What should K-8 math look like?

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Agenda

- Classroom environment
- The math to focus on
- The 3-part lesson
- Critical thinking
- Differentiation of instruction
- Games you might use

The classroom

- Lots of student conversation /debate about mathematics
- Kids working in pairs/groups
- Even as a supply teacher, you could set up debates.
- e.g.., I might ask:

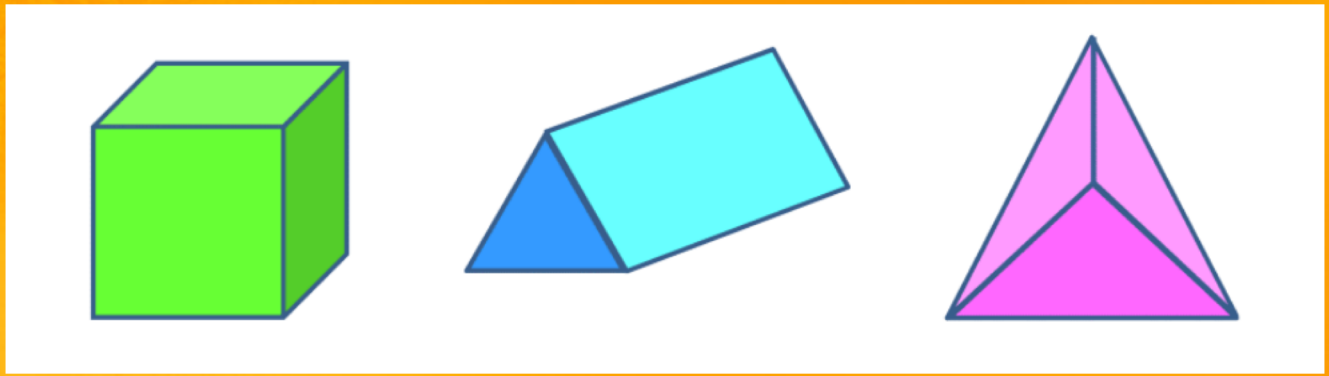
The classroom

- Jason says that you can’t get a little answer when you subtract two big numbers.
- Lia says you can.
- With whom do you agree? Why?

OR

The classroom

- A 3-D figure can have more vertices than edges, sometimes.



OR

The classroom

- Kyle says that when the numerator and a denominator of one fraction are closer together than the numerator and denominator of another, it is greater.
- Lia says that this might not be true.
- With whom do you agree? Why?

OR

The classroom

2 truths and a lie– which is the lie?

1. A number that takes 4 words to say can be greater than a number that takes 7 words to say.
2. A number with more digits is always greater.
3. You can represent 1000 with 19 blocks.

You try

- What might be a fun math debate for you to lead?
- OR
- What might be a good math 2 truths and a lie to try?

The classroom

- A safe environment- mistakes not a big deal
- Lots of use of thinking tools, whether manipulatives or technology

Most valuable tools for number work

- Counters
- Ten-frames
- Cuisenaire rods
- Linking cubes
- Hundred charts
- Base ten blocks
- Hundredth grids
- Thousandths grids
- Fraction pieces
- Pattern blocks
- Integer chips

Counters

- A task might be:

- Take 20 counters.
- How can you arrange them in 3 piles so that the piles are close to equal in size?
- How can you arrange them in 2 piles so that one pile is about double another pile?

Counters

- Take 20 counters.
- How can you arrange them in 3 piles so that there is 1 really big pile and 2 small ones?
- Can can you arrange them into 4 piles so that two piles are equal and small and two are equal and big?

- Make up your own counters challenge.

Counters

- You have red, blue and green counters.
- You have 2 more red ones than blue ones.
- You have 3 more green ones than blue ones.
- How many of each colour might you have?
- How many altogether?

Counters

- You have a bunch of counters.
- Almost half are red.
- How many of what colours might you have?

Counters

- You have 2/3 as many red counters as blue ones.
- You have 1/3 as many yellow ones as green ones.
- How many counters might you have altogether? How many of each colour?

Counters

- You have red, green and blue counters.
- The mean (average) number of counters for a colour is 10.
- How many of each might you have?

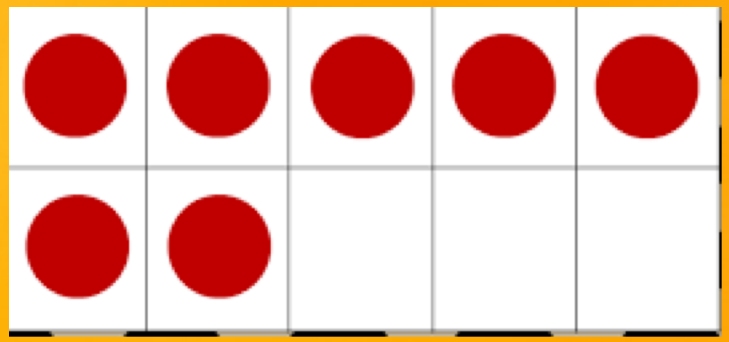
Counters

- You can show a number by putting out counters in the form of a triangle.
- What numbers can you make?
- Which are harder to make?

