Computational and Estimation Strategies for K – 3

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Generally

- Computational fluency comes from a strong sense of number.
- Students need to have a sense of number size.

Generally

- Students need to be flexible in composing, decomposing, and recomposing numbers.
- Students need to be able to relate computations.

Generally

- In terms of calculations, students need to be able to estimate and calculate, but they also NEED to know when operations apply.

So I would ask

- Draw me a picture that shows what 8 + 4 means (not what the answer is).
- Or 8 – 4
- Or 8 x 4
- Or 8 ÷ 4

They need situations of all of these types:

- a + b = []
- a + [] = c
- [] + b = c
- a – b = []
- a – [] = c
- [] – b = c

Addition & Subtraction

- Learning Facts
- Critical pieces are:

Relating addition and subtraction

- Students need to think of subtraction as the opposite of addition.
- Every subtraction is a missing addition.
- That means subtraction facts need not be memorized.

So to model 5 – 2

- I want students to think of having 2 and working up to 5 and not just starting with 5 and taking away 2.

Commutativity

- I need students to understand WHY numbers can be added in any order.

- By realizing this, students need to learn only slightly more than half the facts.

Adding or subtracting 0, 1, or 2

- I need students to think of these situations as counting situations.

The smaller facts

- Students need MANY opportunities to add small amounts on number lines, with counters, on five-frames, with Cuisenaire rods….
- They need to commit those small facts to memory through those experiences.

The smaller facts

- Then these facts can be used to help figure out other ones.

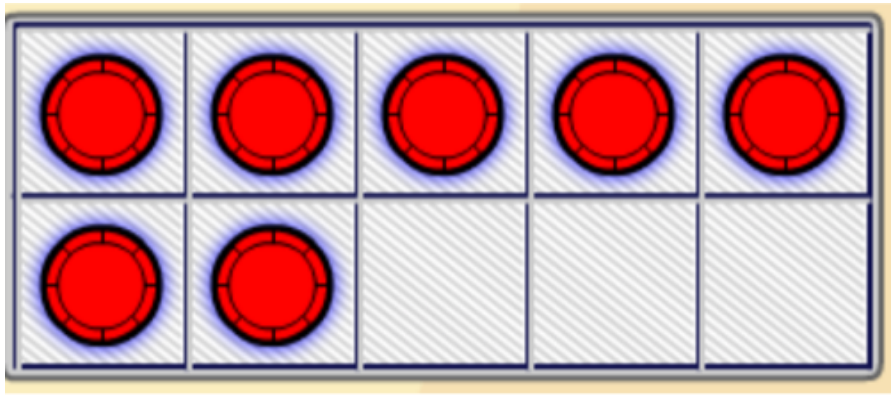
- For example, if I know 2 + 3 = 5, then I know 7 + 3 is 5 more, or 10.

Combinations for 10

- I need students to realize that if 5 + 5 = 10 (and most kids are good with that one), then so are 6 + 4, 7 + 3, 8 + 2 and 9 + 1, just moving an item from one pile to the other, one at a time.

Using the ten-frame

- Using the ten-frame is so important to help students see combinations for 10.



The 10 + facts

- I need students to use what they know about what 13 or 14 or 15, etc. means, to figure out why 10 + 3 is no work- it's what 13 is.

Juggling

- Students need to realize that it is always possible to move items from one addend to the other without changing the sum.
- It’s kind of like juggling the total number of counters in your hands.

So here is where relationships help

- Instead of 9 + 8, I would rather do 10 + 7.

- Instead of 8 + 7, I would rather do 7 + 7 and then 1 more, or I might rather do 10 + 5.

I believe

- That even though children might do 9 + 8 as 10 + 8 and then subtract 1 or 10 + 10 and then subtract 1 and then 2, there are likely fewer errors if students think of 9 + 8 as 10 + 7.

