What does math leadership look like?

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Last year, I asked leads about their needs

- Real-life context problems
- Building confidence (not sure if it’s kids’ or theirs)
- Plan for most “bang for “buck”
- Parent information

I asked leads about their needs

- **Building culture of deep thinking**
- **Making sense of problems**
- **Effective assessment practices/efficient assessment**
- **How to be agents of change**

Building a culture of deep thinking

- Would like to propose we “adopt” from the U.S. their standard for mathematical practice:
- Construct viable arguments and critique the reasoning of others
- I think the heart of math really is reasoning, so this makes sense to me.

So what might this standard look like

- We will look at different grade bands.
- We will look at two strategies that I think might be interesting.
- One is the use of debates.
- One is the two truths and a lie strategy.

Let’s look at some debates

- Jason says that you cannot get a little answer if you subtract two big numbers.
- Lia says you can.
- With whom do you agree? Why?

Let’s look at some debates

- Jason says that if you add two numbers in a row (like 8 and 9 or 123 and 124), the answer is sometimes even and sometimes odd.

- Jill says no.
- With whom do you agree?

Let’s look at some debates

- Mel says: You can add an amount that you can show with 5 coins to an amount that you can show with 6 coins and get an amount that you can show with 9 coins.
- Kyle says: You need 11 coins.
- With whom do you agree?

- LET’S TRY THIS ONE.

Let’s look at some debates

- Sam: It is not possible to multiply two fractions and get a product with a smaller denominator than the ones you started with.
- Kevin: Yes, you can.

- With whom do you agree?
- LET’S TRY THIS ONE.

Let’s look at some debates

- René: If the 5th term of a growing pattern is 20 more than the 2nd term, then the 15th term must be 20 more than the 12th term.
- Justin: It doesn’t have to be 20 more.
- With whom do you agree?

Let’s look at some debates

- Maude: If you halve the area of a rectangle, the new perimeter could be any fraction of the old perimeter.
- (E.g. It is ¾ if you go from a 4 x 4 to a 4 x 2.
- Natalie: I disagree- the fraction has to be more than ½.
- With whom do you agree?

- LET’S TRY THIS ONE.

Let’s look at some debates

- Grace: A 3-D figure can have more vertices than edges sometimes.
- Andrew: There have to be more edges.

- With whom do you agree?

Let’s look at some debates

- Kyle says that when the numerator and a denominator of one fraction are closer together than the numerator and denominator of another, it is greater.
- Lia says that this might not be true.
- With whom do you agree? Why?

- LET’S TRY THIS ONE.

Let’s look at some debates

- Julie: Every function of the form $y = a \sin(x - h) + b$ can be renamed as a function of the form $y = c \cos(x - k) + d$.
- Xavier: Some can, but not every.

- With whom do you agree?

2 truths and a lie

- You can add 3 numbers in a row and get the number 74.
- You can add 4 numbers in a row and get the number 74.
- You can add 5 numbers in a row and get the number 75.

- LET’S TRY THIS ONE.

2 truths and a lie

- There are fewer days in 3 weeks than there are hours in a day.
- There are more weeks in 2 years than there are days in 3 months.
- There are more minutes in 3 hours than there are decades in 2 centuries.

2 truths and a lie

- 1. A number that takes 4 words to say can be greater than
- a number that takes 7 words to say.
- 2. A number with more digits is always greater.
- 3. There are more numbers between 100 and 200 that take 4 words to say than 3 words to say.

2 truths and a lie

- 1. There are more ways to divide up 72 into equal groups than 60.
- 2. There are more ways to write 60 as a product of factors than to write 50 as a product of factors .
- 3. There are more multiples of 8 between 100 and 200 than multiples of 10.

2 truths and a lie

- 1. There is a fraction equivalent to 5/11 where the denominator is between 80 and 100.
- 2. There is a fraction equivalent to 5/11 where the numerator and denominator are 64 apart.
- 3. There is a fraction equivalent to 5/11 where the numerator is even.

- LET’S TRY THIS ONE.

