Gr 9&10 math

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agenda

- Teaching for understanding- What Ideas Do We Want to Emerge?
- Thinking vs application questions
- Value of open questions
- Value of manipulative materials

Teaching for understanding

- What does it look like to teach for understanding as well as knowledge and application?

Teaching for understanding

- Knowledge: Solve $4/x = 3/7.6$
- Understanding: WITHOUT solving, tell what a good estimate for x would be and why.

Teaching for understanding

- Knowledge/Application: What is the volume of a cone with a base radius of 8 cm and a height of 12 cm?
- Understanding: Two cones have the same volume, but one is much taller than the other. What could the radius and height of each be?

Teaching for understanding

- Knowledge: What is the slope of the line $2x – 3y = 12$?
- Understanding: The point (5,3) is on a line with slope –2. Name three other points on that line.

Teaching for understanding

- Knowledge: Solve $100x + 6 = 87x + 2$
- Understanding: WITHOUT SOLVING, tell why the solution to $100x + 6 = 87x + 2$ HAS TO be negative.

Teaching for understanding

- Knowledge: What is $x^2 - 2x - 1 - (-3x + 2)$?
- Understanding: When you subtract a binomial from a trinomial, can your answer be a binomial? A trinomial? Explain.

Teaching for understanding

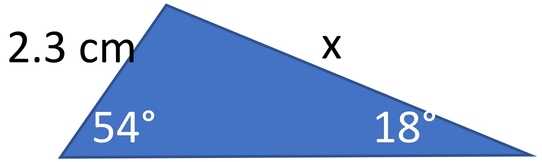
- Knowledge: The cosine of an angle is 0.8. What is the angle?
- Understanding: The cosine of an angle is a bit more than the sine. What angles might make sense to try? Which would not?

You try

- Turn one or two of the following knowledge questions into understanding ones.

You try

- What is $x^3 (2x)^4/x^7$?
- A line through (5, –1) is perpendicular to $y = -2x - 1$. What is its equation?
- Factor $x^2 + 2x - 15$.
- How long is side x?



Maybe for: What is $x^3 (2x)^4/x^7$?

- You multiply and divide some powers and you end up with 16. How could that happen?
- $x^a x^b/x^c = x^{12}$. What could a, b, and c be?
- You multiply some powers and you end up with $1/x^8$. What could you have multiplied?

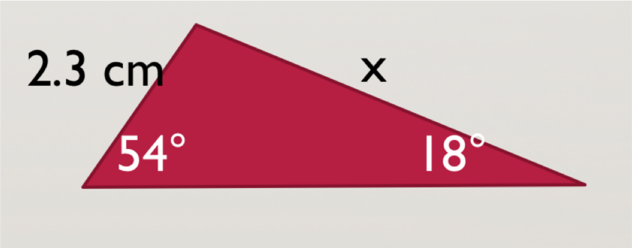
Maybe for the line through (5, –1) perpendicular to $y = -2x - 1$

- A line is perpendicular to $y = -2x - 1$. Tell two things you are sure are true about the line and two that might be true, but you are not sure.
- Two lines are both perpendicular to $y = -2x - 1$. Could they both go through (5, -1)? Explain.

Maybe for Factor $x^2 + 2x - 15$

- What has to be true about a, b, and c if you factor $x^2 + bx + c$ and b is positive and c is negative?
- How do you know that it won't be possible to factor $x^2 + c$ if $c > 0$?
- What does it mean if you say that you can't factor a quadratic expression?

Maybe for How long is side x?



- Without doing any calculations or referring to the picture, how do you know x is longer than 2.3 cm?
- Since 54 is 3 x 18, should x be 3 x 2.3? Why or why not?

Big Ideas

- The goal is to focus instruction on ideas we want students to learn.
- So when we look at expectations, we say to ourselves: What ideas are buried in here that we want to make sure students address?
- We then make those ideas learning goals for instruction.

For example

- Consider this expectation.
- What ideas do you want students to walk away with?
- **Develop, through investigation, the formulas for the volume of a pyramid, a cone, and a sphere**

Maybe

- That you only need one measurement of the sphere to figure out its volume, but you need at least two for the volume of the pyramid or cone.
- That there is a choice of measurements other than volume I could give you and you’d still be able to figure out the volume.

Maybe

- That the volume of a cone relates to a pyramid and to a cylinder.
- That the volume of a pyramid relates to the volume of a prism.
- That cones or pyramids with different measurements can have the same volume, but that is not true about spheres.

For example

- Consider this expectation.
- What ideas do you want students to walk away with?
- **Solve systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination**

Maybe

- That the solution to a system is a set of two numbers that makes both equations true, not just one of them.
- That any system of equations is equivalent to an infinite set of other systems, some of which are easier to quickly see the solution to than others.

Maybe

- That if you have an equation with only one variable in it rather than two, it’s easier to pin down the solution.
- That to get an equivalent equation, you merely maintain the balance.

Maybe

- That sometimes solving algebraically is easier than solving by graphing and sometimes not and when.

