Quel est le rapport? Comparons!

October 2018

I wonder!

- How many times will you send a text message in the next year?

Agenda

- What is proportional reasoning?
- Using manipulatives to develop proportional reasoning

Proportional reasoning

- Proportional reasoning involves the deliberate use of multiplicative relationships to compare quantities and to predict the value of one quantity based on the values of another.

Why is this topic important?

- It is so useful in real life.
- It appears in so many places in math curriculum.

And in real-life

- Exchanging coins
- Exchanging currency
- Changing measurement units, e.g. centimetres to metres
- Choosing a best buy in terms of money…

In the curriculum

Skip counting

Comparing numbers

Finding factors of numbers

Comparing fractions

In the curriculum

- Fractions (represent, compare, changing from improper to mixed, equivalent)
- Multiplying and dividing
- Using measurement units

In the curriculum

Measurement formulas

Graph scales

Probability

Percentage

Averages

Big Idea

- It is often useful to think of one amount as how many of another amount.
- e.g. one loonie as 4 quarters
- 1 week as 7 days
- 1 bookcase as 5 shelves
- 24 shoes as 12 pairs
- 20 eggs as 1 2/3 dozen

Related important ideas

- If you use a bigger unit, you need fewer of them.
- If units are related, you can use that relationship to predict how many of one unit if you know how many of the other.

Comparing changes

Which price changed the most?

Cereal from : $5.99 to $2.99

Car from: $46 945.00 to $44 999.00

Related important ideas

- Any number can be compared to any other number multiplicatively, e.g. 9 can be compared to 2 by thinking of it as 4 ½ twos.

And 2 can be compared to 9 as 2/9 of a 9.

Related important ideas

- Two numbers can be far apart from an additive point of view but not from a multiplicative point of view, e.g. 1000 and 100 are 900 apart, but 1000 is only 10 100s.
- Or vice versa, 4 and 0.5 are less than 3 apart additively, but 4 is 8 sets of 0.5.

Related important ideas

- Using a fraction, decimal or percent is automatically a way of comparing numbers multiplicatively.
- For example, 2/3 tells us that 2 is only 2/3 of a 3 (and that 3 is 3/2 of a 2).
- 0.4 is a way to compare 4 to 10
- 35% is a way to compare 35 to 100

Let’s look at the types of

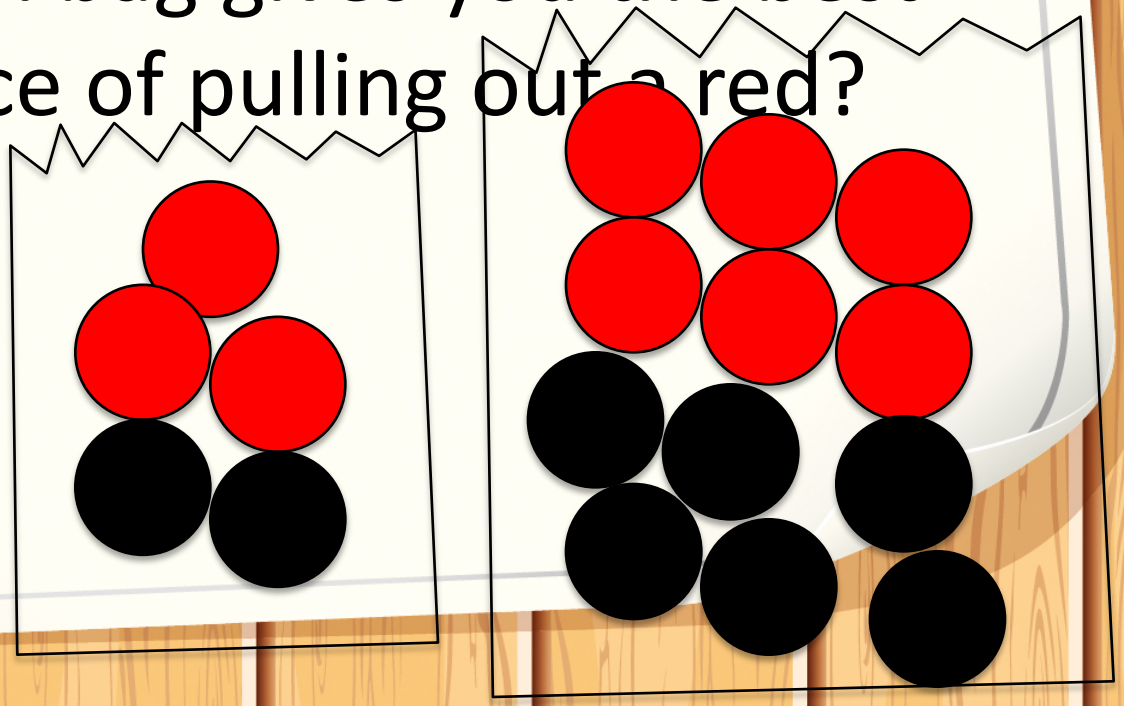
- problems students might solve that involve proportional reasoning.