Ten-frames

You might ask:

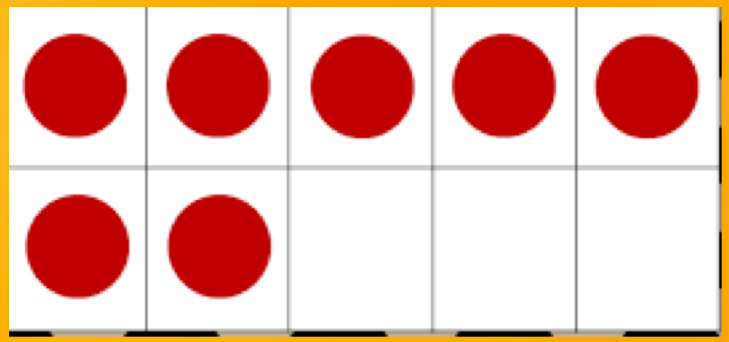
- How could you use 10-frames to help you figure out pairs of numbers that add to 10?
- How could you use 10-frames to show why 8 + 9 = 17?
- What numbers could you show if you fill 3 ten-frames and part of a fourth one?



Ten-frames

You might ask:

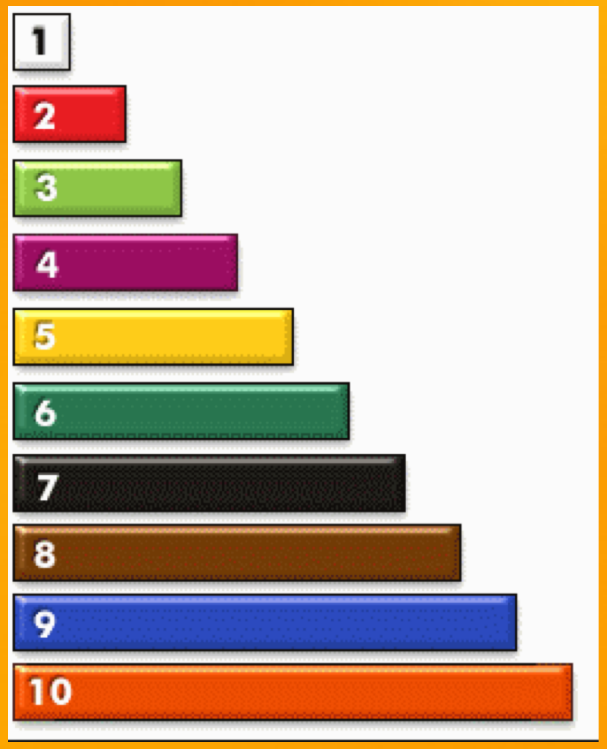
- What ten-frame challenge can you think of?



Cuisenaire rods

You might ask:

- Which pairs of rods show ½?
- If light green is worth 9, what should the other rods be worth?
- How would you show 14 – 8?
- How would you show 4 x 5?
- How would you show 32 ÷ 4?

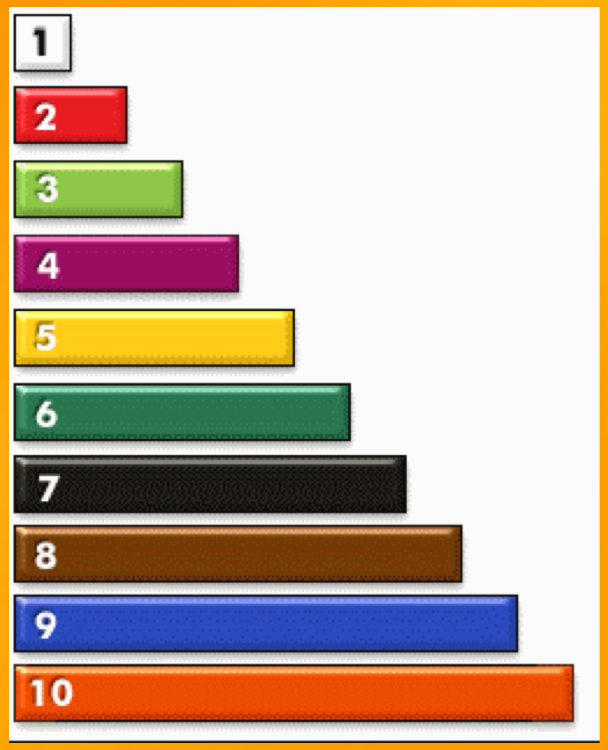


Cuisenaire rods

You might ask:

- How could you use them to find factors of 48?

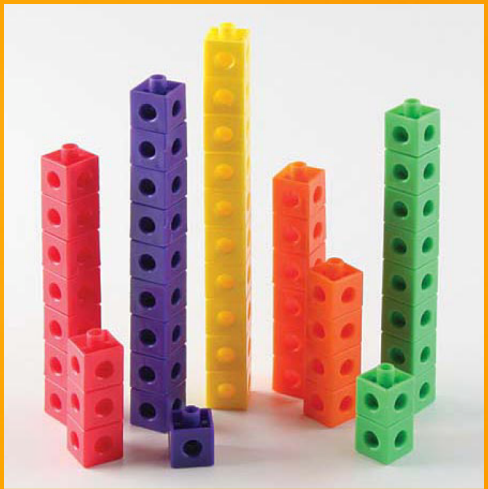
- What challenge could you create?