For example

- Consider this expectation.
- What ideas do you want students to walk away with?
- **Solve for the unknown value in a proportion using a variety of methods.**

Maybe

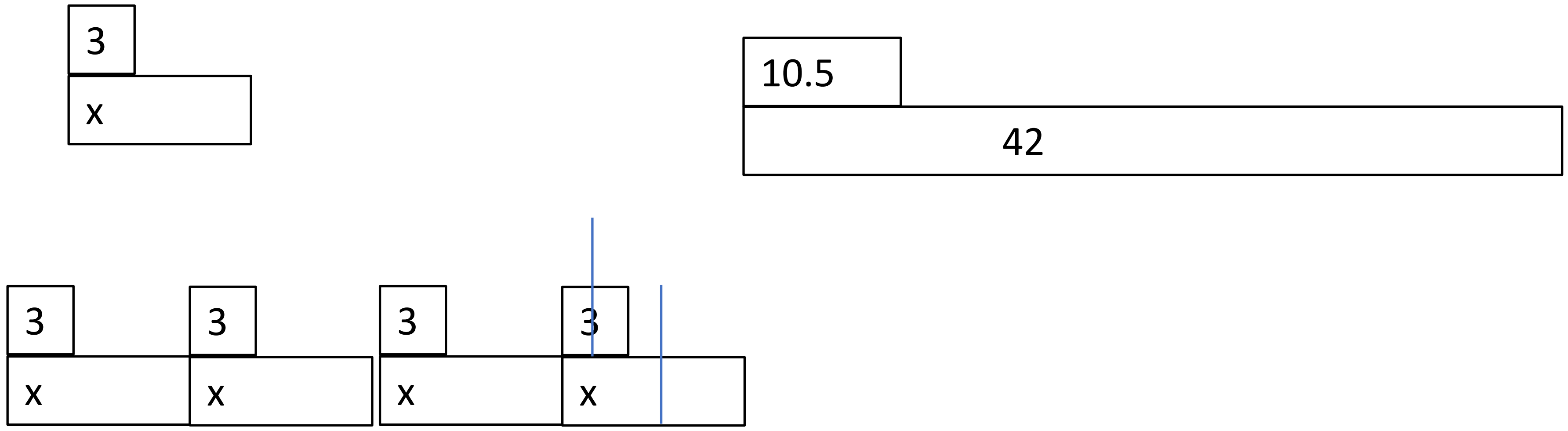
- That a proportion is a statement that two ratios (or rates) are equivalent.
- That equivalence is about a multiplicative relationship, not an additive one. (So 3/6 is equivalent to 6/12 but not to 4/7.)

Maybe

- That there are visual, algebraic, and numerical ways to solve a proportion.

Here is what I mean for 3/x = 10.5/42

- Visual



Here is what I mean for $3/x = 10.5/42$

- Algebraic

- $3 \bullet 42 = 10.5\,x$

Here is what I mean for $3/x = 10.5/42$

- Numeric
- 10.5 is one fourth of 42, so 3 is one fourth of x

Maybe

- That sometimes it makes sense to solve a proportion by looking “across” and sometimes “within”
- E.g. $17/85 = 3.5/x$ (within)
- $17/85 = 34/x$ (across)

Maybe

- That estimating is a useful part of solving a proportion.
- E.g.   8.2/15.3 = x/72.9
- Since 8.2 is a bit more than half of 15.3, x must be a bit more than half of 72.9.

Maybe

- That sometimes it makes sense to solve a proportion by looking “across” and sometimes “within”
- E.g.   17/85 = 3.5/x   (within)
- 17/85 = 34/x (across)

Now you try

- Take two of these expectations and dig into them to figure out ideas students should learn.

Now you try

- Determine the equation of a line from information about the line (e.g. slop and y-intercept; slope and a point; two points; etc.)
- Solve problems that require maximizing the area of a rectangle for a fixed perimeter or minimizing the perimeter of a rectangle for a fixed area
- Solve quadratic equations that have real roots, using a variety of methods (i.e. factoring, quadratic formula, graphing)
- Perform everyday conversions between the imperial system and the metric system to solve problems involving measurement