2 truths and a lie

- 1. A picture with only red and blue circles can show the ratios 3:5 and 3:8 at the same time.
- 2. A picture with only red and blue circles can show the ratios 3:5 and 5:8 at the same time.
- 3. A picture with only red and blue circles can show the ratios 1:4 and 2:3 at the same time.

- LET’S TRY THIS ONE.

2 truths and a lie

- Suppose A = 30% of B.
- 1. A = 60% of 2B.
- 2. 2A = 30% of 2B.
- 3. A/2 = 15% of B.

2 truths and a lie

- 1. A cone and a pyramid cannot have EXACTLY the same volume.
- 2. The volume of a pyramid can be 2/3 the volume of a prism.
- 3. The volume of a pyramid can be 3 times the volume of a hexagonal prism.

2 truths and a lie

- 1. The cosine of an angle close to 45° is very close to the sine of that angle.
- 2. The tangent of an angle close to 90° is very close to the cosine of that angle.
- 3. The tangent of an angle close to 0° is very close to the sine of that angle.

2 truths and a lie

- 1. It is possible to show the number 534 with 21 base ten blocks.
- 2. It is possible to show the number 121 with 13 base ten blocks.
- 3. It is possible to show the number 1048 with 23 base ten blocks.

- LET’S TRY THIS ONE.

Do you see it possible to…

- Teach teachers how to create debate or 2 truths and a lie questions rather than depend on examples we give them?

More on building a culture of deep thinking

- Teachers NEED TO focus less on solutions to problems and more on the thinking that gets to those solutions.

- That means that even when they offer a problem for students to solve, the discussion focuses more on the thinking than the solution.

- And, of course, the problems need to generate thinking.

Here is a problem for you

Trading Coins

I represent an amount of money with 8 coins.

I represent the same amount with 22 coins.

What coins might I have had each time?

Here is a problem for you

- What numbers can you write as combinations of some or all of ONLY the numbers 6, 9 and 20.

- YOU TRY THIS FIRST FOR A LITTLE WHILE

Here is another example

- There are 2 small piles of counters.
- There are 2 bigger ones.
- Each big one is twice as big as a small one.
- How many counters could there be in total?

- YOU TRY THIS FIRST.

Here is another example

- You build a rectangle with square tiles.
- You cut it in half, based on area.
- Could the new perimeter be half of the old one?
- What fraction could it be?

Here is another example

- The problem is: An item that was 40% off cost the same as an item that was 20% off.
- How were the original prices related?

A caution

- I worry a little about the current focus on I notice and I wonder.
- I see the value, but not convinced it fosters deep thinking.

Working against us

- I am not convinced that most teachers actually do value deep thinking as opposed to just being able to ”do things” correctly.

- This may require curriculum changes???

Making sense of problems

- The presumption here is that the obligation is on the student’s part to figure out what we mean and not for us to make it clear what we mean.

- Am I being unreasonable about this?

Effective asssessment practices

- Some of the issues, I think are:
- Collecting LESS data from EACH kid EACH day.
- Using OBSERVATION/CONVERSATION data as a bigger proportion of what we care about so we have to learn what to observe and ask about.
- Asking fewer repetitive questions

Effective asssessment practices

- Focusing more on understanding/thinking questions than knowledge/application questions.

- Here are examples.

Some examples

- **Adding small numbers questions**
- **Knowledge:** What is 28 + 36? Show your work.
- **Understanding:** You add two 2-digit numbers and the answer is 3—digit. How can that happen? Does it happen a lot?
- **OR**
- You add a number to its double and the answer is close to 50. What could the numbers be?

Some examples

- **Equivalent fraction questions**
- **Knowledge:** Write three equivalent fractions for 5/6.
- **Understanding:** Describe a problem where it would be useful to know an equivalent fraction for 5/6.
- **OR**
- How do you know that 12[]/38[] cannot be an equivalent fraction for 5/6 without actually doing any work?

Some examples

- **Subtraction of integers:**
- **Knowledge:** What is –42 – (–17)?
- **Understanding:** You subtract a negative number from a negative number? Could you get a positive answer? How or why? OR
- Draw a **picture** that shows **why** –8 – (–3) has to be –5?