Linking cubes

You might ask:

- How could you show 43?
- How could you show that 12 – 4 = 13 – 5?
- How could you show that the least common multiple of 3 and 4 is 12?
- What total lengths can you have if you use 3 colours and consecutive amounts for the colours?



Hundred chart

You might ask:

- What number might be in the bottom left quarter of the chart?
- What numbers is 34 near?
- How could you use the chart to add 43 + 25?
- How could you use the chart to figure out 62 – 18?
- What would happen if you coloured all numbers including the digit 2?

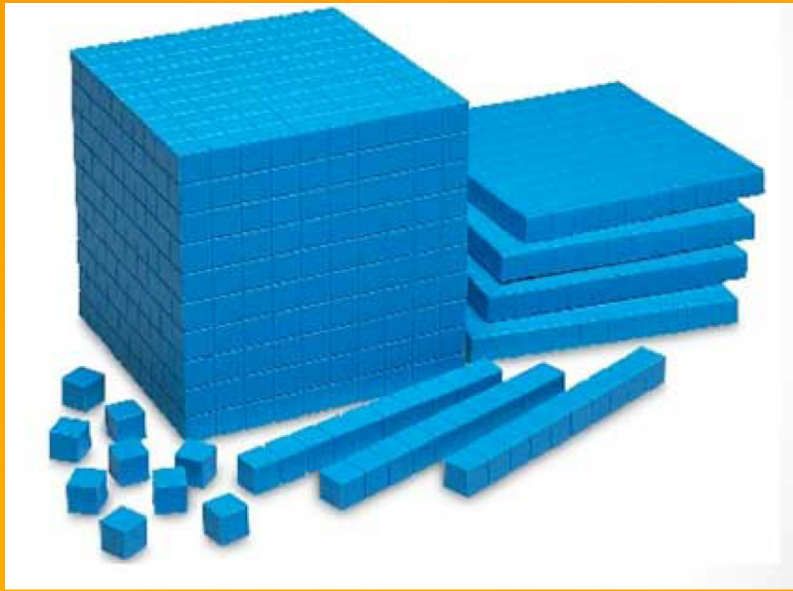
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Base ten blocks

You might ask:

- What numbers can you represent with 12 blocks?
- What numbers can you represent with a lot of sticks and only a few small cubes?
- How can you use the blocks to figure out 132 – 48?
- How can you model :

- 4 x 32?
- 24 x 32?

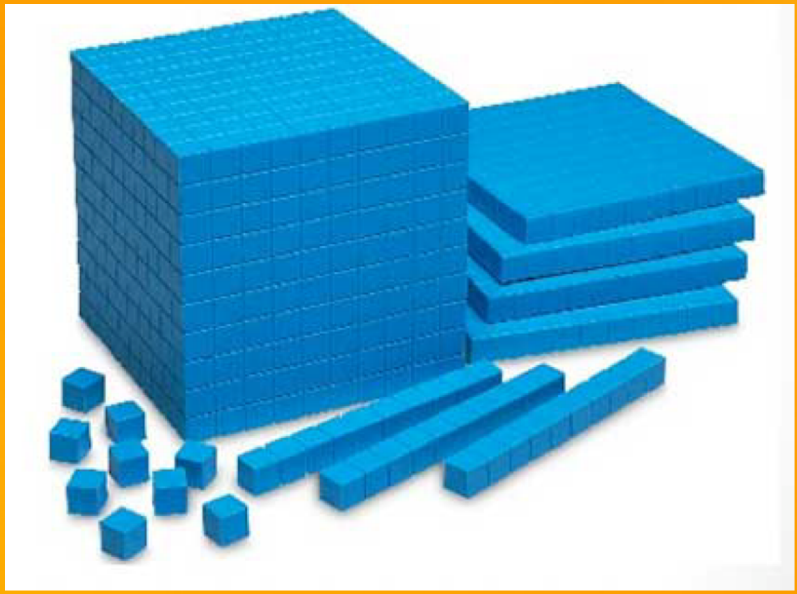


Base ten blocks

You might ask:

- You model a multiplication with exactly 18 blocks. What numbers could you have been multiplying?
- How might you represent

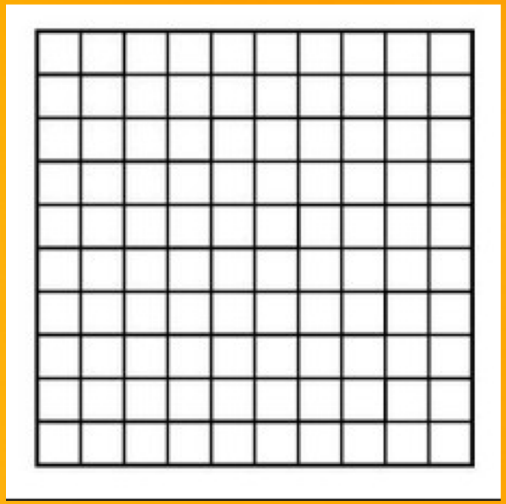
  10 000?