MAYBE: EQUATION OF LINE

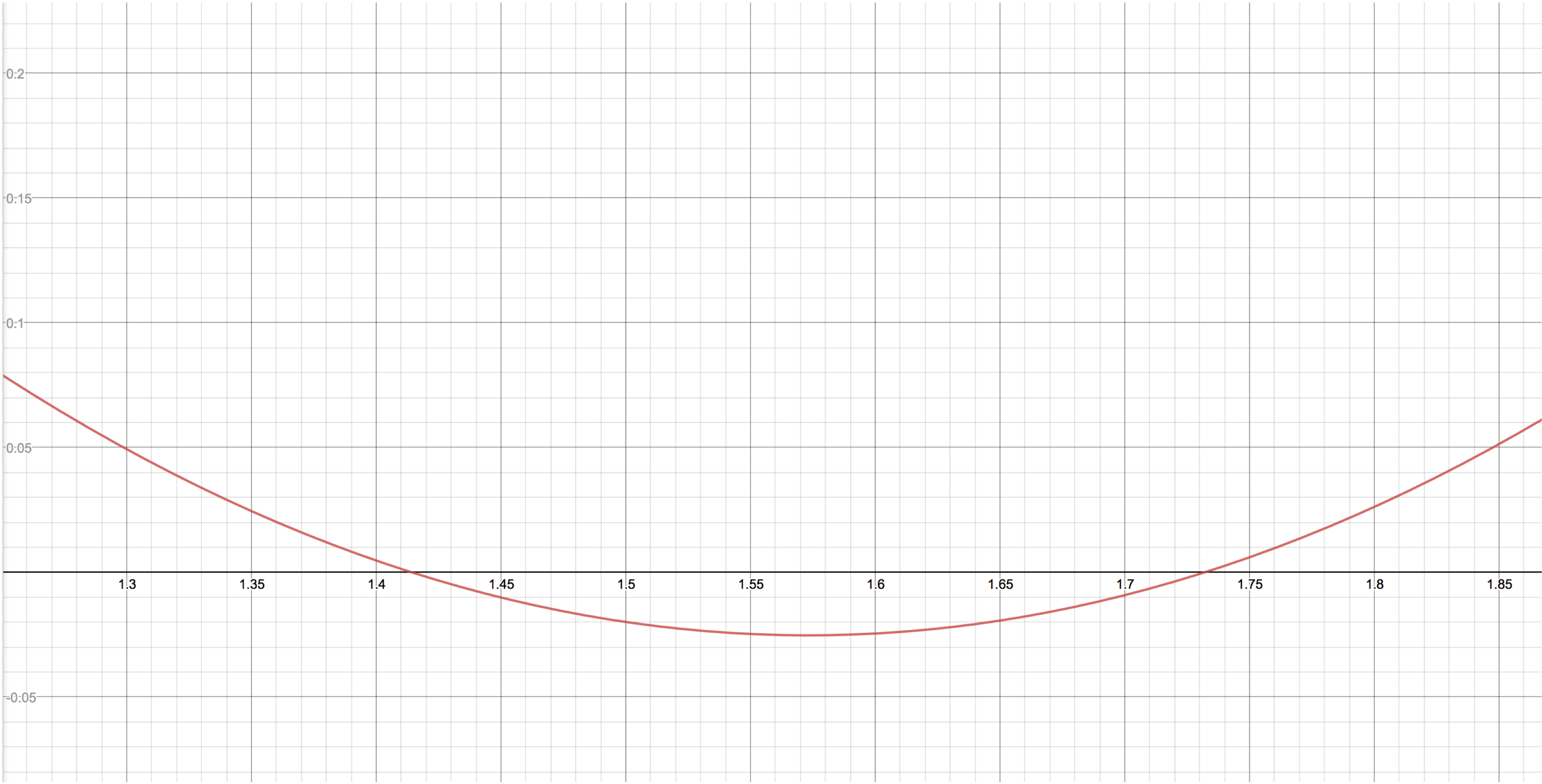
- That to pin down a line, you need two pieces of information that are “independent”, but that there are a lot of choices.
- That knowing a slope and a point is the same as knowing two points and why
- That knowing two points (maybe intercepts) is same as knowing slope and one point.

MAYBE: OPTIMIZATION

- That perimeter and area are independent measures of a rectangle
- Why you have more perimeter for a given area if it’s long and skinny
- Why you have less area for a given perimeter if it’s long and skinny

MAYBE: SOLVING QUADRATIC EQUATIONS

- That if you have the right graphing software to solve an equation, it's probably the easiest way, if you can live with an estimate. (e.g. if roots are $\sqrt{2}$ and $\sqrt{3}$, then graphing software gives decimal estimates).


0.2
0.15
0.1
0.05
-0.05
1.3
1.35
1.4
1.45
1.5
1.55
1.6
1.65
1.7
1.75
1.8
1.85

MAYBE: solving quadratic equations

- That sometimes factoring is easy to see, but not always
- That the quadratic formula is a “guaranteed” method; no thinking required

MAYBE: IMPERIAL/METRIC

- That if one of the units is smaller, the numerical value should be greater
- That units should be the same “type”, e.g. miles and metres is ok, but miles and cubic metres is not

MAYBE: IMPERIAL/METRIC

- That conversions for square or cubic units are different than conversions for “linear” units
- That some imperial units are REALLY CLOSE to some metric units and some are not and which are which

Thinking vs application

- There is also a difference between application problems, where students solve problems very much like what they have already seen and ones where they have to think.

Thinking vs application

- I can’t show you an example that will apply to everyone since it will always depend on what the teacher has shown students, but it might be something like this.

Linear relations

- It is likely that a teacher has asked students to take some straightforward problems and write the linear relation to describe them, e.g.
- Construct a table of values and a graph to represent a monthly cellphone plan that costs $25 plus $0.10 per minute of airtime.

Linear relations

- So a similar problem, just changing the numbers or even a slight change to the context (e.g. cost of a hall for a banquet and per dinner cost or cost of a pizza that is plain plus per topping costs) is an application.

What would a thinking problem be?

- A gym has a choice of payment plans :
- A fee to join and a per month cost OR
- No fee to join and a higher per month cost

What would a thinking problem be?

- What might the values be if the first plan is better if you stay with them for more than a year, but the second plan is better if you stay less than a year.

Or consider this application problem

15. A baseball is thrown from the top of a building and falls to the ground below. The height of the baseball above the ground is approximated by the relation $h = -5t^2 + 10t + 40$, where h is the height above the ground in metres and t is the elapsed time in seconds. Determine the maximum height that is reached by the ball.