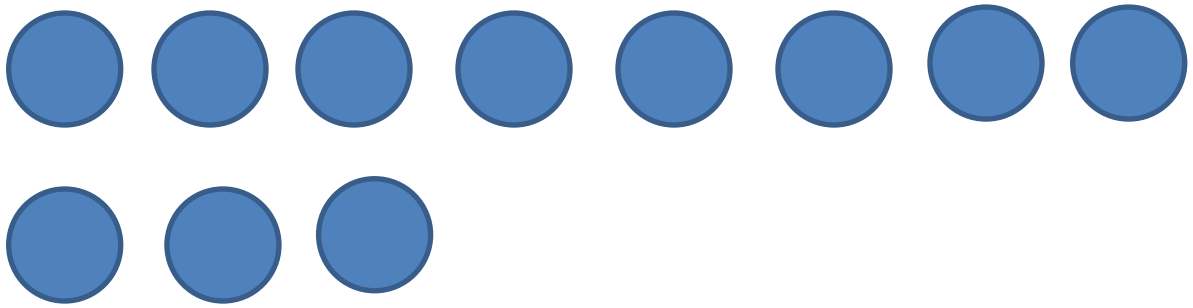
For subtraction

- I want students to realize that the difference does not change if I increase both amounts by the same value.

- I can show this on a number line or with counters.

Constant difference: counters

- 8 – 3 is the same as 10 – 5.



Constant difference: counters

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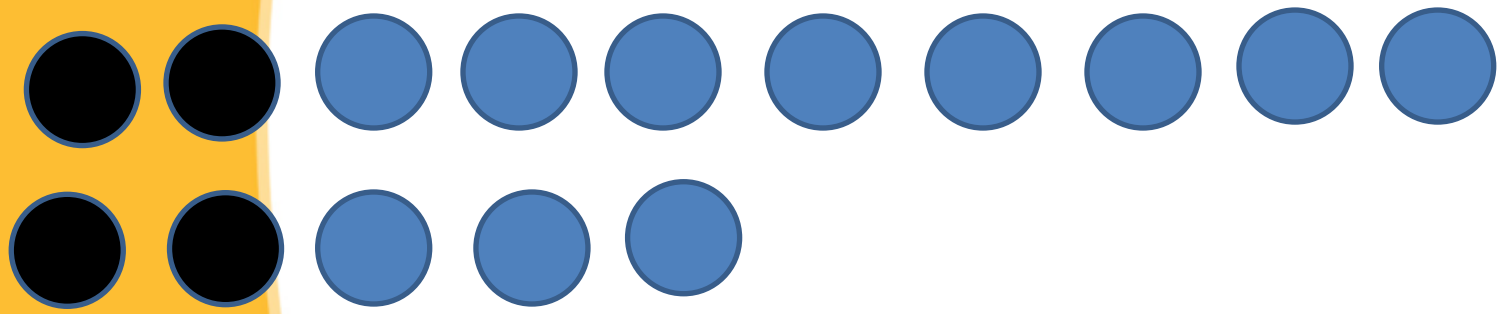
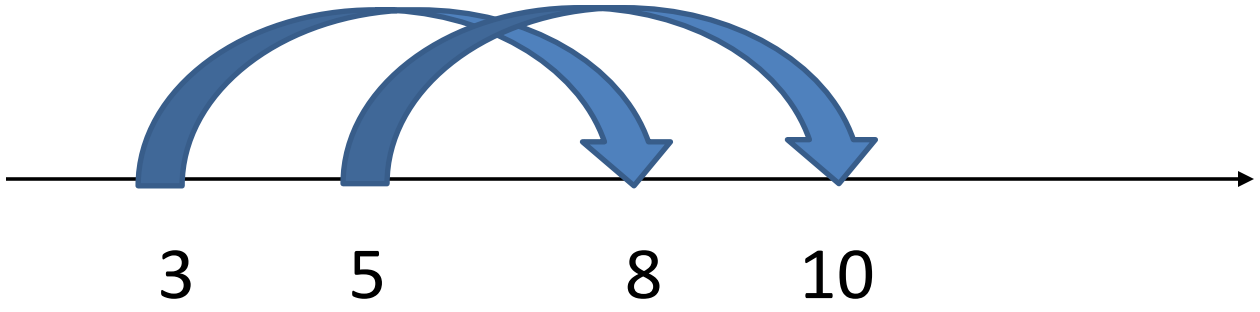


Constant difference: number line

- 8 – 3 is the same as 10 – 5.



So I might ask

- If you know that 8 + 4 = 12, what else do you automatically know?

So I might ask

- Draw a picture to show why
- $12 - 8 = 14 - 10$.

How do I practise?

- Rather than worksheets, which you might occasionally use, you might use interesting problems and games, as well as allowing students to create their own practise.

Creating my own practise

- You might say:
- Choose two numbers under 10.
- Add them and subtract them.
- Repeat with 5 other pairs of numbers.