Hundredth grid

You might ask:

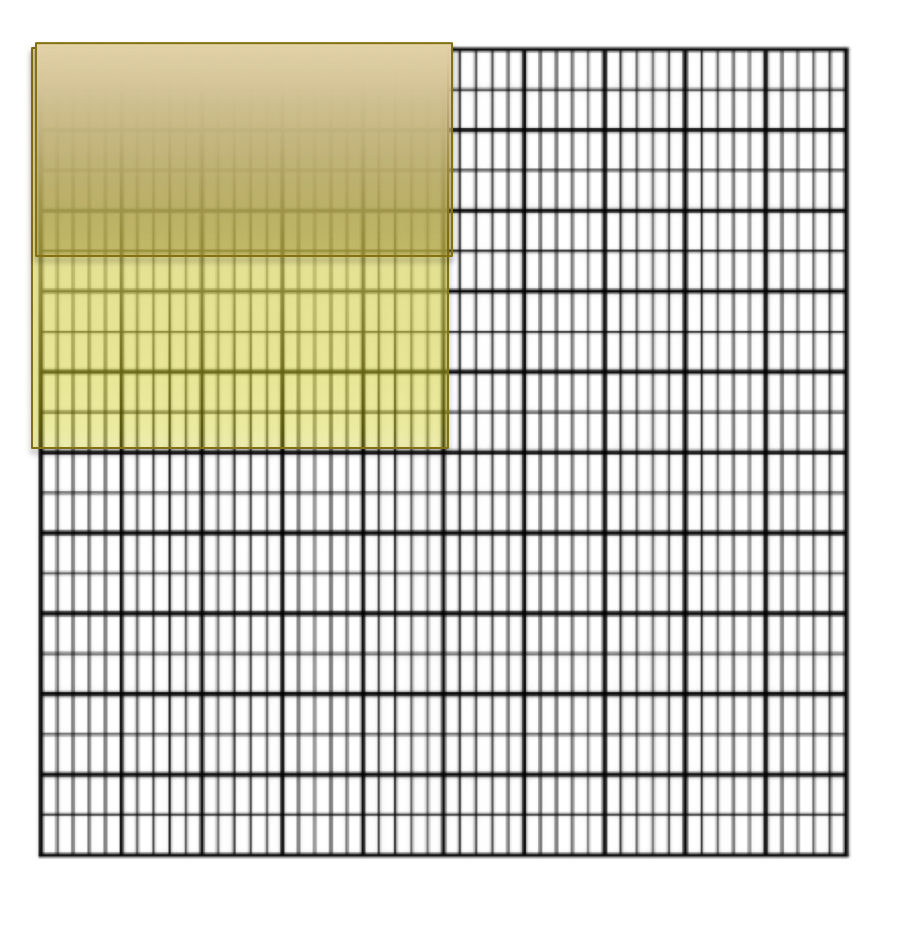
- What decimal is the same as 0.4?
- What decimal is just slightly less than ½?
- What decimal is close to ¼?
- What other challenges might be interesting to kids?



