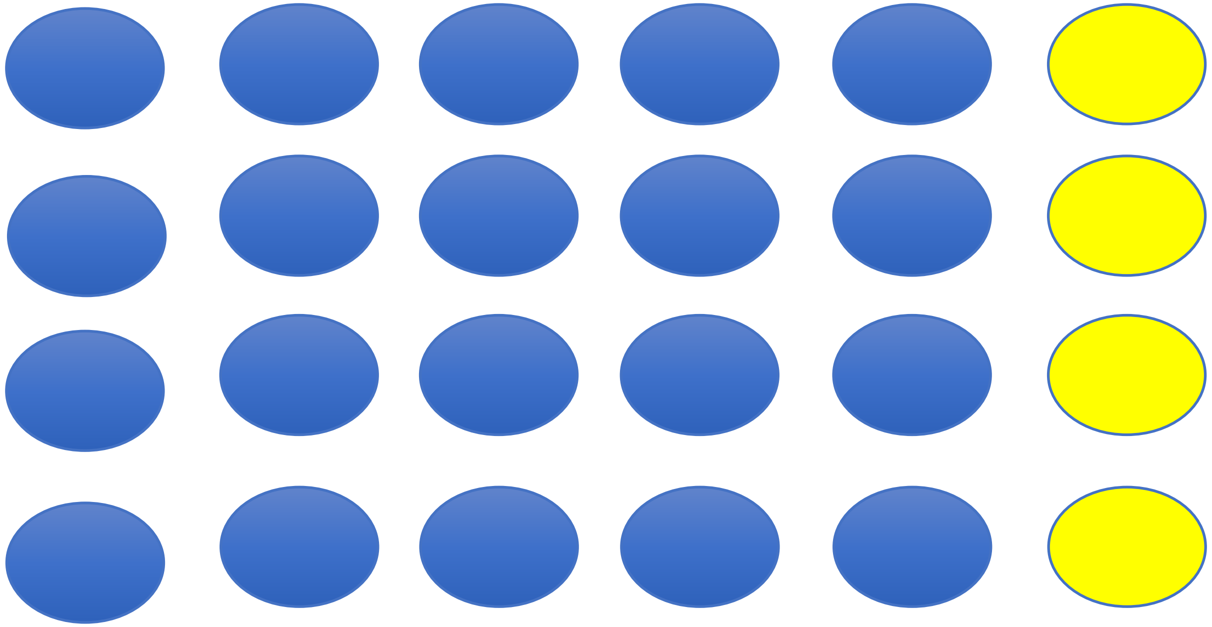
Visuals



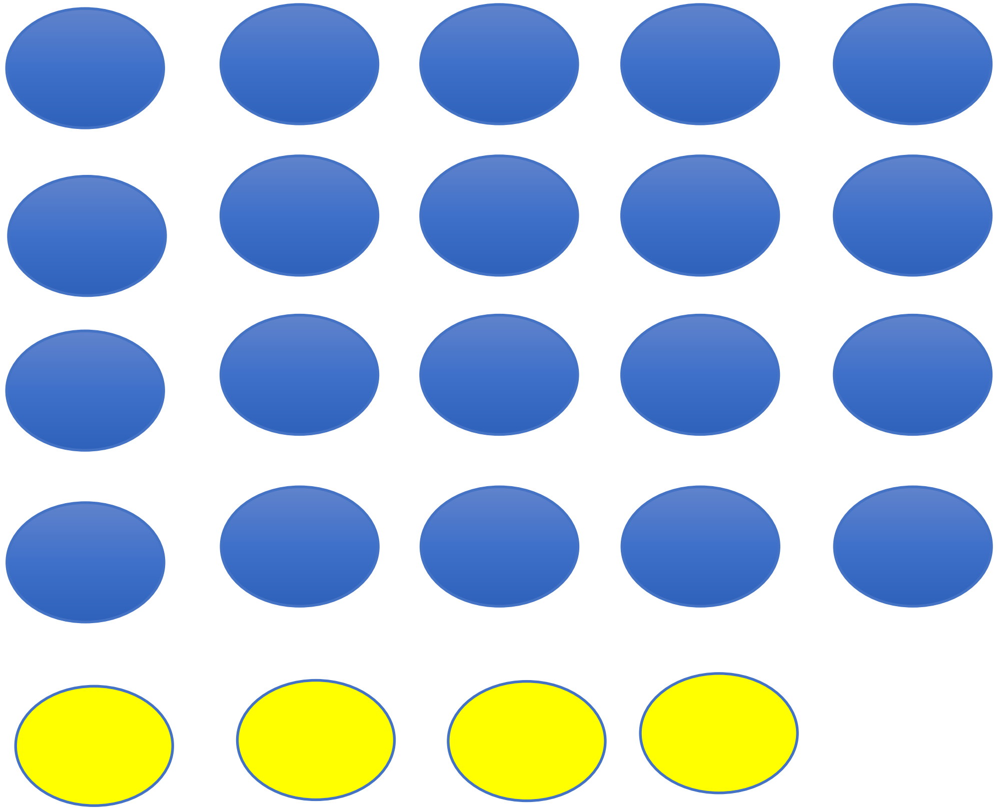
Visuals



Visuals



Visuals



Grade 11

- Factoring trinomials

Grade 11

- I might ask:
- Evaluate $2x^2 – 5x – 3$ for $x = 0, 1, 2, 3, 4, 5$.
- Factor the results.
- Use that to help you factor the expression.