Creating my own practise

- You might say:
- Choose a number for the blank and then answer the question.
- Two numbers add to ____.
- What are they?

Creating my own practise

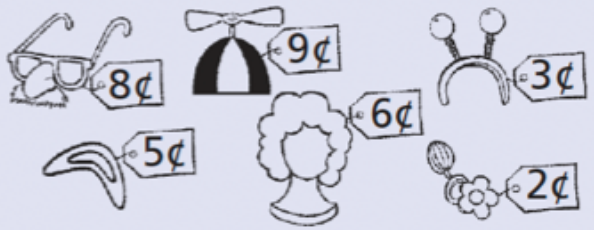
- You might say:
- Choose a number for the blank and then answer the question.
- The difference between two numbers is _____.
- What are they?

Sc oblems

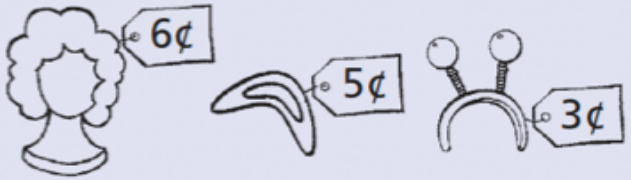
ACTIVITY 9.1

Students might be asked to solve problems involving spending cents at a yard sale.

The Silly Stuff Sale



Kim goes to the Silly Stuff sale.
She takes 20¢.
She wants to buy



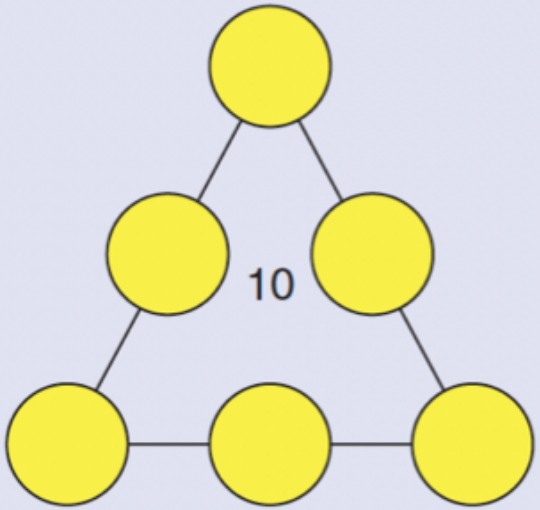
How much money will she have left?

What could Kim buy with the money she has left?

Some activities/problems

ACTIVITY 9.5

Provide students with small circles with numbers from 0 to 10 on them. Ask them to arrange six of the circles on the triangle so that each side sums to 10.