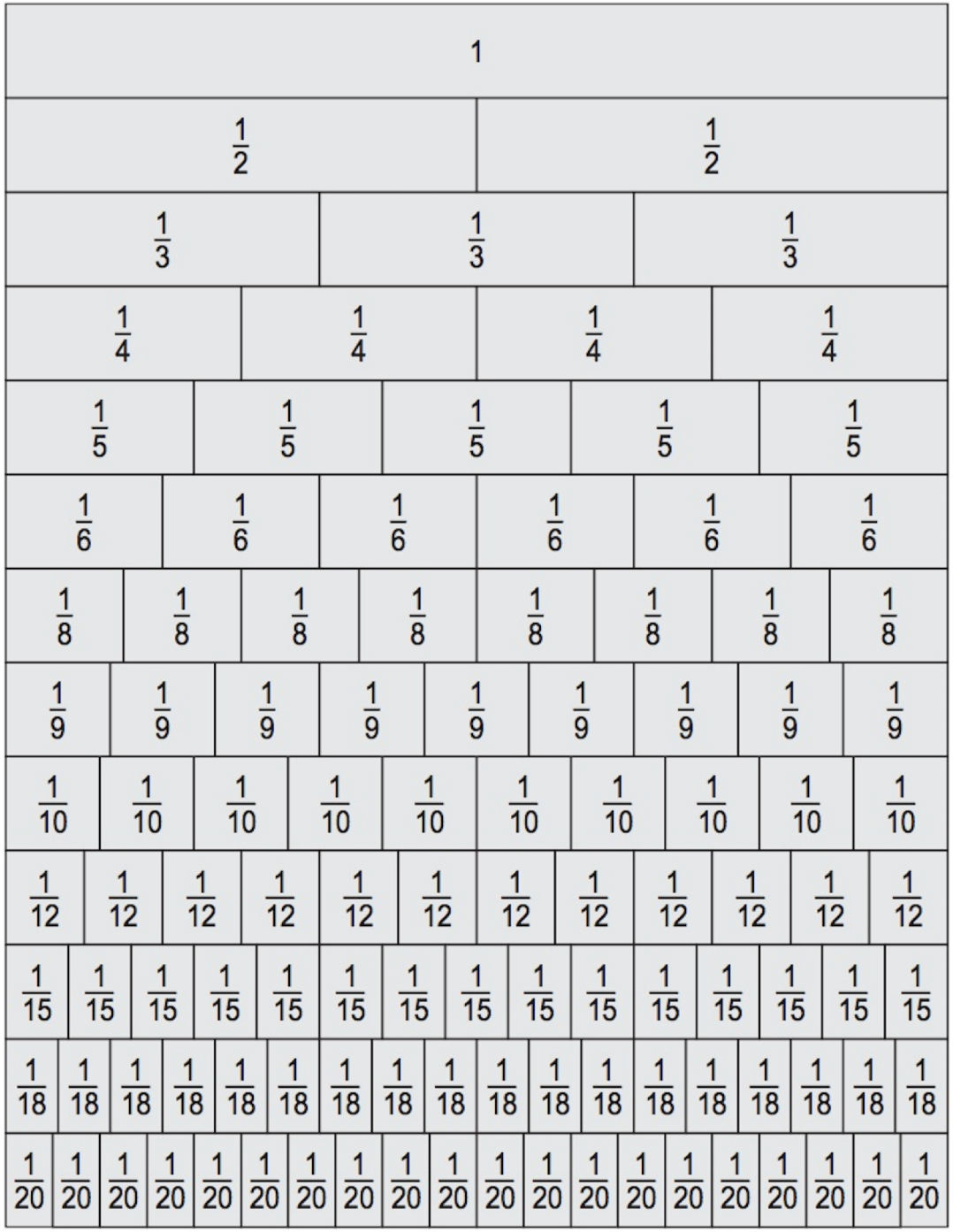
Thousandth grid

You might ask:

- What fraction is 0.125?
- Why does 0.100 = 0.1?
- What decimal is about 2/3?



Fractior


1
1/2 1/2
1/3 1/3 1/3
1/4 1/4 1/4 1/4
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1/6 1/6 1/6 1/6 1/6 1/6
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Fraction Strips

You might ask:

- What equivalent fractions do you see?
- Name fractions a little more than 2/3.
- What fraction is about half as big as which other one?
- What fraction is about 1 1/2 times as big as which other one?

Pattern blocks

- You might ask:
- If red is 1, what fraction is blue?
- If yellow is worth 6, make a design worth 20.
- What blocks could you put together to make your initial? How else could you make your initial?

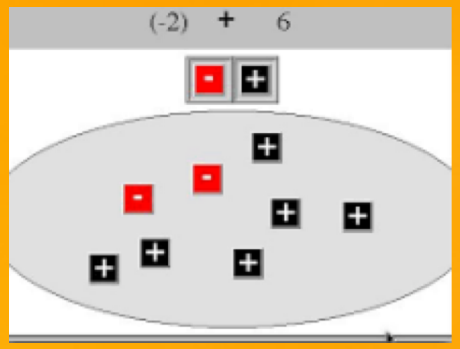
Pattern blocks

- Make a design that is half yellow.
- Make a design that is 2/3 red and 1/3 green.
- Make a design that is ½ blue and 1/5 green.

Integer chips

You might ask:

- If you have a few more –1 chips than +1 chips, what value might your collection be?
- How would you model: 4 + (–2), (–4) – (–2), (–4) – (3)?
- If you add a number you can show with 8 chips to one you can show with 6 chips, how many chips might you need to show the sum?



The classroom

- Teachers leave the “work” to the students.
- High expectations

The classroom

Curiosity is piqued.

- How long is long hair?
- How many text messages does a typical 12 year old send in a year?
- How much recycling does your school do?

- Come up with another mathematical curiosity to investigate.

The classroom

- Accessible tasks with high ceilings

I might ask:

- Choose two numbers so that when you add them, the answer is double when you subtract them.
- Come up with at least a few answers.

- What if the sum is triple the difference?

The classroom

- Feedback – more questions, less marks

The math to focus on

- The seven processes require attention no matter the grade level.
- Students need opportunities to:
- Problem solve
- Communicate
- Reason
- Reflect
- Connect
- Represent
- Select tools and strategies

The math to focus on

- Each expectation requires “interpretation”.
- Teachers have to decide they are focusing on what is important.
- For example… Grade 1:
- Estimate, measure and record lengths, heights and distances
- What are the most important markers for you that the student has been successful on this expectation?

It might be

- How reasonable are their estimates? (i.e. can they relate two measurements to each other)
- Do they use unit size as a factor?
- Do they recognize bad measuring vs good measuring?

It might be

- Do they have a good sense of which attributes matter and which don’t when they measure?
- Do they realize units need to be the same size and why?
- Do they choose appropriate units?

It might be

- Do they know what to do if they don’t have enough units?
- Do they know what to do if the fit is not perfect?