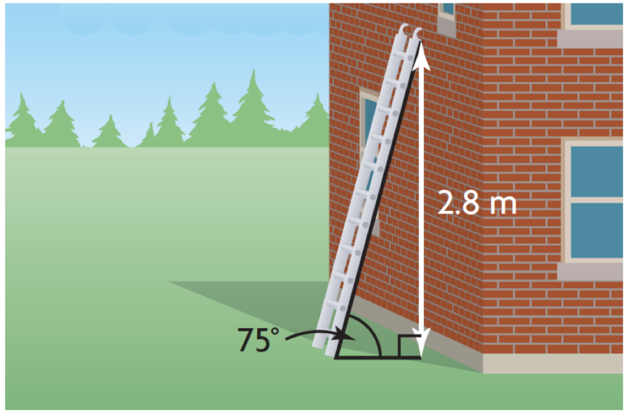
A thinking problem might be

- A baseball is thrown from the top of a building .
- What is a reasonable equation that might describe the height of the ball at time t?
  Why is it reasonable?

Or consider this application problem

11. A ladder leans against a wall, as shown. How long is the ladder, to the nearest tenth of a metre?

A



A thinking problem might be

- A window washer puts an 8m ladder against a building.
- What is a likely angle for it to be set at?
- What is a likely maximum height it might reach?

The value of open questions

- We are going to look at the value of open questions at the Grade 9 and 10 level.

What are open questions?

- They are questions with multiple reasonable responses, not just different ways to get the same answer.

Why are they valuable?

- We will talk about this more after you see them, but….
- They are accessible to struggling students.
- They extend strong students.
- They facilitate rich math conversations.

numerical

- A numerical expression that includes the number π is worth about 20.
- What could the expression be?

Maybe

- $\pi + 17$
- $3\pi + 10$
- $2\pi^2$

Proportional thinking

- Choose a car speed.
- Write that speed using as many equivalent rates as you can.

Maybe

- 50 km/h
- 50 000m/h
- 833.33 m/min
- 100 k per 2 h

Proportional thinking

- Write a proportion by choosing values for the blanks.
- []/[] = []/x.
- Write a story for that proportion.

Maybe

- $4/20 = 8/x$

- I used 4 cups of flour to make a recipe for 20 people.
- If I have 8 cups of flour, for how many people can I make that recipe?

powers

- You have an expression involving a bunch of powers.
- When you simplify it, the result is 5^{-3}.
- What could the expression have been?

Maybe

- $5^4/5^7$ OR
- $(7^3 \times 2^3 \times 5)/(7^3 \times 2^3 \times 5^4)$

algebra

- You simplify an algebraic expression to $14x - 8y - 37$. What could the original expression have been?

algebra

- Create an algebraic expression that is :
- ALWAYS more than $2m + 1$
- SOMETIMES more than $-2m$

algebra

- The equation $4x - 5 = 15$ describes two very different situations.
- What might the situations be?

Linear relations

- The lines of two equations intersect at (4, –8).
- What could the lines be if they are not horizontal or vertical?

Maybe

- $3x - 5y = 52$ and
- $2x + 8y + 56 = 0$

algebra

- You add an algebraic expression with 4 terms to one with 5 terms and the result has 3 terms. Is that possible? If so, how?
- If not, why not?

Maybe

- $(3x + 2x^2 - y + 7) + (5x - x^2 + y - 7 + 3z)$

Linear relations

- A line goes through (4,2) and slants up and to the right.

- Name at least one other thing that you are sure is NOT true about that line.

Maybe

- It does not go through (4, 3).
- The slope is not negative.
- It does not go through (5, 1).