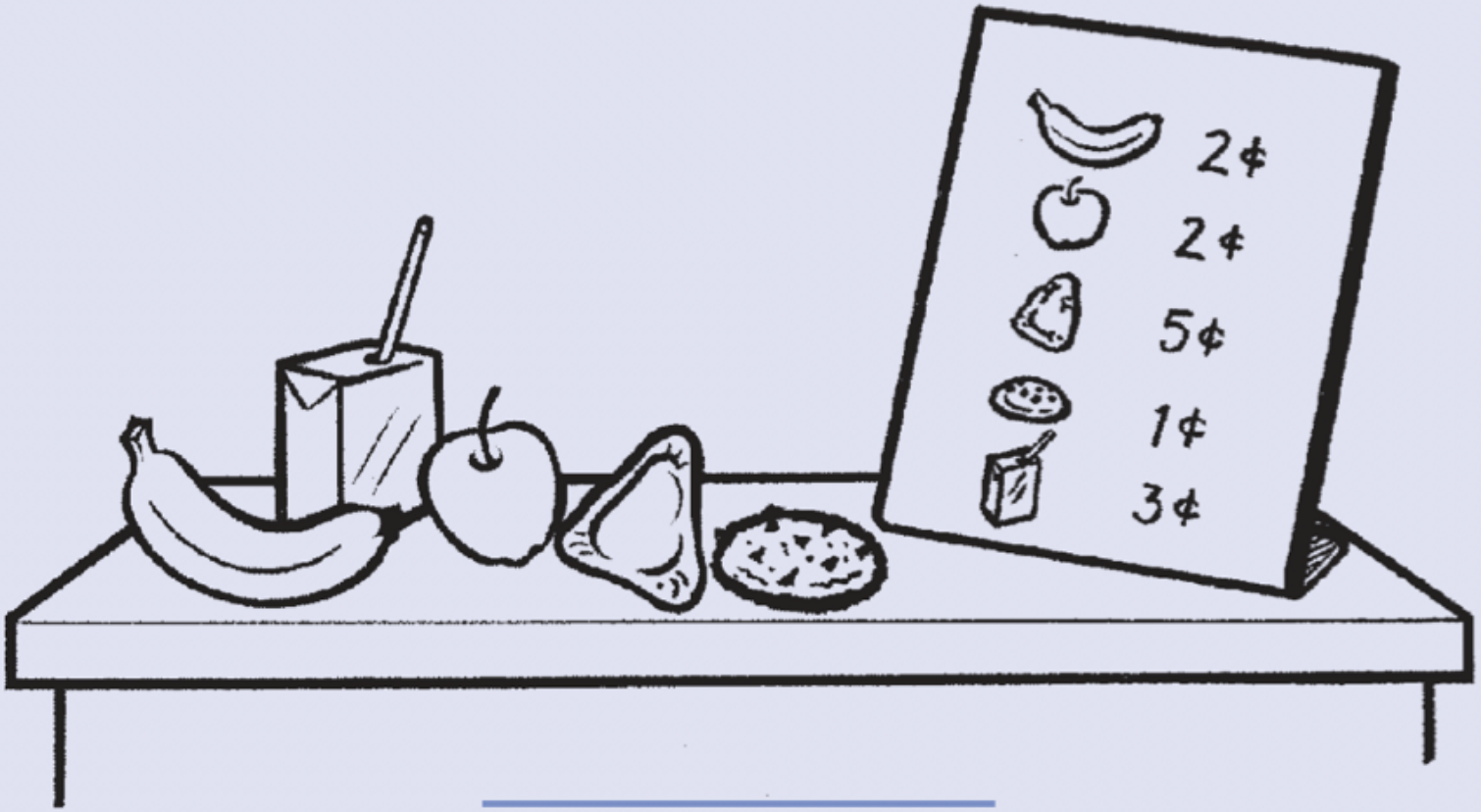


Building Number Sense

ACTIVITY 9.4

The activity not only promotes mathematical thinking, but also provides an opportunity to practise facts.

Show 3 ways to spend exactly 10¢.



Practice problems

- Two sums on the addition table are 5 apart. What could the numbers that were added be?
- Put 8 in one box. Put numbers in the other boxes to make sense.

Practice problems

Q Choose numbers and finish each story. Do each story two more more ways.

a) ___ frogs jumped into a pond. Then, ___ more jumped in. There were ___ frogs in the pond.

b) I need ___ spoons. First I put out ___ spoons. I needed ___ more.

c) There were ___ birds. ___ flew away. There were ___ birds are left.

Games

- You can use tons of digital games that involve fact practise or make up your own games, e.g.
- Spin a spinner (digits 1 – 9) twice. Add the numbers. If the answer is 9 or 12 or 15, you get a point. The first person with 4 points wins.

What about larger numbers?

- I need to practise decomposition.
- So I might use tasks like these.

Decomposition

- How might you fill in the blanks each time?

35		

Decomposition

- How might you fill in the blanks each time?

152		

Decomposition

- What numbers between 50 and 60 can you write if they have to be made up of only 3s and 4s?

What about larger numbers?

- How would students model 28 + 7?
- How would they do it numerically?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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Numerically

- 20 + 8
- + 2 +5
- 20 + 10 + 5 = 35

What about larger numbers?

- How would students model 28 + 48?
- How would they do it numerically?

With models

- On a 100 chart, start at 28, go down 5 rows and back 2

- With base ten blocks, model 28 with 2 rods, 8 ones; add 5 rods and take away 2 ones

Symbolically

- 28 = 20 + 8
- 48 = 40 + 2 + 6
- 60 + 10 + 6 = 76

What about larger numbers?

- How would students model 328 + 348?
- How would they do it numerically?

What about subracting?

- How would students model 22 – 9?
- How would they do it numerically?

With models

- I would lay down 2 rods and 2 ones and lay 9 ones on one of the rods and see what was left.

- I would see 1 full rod, 1 one and another 2 ones, so 13.

What about subtracting?

- How would students model 52 – 27?
- How would they do it numerically?

With models

- How would students model 52 – 27?
- I would put down 5 rods and 2 ones. I might put 2 rods and 7 ones on top of 3 of the rods and see 2 rods, 3 ones and 2 ones or 25 left.