Dogs

- 1 out of every 3 Canadian households has a dog.
- About how many dogs would you predict for the students in your class?

Probability

- You are pulling out a counter from each bag.
- Which bag gives you the best chance of pulling out a red?



Speeds

- A car goes 246 km in 3 hours.
- How far, at that speed, will it go in another 2 hours?

Fraction comparisons

- How do you know that $\frac{18}{37}$ is a bit less than $\frac{1}{2}$?

Length

- How long is a line of 1 000 000 pennies?

19 cm

Getting to 1000

- Which sequence will get to 1000 first?

15, 25, 35, 45,….

500, 502, 503,…

Estimation

A Fermi problem, e.g.

Estimate the number of square centimetres of pizza that all of the students in Ontario eat in one week.

What can you do to build proportional reasoning?

- Many suitable tasks are suggested as models.

You could ask

- I looked at a bunch of animals. There were twice as many rooster legs as dog legs.
- How many of each animal might there have been?

You could ask

- How many numbers would I need to write (say) to continue this way to get to 100?

  12, 14, 16,

You could

regularly use multiplicative language such as:

- Twice as much
- Four times as big
- Half as many
- Two thirds as heavy

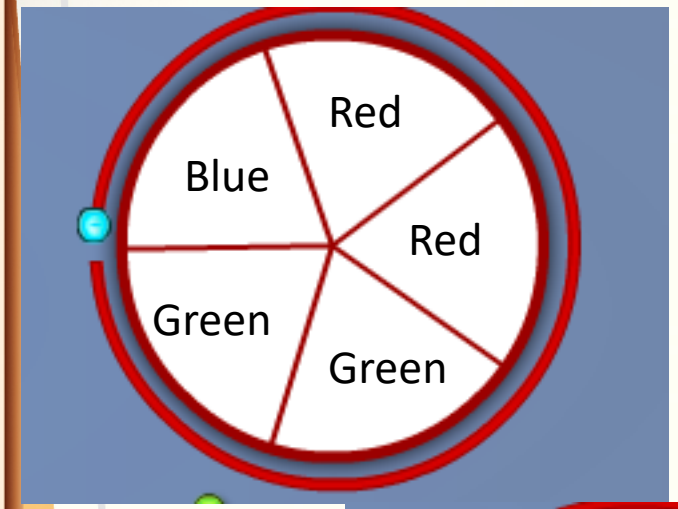
You could ask

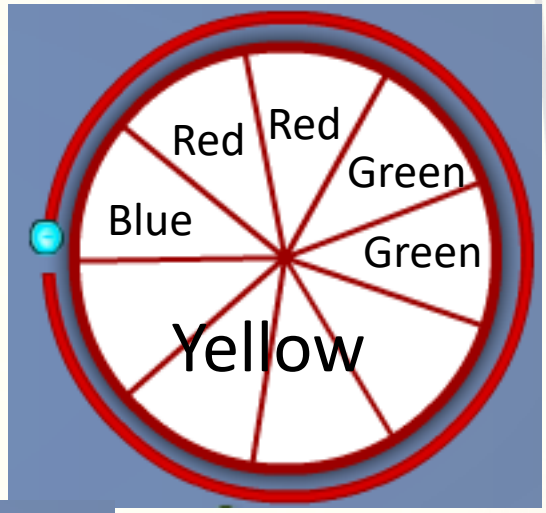
- My brother has 2.5 times as many games as I have. How many might we each have?
- Do you think I have 9 games?