quadratics

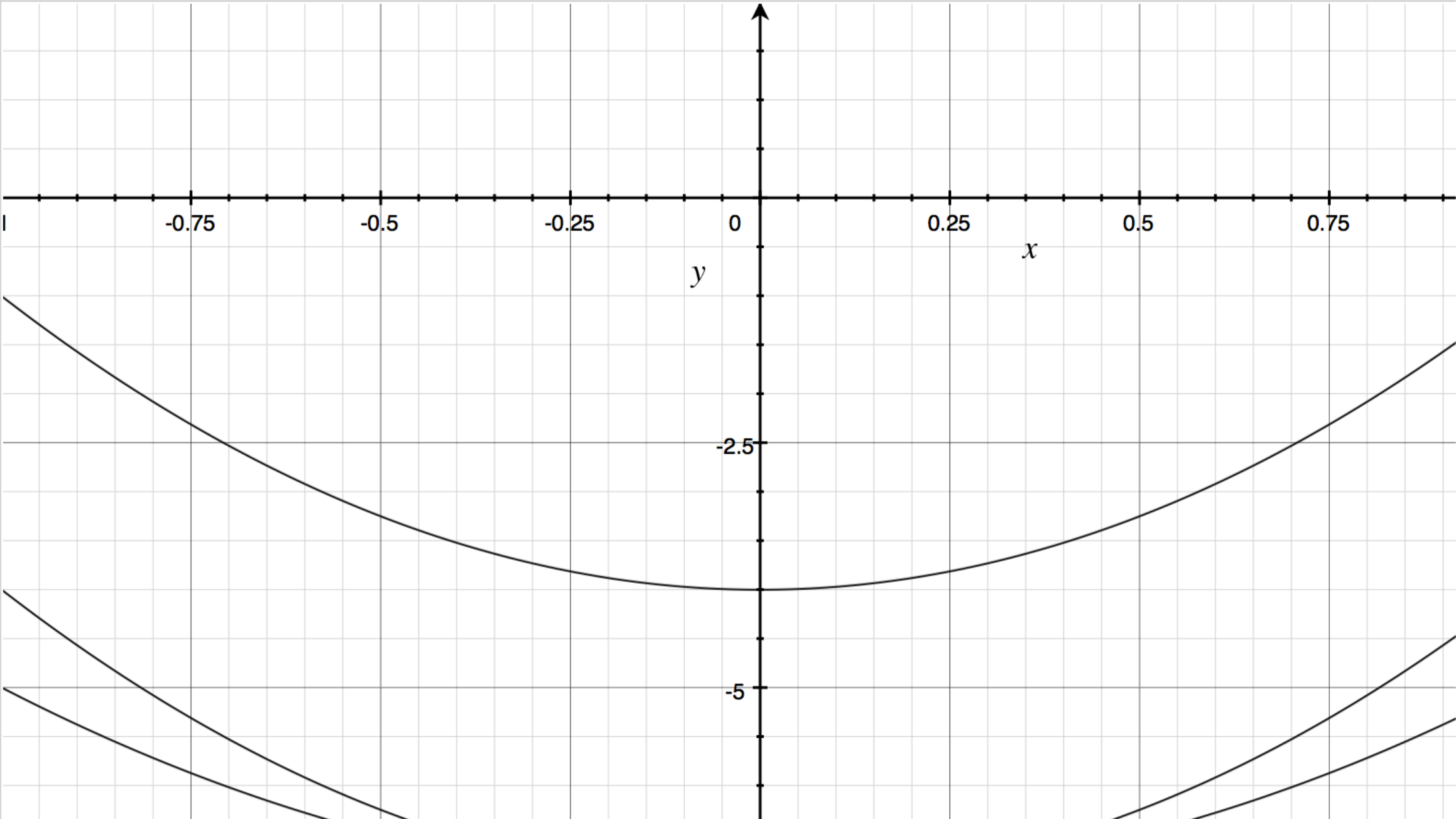
- The roots of a quadratic are two apart.
- What could the equation be?

Maybe

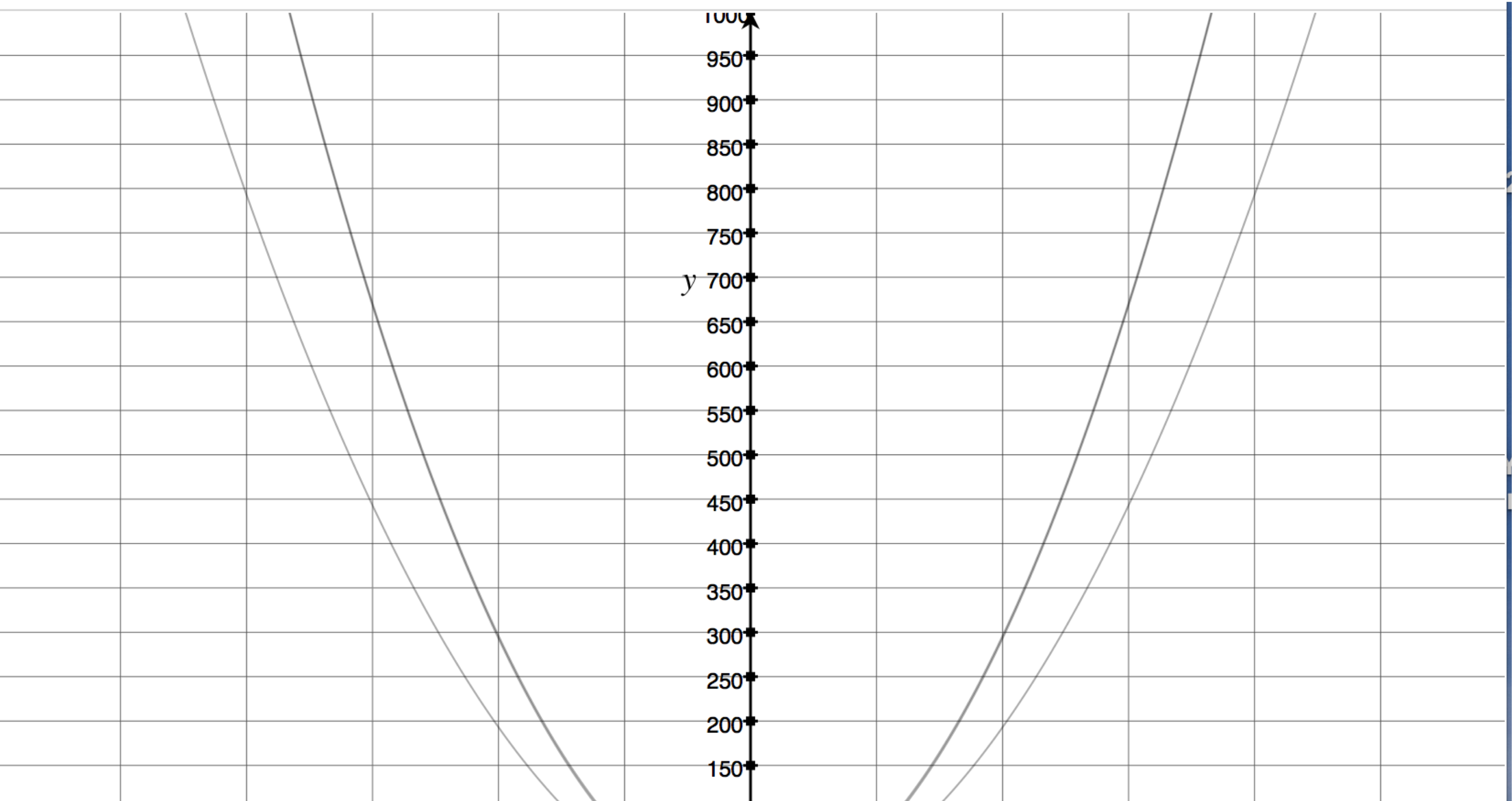
- $(x - 3)(x - 5) = x^2 - 8x + 15$ OR
- $(x - \frac{1}{2})\ (x - 5/2) = x^2 - 3x + 5/4$