Some examples

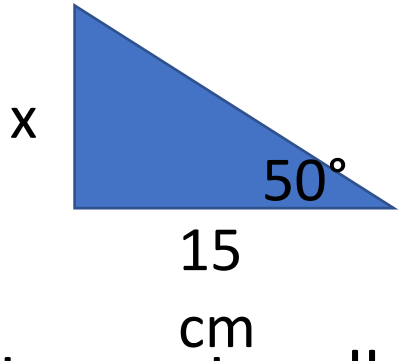
- **Solving percent problems**
- **Knowledge:** 35 is 45% of a number. What is the number?
- **Understanding:** The number A is 35% of the number B. What percent is it of the number 2B?
- OR
- 120% of one number is 30% of another number. What could the two numbers be?

Some examples

- **Solving linear relations problems**
- **Knowledge:** What is the equation of a line with a y-intercept of 5 that goes through (2,5) and (3, 6)?
- **Understanding:** A line goes through Quadrants II, III and IV. What could its equation NOT be? Why? OR
- Describe a real-life situation that can be represented by the equation $y = 50x + 400$.

Some examples

- **Trigonometry**
- **Knowledge:** What is the length of x?



- **Understanding:** For what angle is the tangent really close to the sine? Why does that happen? OR
- Suppose you did not know that the base in the triangle (above) was 15 cm. Could you find the length of x? Why or why not?

NOW LET’S LOOK BIGGER PICTURE

- Suppose we looked at a tool you might use to evaluate student knowledge in a topic.

- Create something that you think might be appropriate for
- ONE OF
- Grade 3 place value
- Grade 7 addition/subtraction of fractions
- Grade 9 measurement- volume and surface area

NOW LET’S LOOK BIGGER PICTURE

- Compare what you did with what you will see me do.

- What do you think about my suggestions?
- What about them do you like?
- What don’t you like?
- What would you change?
- How close is it to yours?

Grade 3 Place value

- 1. List three 3-digit numbers for each situation:
- A) a number more than 32 tens.
- B) a number with one 4 digit in it that is worth ten times as much as the other 4 digit
- C) a number you can represent with 18 base ten blocks.
- 2, A46 < BC7 . A, B and C are digits (0, 1, 2,….9).
- Which of these can be true? Explain.
- A < B < C A= B = C
- B < A < C C < A < B

Place value

- 3. Model the number 512 on a place value chart or using base ten blocks in 3 different ways.

Grade 7 Fraction + and –

- Choose two fractions less than 1. The denominators have to be different. The sum has to be more than 1. Figure out their sum and their difference.
- Repeat Question 1.
- Draw a picture that shows why 3/5 + 1/3 = 14/15.
- Create a problem that would be solved by figuring out 2 1/3 – 1 ¾.
- Explain why 3 ½ – 1 2/3 = 1/3 + 1 ½ without actually getting the answer.
- Two fractions add to 1/18, What could they be?
- You add two fractions with different denominators. and the numerator turns out to be 3. What could the fractions be?

Grade 9 Volume & surface area

- 1) Which of these is true and which is not? Explain your reasoning.
- - If you know the height of a cone and the circumference of the base, you can figure out the surface area.
- - If you know the diagonal length of a square prism and its height, you can figure out its volume.
- - If you know the surface area of a sphere, you can figure out its volume.

Volume & surface area

- Choose ONE of questions 2 and 3.
- 2) A sphere, a cone and a cylinder have the same volume. Decide what the volume is and tell the radius of each and the heights of the cone and cylinder.
- 3) A cylinder and a prism have the same surface area. Decide what the area is and tell the radius and height of the cylinder and the area of the base and height of the prism.

- 4) A cylinder has a surface area that is a WHOLE LOT of square centimetres, but a volume that is not many cubic centimetres. What could the radius and height be? Explain.

So …

- Which of these things do you think we can convince teachers to change?
- Moving toward more understanding/less knowledge questions
- Using more observation/conversation data
- Collecting less data
- Focusing on understanding and thinking

Being an agent of change

- I am not sure math leads are expected to be an agent of change.
- I am not sure this is even possible without the support of department head and actually the admin.
- I think we can model, but we can’t force.
- I think we have to offer options, and not make demands.

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