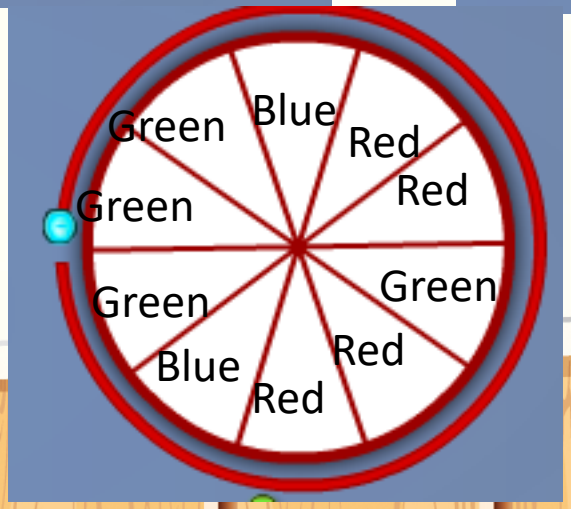
A Colourful Spinner

- I spin a spinner.
- I am twice as likely to get red as blue.
- I am half as likely to get blue as green.
- What could the probability of green be?

Possibilities







You could ask

- A sentence has 40 letters in it. What number of words do you think it probably has? Why?

You could ask

- About how many ceiling tiles are there in the whole school?

You could ask

- The perimeter of one square is 1/3 as long as the perimeter of another. What do you know about the side lengths?

You could ask

- Jane is 8. Her mom is 38.
- When did or will her mom be twice as old as Jane?
- When would it happen again (or would it)?

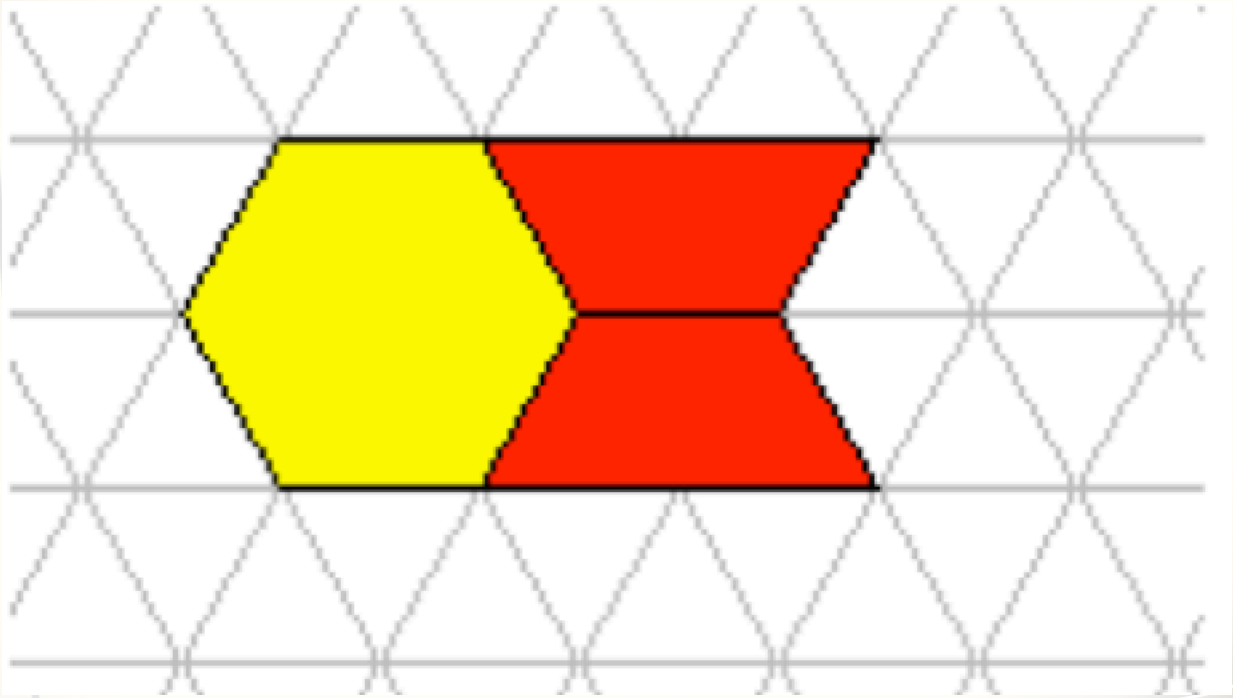
More problems

- Often using manipulatives effectively can support proportional reasoning.
- Here are some examples.