quadratics

- The graphs of two of these are most alike. Which two? Why?
- $y = 3x^2 – 4$
- $y = 3x^2 – 7$
- $y = 2x^2 – 7$



-0.75
-0.5
-0.25
0
0.25
0.5
0.75
x
y
-2.5
-5



1000
950
900
850
800
750
y
700
650
600
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400
350
300
250
200
150

trig

- An angle in a right triangle has a sine of 0.2.
- What else do you know about that triangle?

trig

- One of the trig ratios of an angle in a right triangle is really close to 1.
- What could the angles in the triangle be?

algebra

- You evaluate an algebraic expression when $x = 2/3$ and the value is $1\frac{1}{4}$.
- What could the expression have been?

Maybe

- $3x - \frac{3}{4}$
- $x + 7/12$
- $6x - 2\frac{3}{4}$
- $x^2 + 29/36$

measurement

- A pyramid has a surface area of $100cm^2$.
- What could its dimensions be?

Linear systems

- I was solving a system of two equations in two unknowns.
- After elimination, I was left with $13y = 22$.
- What could the original two equations have been?

Maybe

- $2x + 5y = 12$ and $3x - y = 7$
- $x + 5y = 6$ $7y + 4x = 2$

trig

- You used the sine law to solve a triangle.
- The solution told you that one side length of the triangle was 4.5 cm.
- What information might you have known about the triangle?

measuring

- An imperial measure and an equivalent metric measure are 10 apart.
- What might the two measurements (with their units) have been?

talk

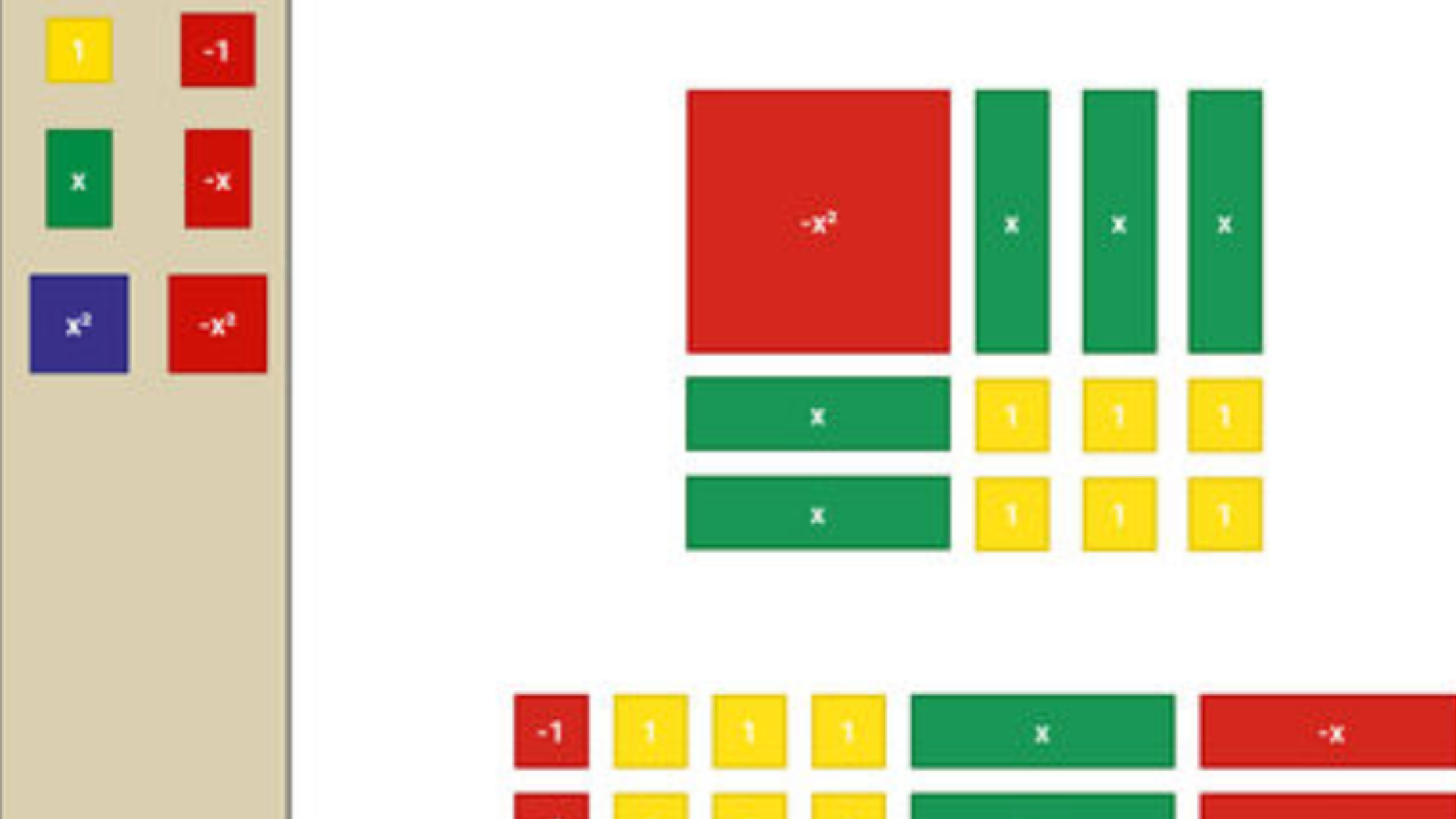
- How easy do you think these might be to create?
- How would you create them?
- Why might they be good to use?