- x $2x^2 - 5x - 3$
- 0 -3
- 1 -6
- 2 -5
- 3 -6
- 4 9
- 5 22

x	$2x^2 - 5x - 3$	
0	-3	= -3 x 1
1	-6	= -2 x 3
2	-5	= -1 x 5
3	0	= 0 x
4	9	= 1 x 9
5	22	= 2 x 11

- x $\quad 2x^2 - 5x - 3$
- 0 $\quad -3 \quad = -3 \times 1 \quad (x-3)(2x+1)$
- 1 $\quad -6 \quad = -2 \times 3$
- 2 $\quad -5 \quad = -1 \times 5$
- 3 $\quad 0 \quad = 0 \times 7$
- 4 $\quad 9 \quad = 1 \times 9$
- 5 $\quad 22 \quad = 2 \times 11$

Grade 12

- Recognizing composed functions; operations on functions

Instead of..

- What is the value of $g(x)$ if $g(x)= f(h(x))$ and $f(x) =$ … and $h(x) =$ ….?

I might ask

- How can you write $f(x) = 3x^2 - 6$ as a composition of two functions in different ways?

- What about $f(x) = x$?

Other reasoning situations

- Variables vs. unknowns

How are these equations alike and different?

- $2x + 1 = 5$
- $2x + 1 = y$
- $2x + 1 = x + x + 1$

Other reasoning situations

- Generalizations

Calculate and simplify.

What generalization do you see?

Why does it make sense?

Pattern

- Calculate and simplify.

2/3 x 3/4

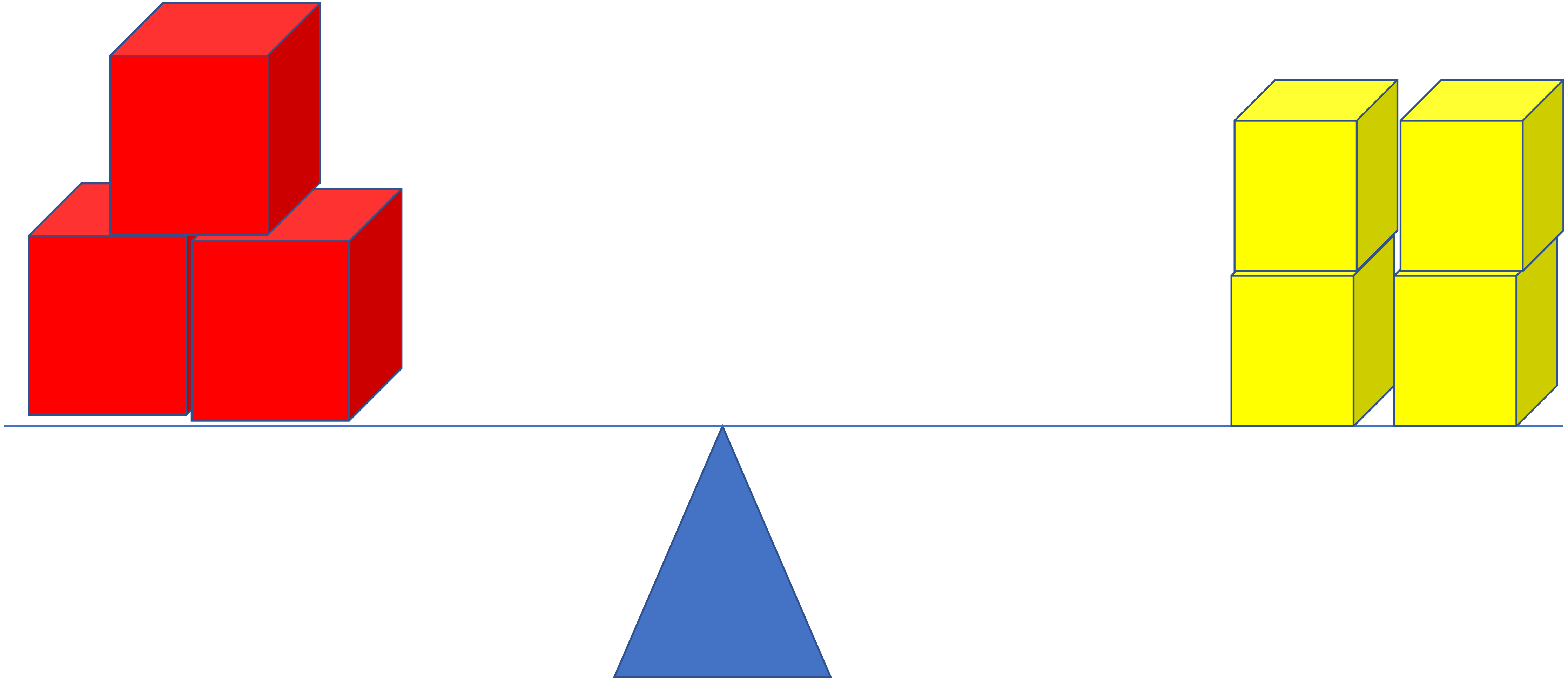
4/6 x 6/9

3/8 x 8/15

2/10 x 10/12

Equality

- What do you know about the relationship between the mass of the yellow and red boxes?



Estimating solutions to equations

You could ask for a reasonable estimate for these:

- $42 + 2x = 150$
- $3x - 40 = 2x + 1$
- $8x = 4 - x$
- $3/4\ x + 40 = x$

There is always more than one way.

- One way to describe the perimeter of a rectangle where the length is double the width is:
- P = 3L.
- What is another way?

Relating algebra to number

- Create two algebraic expressions that fit one of these rules when you substitute in whole numbers.
- The values are always even negative integers.
- The values are always positive multiples of 3.

Relating algebraic situations

- Suppose you know that $3x + 4 = 10$.
- Without solving the equation, tell what else you know about x.

So…

- Rather than thinking about the skills as the end game,
- In this scenario, the skills are a means to the end.
- The end is really about algebraic reasoning.

In summary

- There are skills, but it is easy to embed them in algebraic activities that focus on reasoning.

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