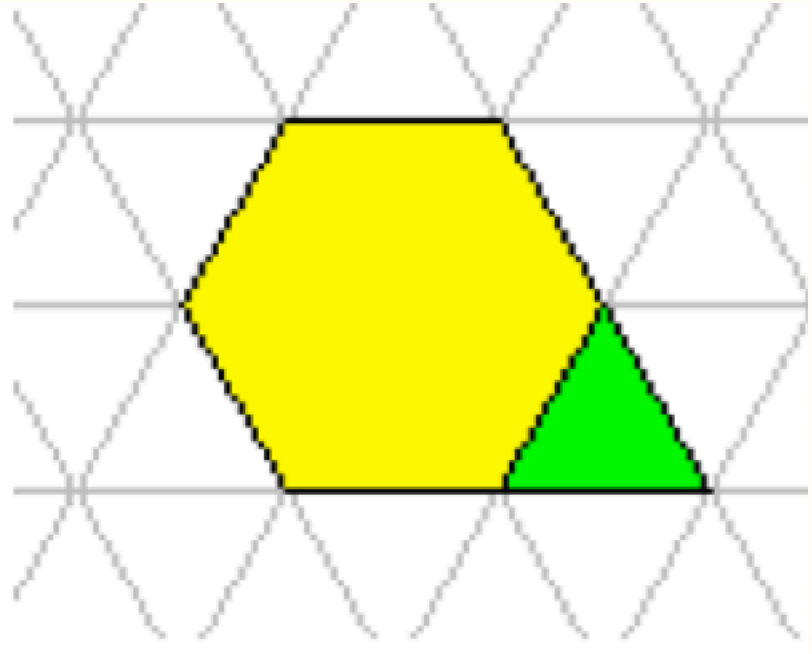
A problem to try

- Use pattern blocks.
- Create a design that is half yellow.

Possibilities



Possibilities



Questions about ½ yellow

- What makes a design half yellow?
- Were half the blocks (or half the area, depending on which students did) yellow?

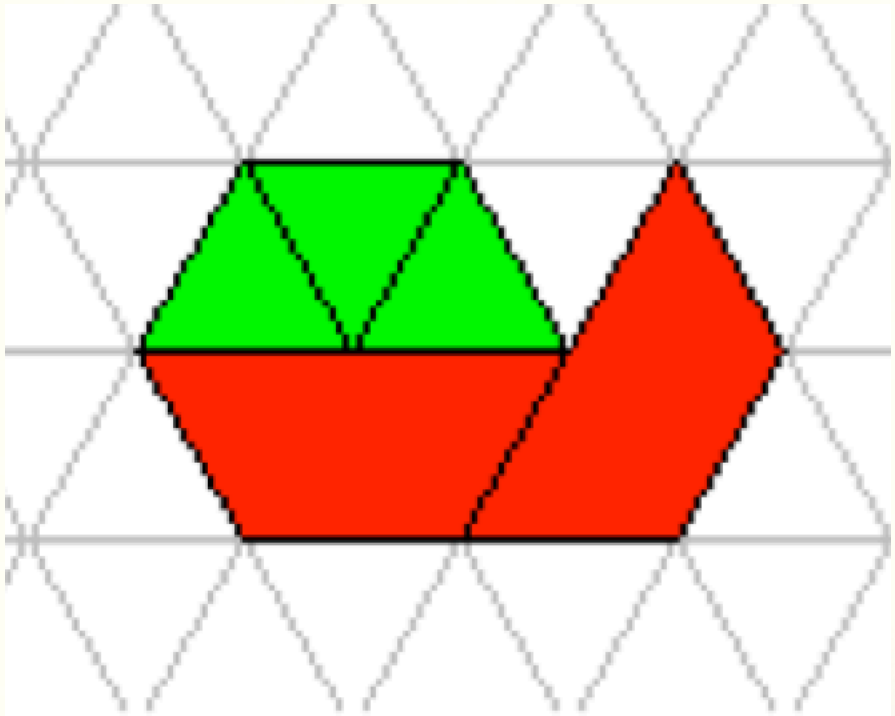
Questions about ½ yellow

- Suppose you had copied your design next to the original one. Would it still be half yellow? Why or why not?