So a task might be

- Maybe we provide some paper clips–maybe 4 of one size and 5 of another.
- Ask them to predict something that will be about 20 paper clips long.
- Explain their prediction.
- Test it.
- Then ask how the answer would change with this paper clip? (show a giant one)

Or it might be Grade 4

- Describe relationships that involve simple whole number multiplication
- What are the important markers?

It might be

- That some numbers are whole number bunches of other numbers, but some are not.
- That one number can be, e.g. twice another and A LOT more than it or NOT A LOT more than it.
- That visually saying one number is twice or three times or four times another is saying that if the first number of counters is in each box, I have 2 or 3 or 4 of those boxes.

So my task might be

- You add a number to its triple.
- List a bunch of possible sums- some big and some small.
- List a bunch of numbers that could not be the sums- some big and some small.
- Why do these answers make sense?

Or Grade 7

- Make predictions about linear growing patterns, through investigation with concrete materials

I might consider

- Maybe that they recognize how multiplication is useful in predicting future terms in patterns
- Maybe it’s relating this pattern to an easier one to help predict future terms in the pattern
- Maybe it’s recognizing that prediction might be enhanced by visual representations

So I might ask

- I created a pattern by adding the same number of items to the figure each time to get the next one.
- One figure used 35 items.
- Another used 59 items.
- What could the pattern have looked like?
  What could the increase have been? NOT been?
- How could you predict the 100th term?

The 3-part lesson

- Getting started
- Action
- Consolidation

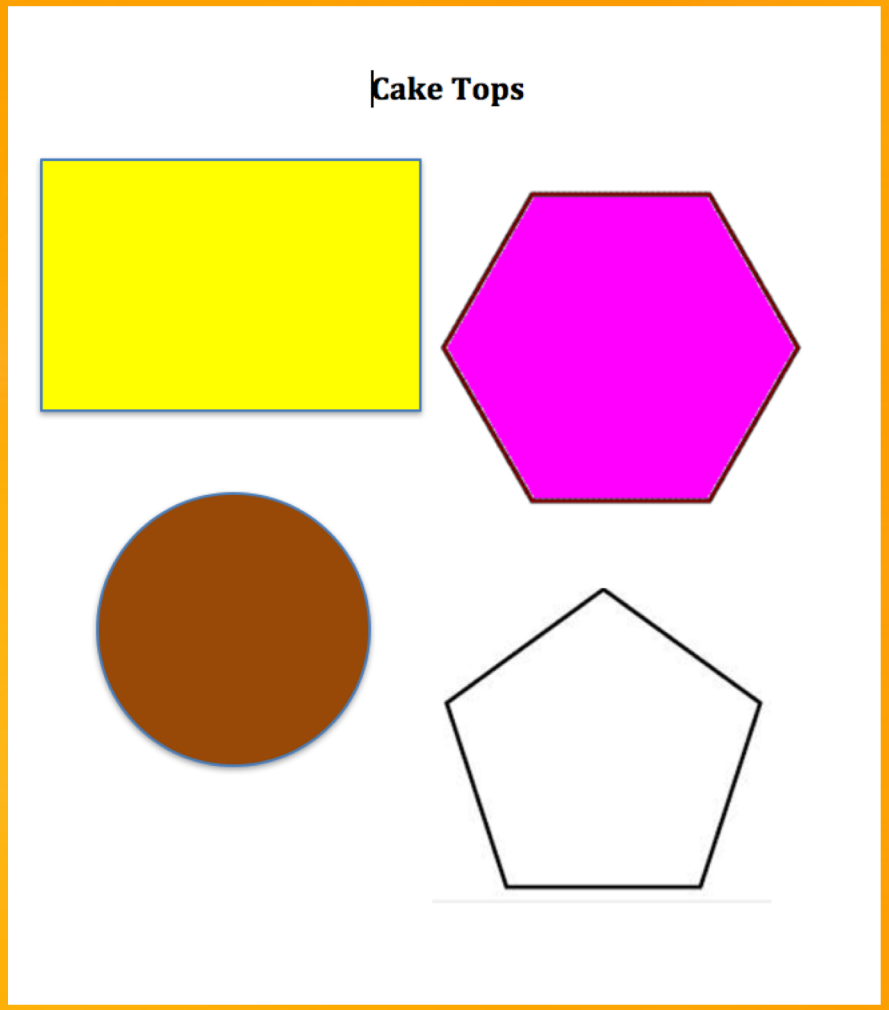
- The action is a problem that is focused on a meaningful learning goal.
- The consolidation is planned to bring out the big ideas based on the work on the action but about the learning goal.
- The getting started is planned last to provide a way into the action.

An example

- **Getting Started:**
- <We provide kids with some fraction materials, e.g. four colour square tiles, fraction strips, fraction circles; pattern blocks; a number line from 0 to 1>
-
- Ask: What are some things you know about the fraction two thirds?
- Show what two thirds might look like.