The value of manipulative materials

- We are going to look at the value of using algebra tiles, at the Grade 9 and 10 level.
- Other materials we don't have time for today that have great value are fraction strips .

Algebra tiles

- Let’s look at them for representing polynomials, adding and subtracting polynomials, multiplying and dividing polynomials, factoring, solving equations.


1
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-x²
-x²
x
x
x
x
1
1
1
x
1
1
1
-1
1
1
1
x
-x

Some examples

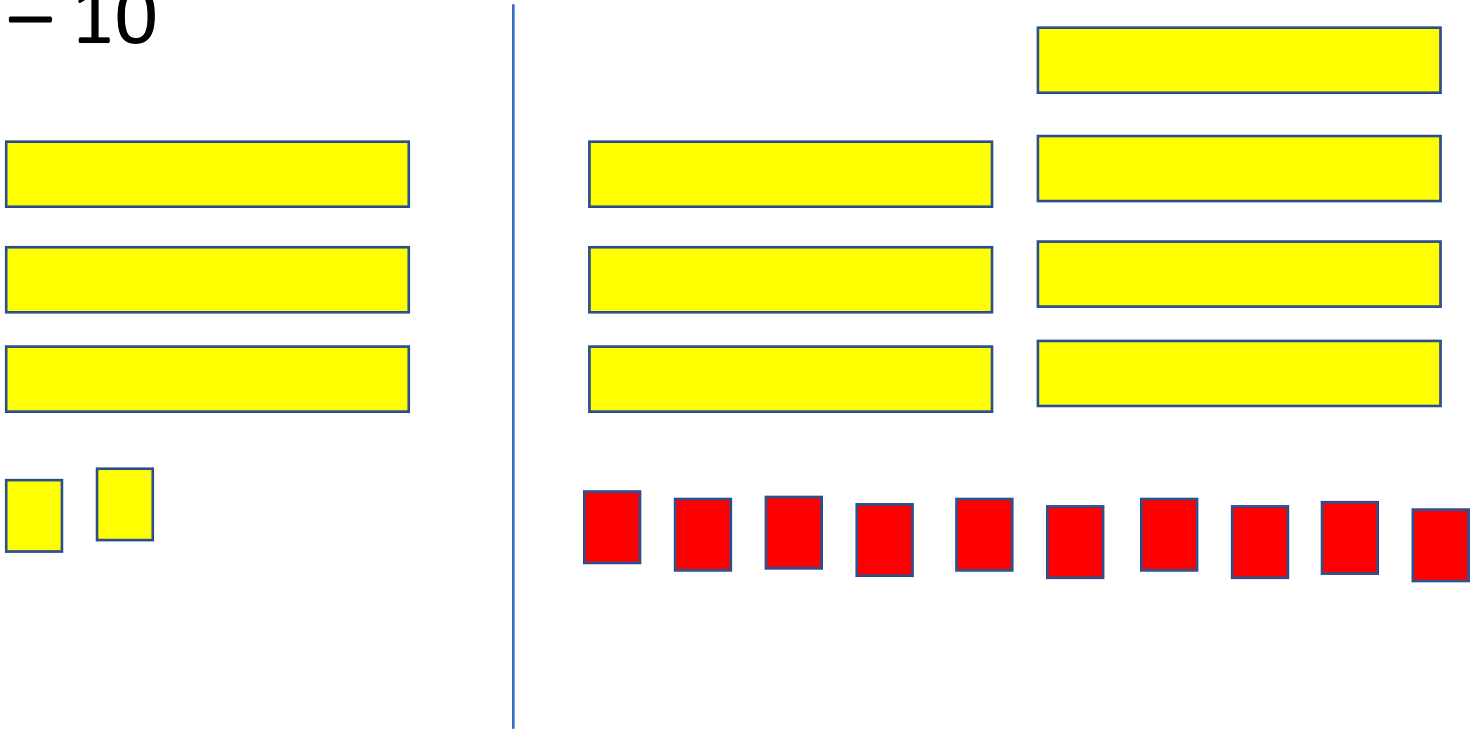
- What polynomials use 4 tiles?
- What is $3x^2 - 2x + 5 + (-2x^2 + x - 3)$?
- What is $3x^2 - 2x + 5 - (-2x^2 + x - 3)$?
- What is $(x + 2)(x + 4)$?
- Factor $6x^2 + 9x + 3$.