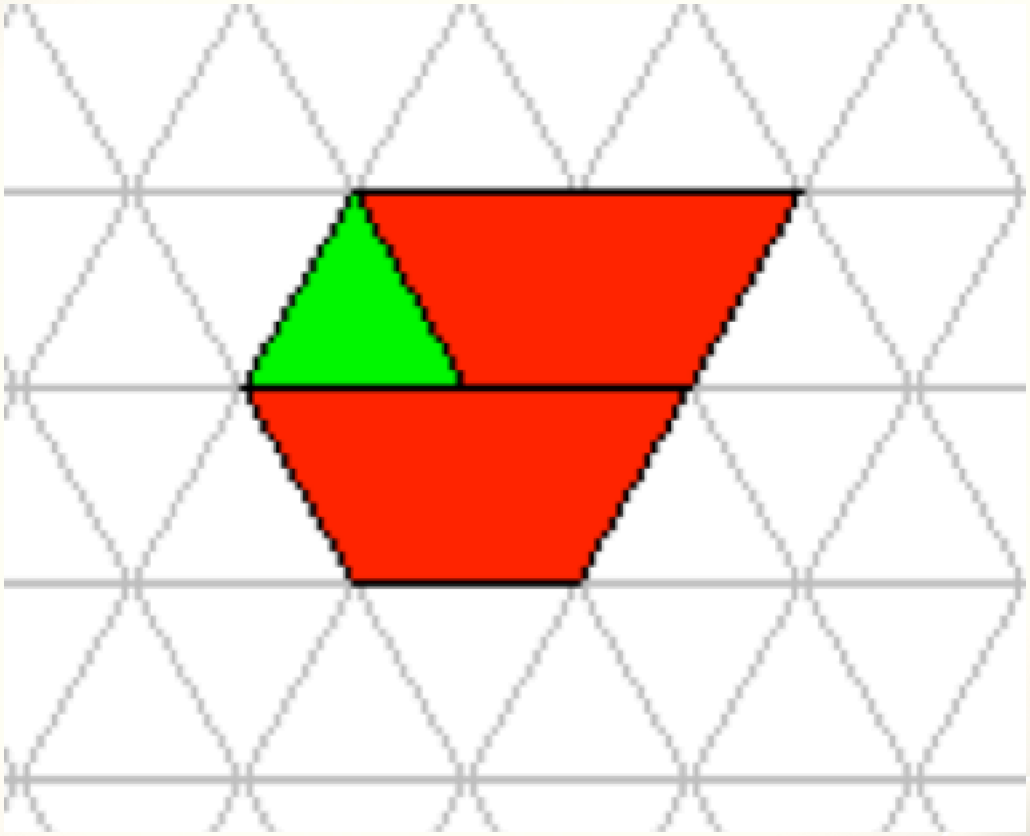
Another design

- Create a design that is 2/3 red and 1/3 green.

2/3 red, 1/3 green



2/3 red, 1/3 green



Questions about 2/3 red

- Were there twice as many red blocks as green? Why or why not?
- What about the design is 2/3 red? Is it area or something else?
- What fraction might someone else use for red, if not 2/3?