An example

- **Main Task:**



An example

- Cut each cake into a DIFFERENT NUMBER of slices.
- For each cake, decide on how many slices to keep and how many to give away, but:
- You have to give away MOST of the cake.
- You have to give away a DIFFERENT NUMBER of slices for each cake.
- Tell what fraction of the cake you gave away and what fraction you kept. And how you know those fractions are correct.

An example

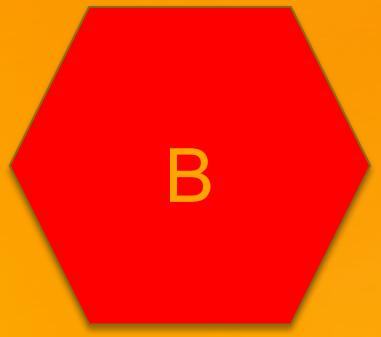
- Did the shape of your cake make any different in deciding how many slices to cut it into? Explain.
- Look at the fractions you wrote.
- What does the denominator of each fraction tell?
- What the numerator tell?
- Were the numerators and denominators closer together for the slices you kept or the slices you gave away? Why?

An example

- Look at the pairs of fractions you wrote for each shape.
- What do you notice about the numerators in each pair?
- Suppose you cut the cakes into a different number of equal slices. Would your fractions change? How?
- What fractions would you use if you gave away the whole cake?

Critical thinking

- A third shape is more like shape A than shape B.
- What might it look like?



Critical thinking

- There is a plate with a LOT of cookies.
- You think it is VERY EASY to share them equally onto either 3 plates or 5 plates.
- How many cookies might there have been?

Critical thinking

- You show a number with a LOT more ten rods than one cubes.
- What could the number be?

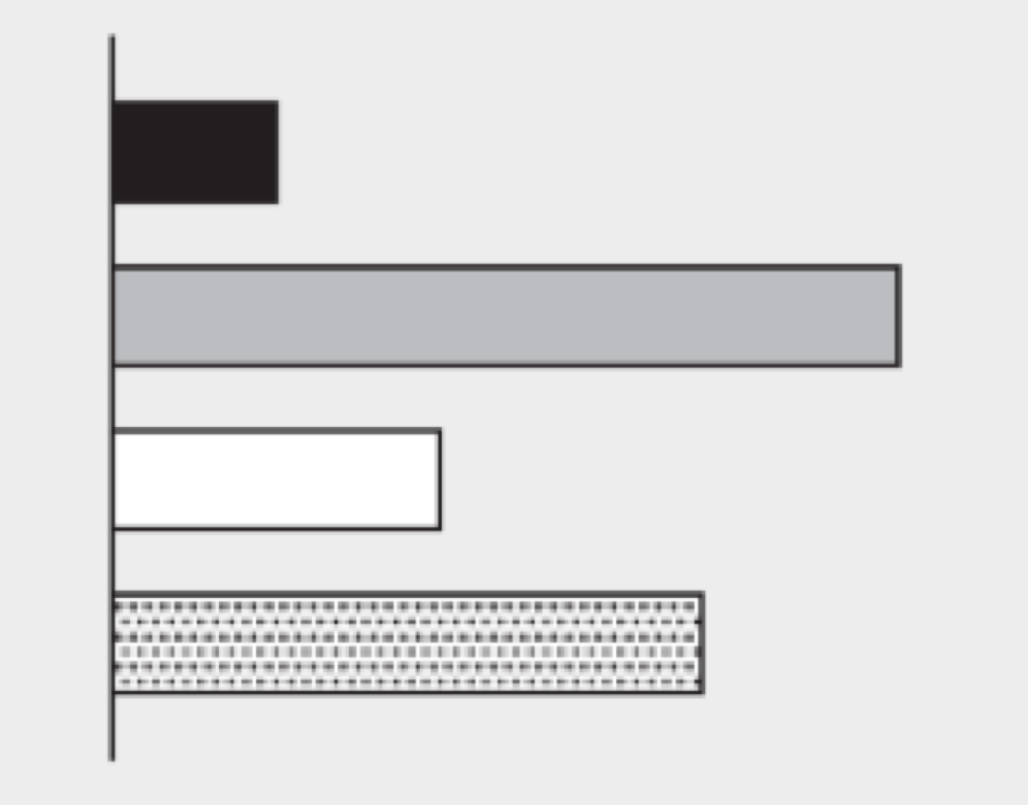
Critical thinking

- An amount you can show with 6 coins is added to an amount you can show with 3 coins.
- How many coins might you NEED to show the sum?

Critical thinking

- Which is a better description of how far away your birthday is?
- Is it 50 days?
- Is it about 7 weeks?
- Is it almost 2 months?
- Why is it better?

Critical thinking



Critical thinking

- How could 1000 be a lot?
- How could it be a little?

Critical thinking

- Who doesn’t belong?

- 11
- 14
- 23
- 17

Differentiation

- The strategy I count on the most is the use of open questions.
- I also use parallel tasks.

Open questions

- The answer is 100.
- What might the question have been?

Maybe

- How old is really old?
- What is the first 3-digit number?
- What is a number with a digit sum of 1?
- What is 10 x 10?
- What comes after 99?
- What is a perfect score on a test?
- What is an easy number to multiply by?