Algebra tiles

- $3x + 2 = 7x - 10$

Algebra tiles

- $3x + 2 = 7x - 10$

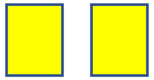


Algebra tiles

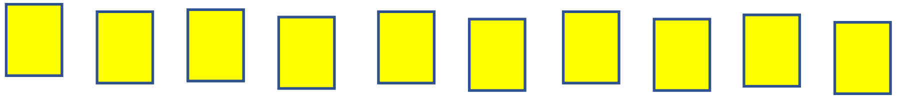
- Remove 3x from both sides

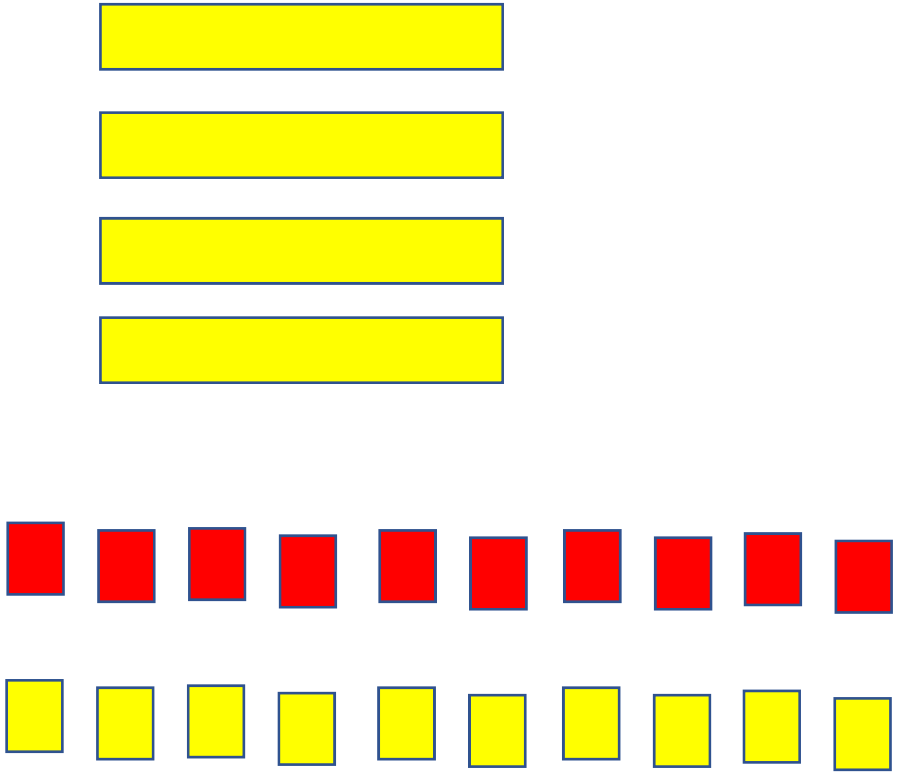
Algebra tiles

- $3x + 2 = 7x - 10$



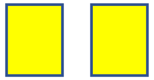
Add 10 to both sides.



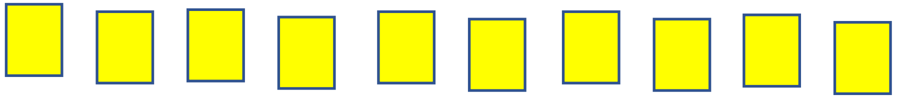


Algebra tiles

- 3x + 2 = 7x – 10



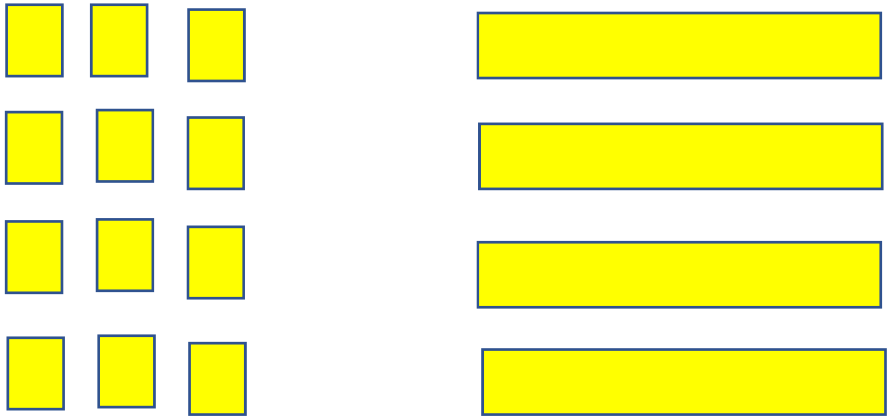
Simplify



Algebra tiles

- $3x + 2 = 7x - 10$

Rearrange ones to match same number of ones to each x.



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