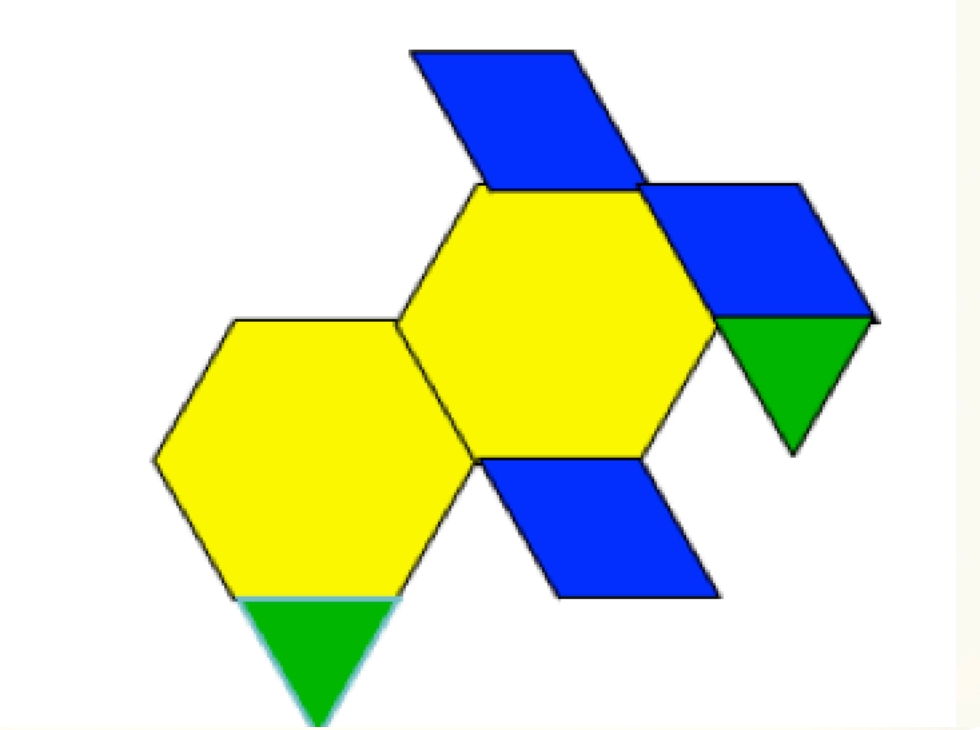
Questions about 2/3 red

- How did you know there were not any other colours?
- How could you use what you did to make another correct design?

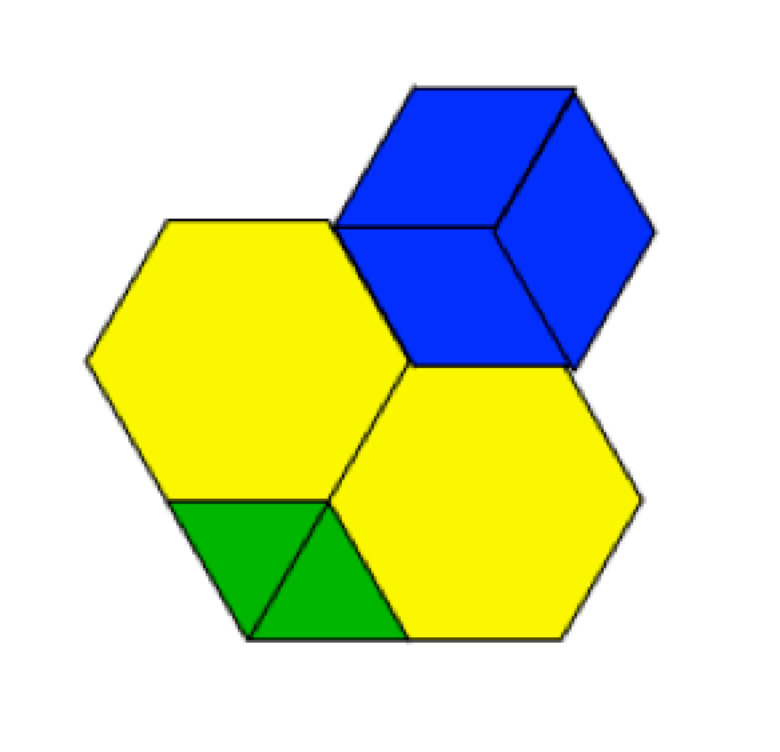
New problem

- Make a pattern block design where:
- The blue area is 3 times as big as the green area AND
- The yellow area is twice as big as the blue area.

Possibility



Possibility



Questions

- What do you notice about the yellow area compared to the green? Why did that happen?
- Was the number of green blocks even or odd? Why?

Questions

- What did you notice about the number of blue blocks?
- Why did that happen?
- What is the ratio of green to blue to yellow blocks? Why?

Another problem

- Start with 15 counters.
- Arrange them into piles so that one pile has twice as many as another.
- How can that be done?

You might ask

- How do you know the piles are not all the same size?
- Could any of the piles be 14?
- Could one of the piles be 1?
- Is there more than one possibility?