More examples

- There are a LOT of people in a car. How many might that be?

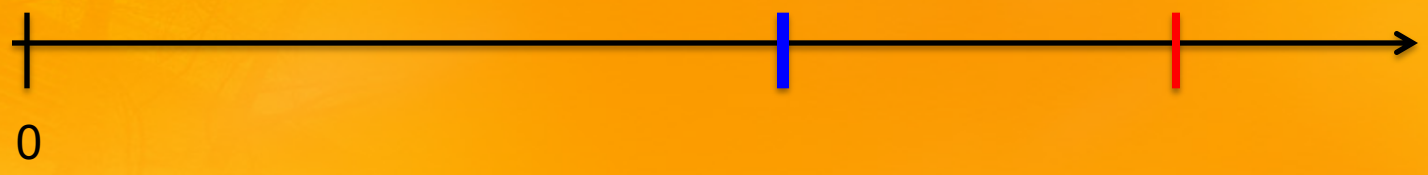
- There are NOT MANY students in a school. How many might there be?

More examples

- The 10th shape in a pattern is a red triangle.
- What could the pattern look like?

More examples

- Choose a number for the red mark. Then choose an appropriate number for the blue mark.



More examples

- A number is just A LITTLE more than ½.
- What might it be?

More examples

- ____ is 4 times as much as _____.
- What could go in the blanks to make this true?

- Then, ask:
- Was the first number bigger or the second one?
- Could they have been equal?
- Was the second number more or less than half of the first?
- Could the first number have been 20?
- Could the second one?
- How about 21?

More examples

- ____ is 2/3 of _____.
- What could go in the blanks to make this true?

- If you used whole numbers….
- Was your first number even or odd? How about your second one?
- Could your second number have been 8?
- Could your second number have been 9?

More examples

- Two fractions are equivalent.
- One has a numerator and denominator that are fairly close together.
- The other has a numerator and denominator that are really far apart.
- What could they be?

More examples

- In a certain number, there are two 5 digits and two 8 digits.
- One of the 5 digits represents a number 10 times the amount of the other.
- One of the 8 digits represents a number 100 times the amount of the other.
- What could the number be?

More examples

- Two decimal numbers are not that far apart.
- Their sum is 95.1.
- What could they be?

More examples

- You buy an item and give the clerk one bill. Your change is one bill and 6 coins.
- What might the price have been?

Parallel tasks

- Very similar tasks that differ only in complexity, but address the same ideas.
- They can be debriefed using common questions.
- For example…

Choice 1: Choose a number to subtract from 20. Draw two different pictures that would help you figure out the result.

Choice 2: Choose a number to subtract from 5. Draw two different pictures that would help you figure out the result.

- *How did you decide what number to choose?*
- *Could you have predicted about how much your answer would be?*
- *How do the pictures show the subtraction?*
- *How are your pictures alike? How are they different?*

Choice 1: Choose two 3-digit numbers to add for a total close to 400. How could you use base ten blocks to help you add?

Choice 2: Choose two 2-digit numbers to add for a total close to 40. How could you use base ten blocks to help you add?

- *How would you model the total you were trying to get close to?*
- *What would you do to show a number close to that number?*
- *How did you select the two numbers to make that total?*
- *How did using the blocks make the problem easier to solve?*

Lisa's dad was driving 16 miles every 15 minutes.

Choice 1: How far would he drive in 20 minutes?

Choice 2: How far would he drive in 2.5 hours?

- *Why was it more helpful to know the distance for 15 minutes than for, say, 17 minutes?*
- *What other numbers of minutes would it be very easy to figure out the distance for? Why those amounts of time?*
- *Is it possible to figure out the distance for any number of minutes or only certain numbers?*

Math Games

- You want a combination of strategy and luck sometimes and just luck other times.

Simple game

- You want a combination of strategy and luck sometimes and just luck other times

- Roll two dice.
- Double one value and add the other.
- The first person to get to 50 wins.

Simple game

- Roll four dice.
- Create two 2-digit numbers.
- Subtract.
- Your point value is the tens digit of the difference.
- First person to 50 points wins.

Simple game

- Three players each roll two dice and make a 2-digit number.
- The number that is in the middle wins.

Simple game

- Seventy percent of Earth is covered with water. Test this statistic by having students stand in a circle and toss an inflatable globe to one another. When a student catches the globe, record whether the student's left thumb is touching land or water. That student tosses the ball to a classmate and then sits down. Once everyone is seated, determine the ratio of the number of times students' thumbs touched water to the number of times they touched land. Record the ratio and repeat the activity on other days. (Over time, the ratio should be fairly close to 7 to 3, or 70 percent.)
  Challenge: Predict the probability that someone's thumb will land on any of the continents based on the ratio of the area of each continent's landmass to that of

The Fa

Four in a Row

With factors 1-9

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

1 2 3 4 5 6 7 8 9

Your turn

- Make up a simple game focused on one of:
- Multiplying little numbers
- Integers
- fractions

Any questions

- Do you have additional questions you would like to discuss?

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- www.onetwoinfinity.ca
- Recent presentations
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