Another problem

- Choose one colour of Cuisenaire rod.
- Make a “train” 3 rods long.
- Predict how long a 10 rod train will be.
- Test your prediction.

Questions to ask

- Would it have been easier to predict a different length train?
- Which one? Why?
- About how does your final train compare to the original one?

Another problem

Choose one of these:

- A: If a light green rod is worth 15, what should the other rods be worth?
- B: If an orange rod is worth 15, what should the other rods be worth?

Questions to ask

- What rods were easiest to figure out once you knew the green one? (or orange one)
- Why were they easiest?
- How much was the white worth?
- How did knowing that help you figure out the red?

Questions to ask

- Suppose you knew the black rod was 35. How would that help you figure out the brown rod value?
- Suppose the green rod had been worth 8. What rods would be easy to figure out? Which would not be as easy?

New problem

- Make a rectangle. Figure out its perimeter.
- Then make a rectangle with half the area.
- Figure out that perimeter.
- What fraction of the big perimeter is the small one?
- Try more times. What fractions are possible and which are not?

Questions to ask

- What did you notice about the fractions for the new perimeters?
- Why does that make sense?
- Could the fraction be really close to 1? How?

Possibility

- 4 x 6 and 2 x 6 perimeter fraction is 8/10
- 12 x 4 and 12 x 2 perimeter fraction is 14/16
- 4 x 4 and 4 x 2 perimeter fraction is 6/8

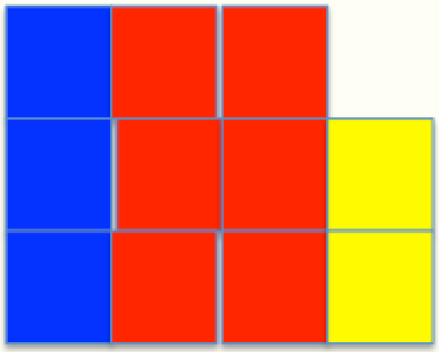
New problem

Use square tiles to build a design where the following are true.

- There are TWICE as many red as blue squares.
- There are 1/3 as many yellow as red squares.

Possibility

- Blue is 3/11, red is 6/11 and yellow is 2/11



Questions to ask

- How many red, blue and yellow tiles are possible?
- What fraction of the design is each colour?
- Are there other solutions?

New problem

- You model a number with base ten blocks.

New problem

- There are twice as many rods as flats.
- There are 1/4 as many rods as unit blocks.
- What could the number be?
- Think of as many numbers as you can that are less than 1000.

Possibilities

- 128
- 256 (2 flat, 4 rods, 16 units)
- 384 (3 flats, 6 rods, 24 units)

Questions to ask

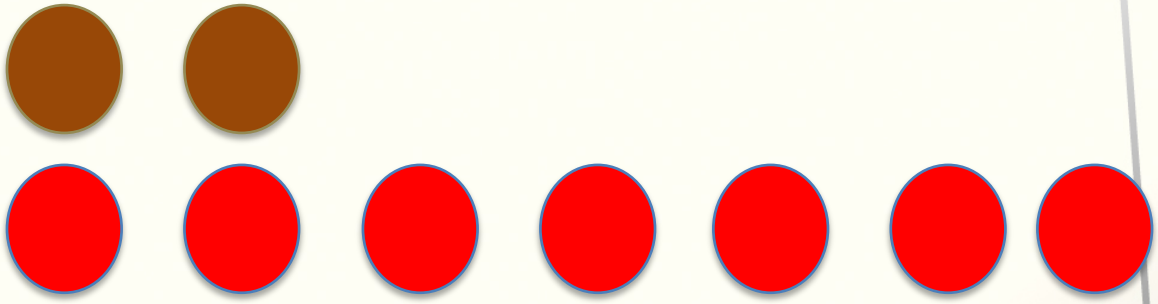
- Is the number of rods even or odd? Why?
- What do you notice about the number of units?
- What is the greatest number of flats possible? Why?

New problem

- Use two colours of counters.
- Create a model where there are 3 ½ times as many of one colour as the other.
- What are all the other fractions you see in that model?

Possibilities

- I also see 2/9, 7/9, and 2/7



Your turn

- This is an opportunity for you to raise and have me respond to your questions about teaching PR or more general questions re math instruction.

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