With models

- How would students model 52 – 27?
- Or I might show 52 with base ten rods and remove 3 rods and add back 3 ones.

With models

- Or I might start at 27 on a number line, add 3 to get to 30 and 20 to get to 50 and 2 to get to 52 for a total addition of 25. (I could do this with blocks, too.)

Numerically

- 52 = 40 + 10 + 2
- -27 = 20 + 7
- 20 + 3 + 2 = 25

Numerically

- 52 = 47 + 5
- -27 = –27
- 20 + 5 = 25

What about subracting?

- How would students model 352 – 146?
- How would they do it numerically?

Estimation

- This is critical with work even with small numbers, but especially with bigger ones.
- But we have to make students willing to estimate.

Estimation

- Instead of just asking for estimates (where students are likely just to calculate and round), I might ask:
- I estimated by using 30 + 70. What might I have been adding?

Or

- Is 152 – 48 closer to 50 or 100 or 150? Tell me why without referring to the actual answer.

- I added two 2-digit numbers and got a 3-digit answer. What numbers could I NOT have added?

Or

- You subtract 53 – 27. Which is a good estimate? Why?
- 50 – 20
- 50 – 30
- 53 – 30
- 57 – 27

Or

- Choose a 2-digit number and a 1-digit number so that the sum is between 30 and 50. Add them.

- Choose two numbers to subtract so the answer is close to what you subtracted.

Practise

- Again, I can practise with problems, games and student-created practice.

Student-created practice

- Choose two 2-digit numbers where the sum is between 50 and 60. Add them.
- Repeat several more times with numbers quite different in size.

Student-created practice

- Choose values for the digits and add.
- []4 + []8 =
- Repeat several times.
- What do you notice?

Problems

- Imagine tossing a pair of dice again and again to create two 2-digit numbers, which are then added. What is the least number of tosses needed for a sum greater than 200? How do you know?
- A triangle has a perimeter of 44 cm. What could the lengths of the sides be, if each side is a whole number of centimetres?

Prob

ACTIVITY 11.10

Students can solve interesting problems involving addition and subtraction.

Examples include:

- Five numbers add to 100. The difference between the least one and greatest one is 12. How do you know that none is greater than 50? What could the numbers be?
- Which numbers between 50 and 100 can be expressed as the sum of four consecutive numbers?

Critical Thinking

Games

- Again, there are digital games, games from companies like Box Cars and One-Eyed Jacks, or kid or teacher created games.

For example…

- Spin the spinner to fill in the blanks: []][][] + [][].
- Your points is the number in the tens place.
- The first person to get 50 points wins.

Multiplication and Division

- Relating multiplication and division.
- Every division is a missing factor multiplication.
- Division facts need not be separately memorized.

Commutativity

- a x b = b x a so only half the facts have to be memorized, but why is that true?

Multiplying by 0 and 1

- Once students learn that 0 x a = 0 and 1 x a = 1, they need not learn a lot of separate facts.

- But how do they understand why these generalizations are true?

The distributive principle

- M groups of 4 and n groups of 4 is (m + n) groups of 4.
- OR
- 4 groups of M and 4 groups of N is 4 groups of (M+N)

Why x 2 and x 5 are fundamental

- Actually, if students know the 2 x facts and the 5 x facts (which, of course, relate to skip counting), they really know everything they need to know.

For example

- 7 x 4 is 5 x 4 + 2 x 4
- 9 x 4 is 2 x 2 x 2 x 4 + 4
- 6 x 4 is 5 x 4 + 4

Practice

- Can be problems, games, or student-created practice.

Some possible problems

Julia's school is having a read-a-thon. Julia's goal is to read 100 pages. If she starts on April 1 and reads 5 pages every night, will she get to her goal by the end of the month or before that? Show how you know.

Some possible problems

- Are there more products like 2[] or 1[] when you multiply two numbers less than 10?

Some possible problems

- Use the digits 0-9, each once to make these true:
- 4 x [] = 2[]
- [] x [] = 2[]
- 8 x [] = [][]
- [] x 3 = []7

Some possible problems

- Use the digits 0-9, each once to make these true:
- 4 x 5 = 20
- 3 x 7 = 21
- 8 x 8 = 64
- 9 x 3 = 27

Some possible problems

- Use the digits 0-9, each once to make these true:
- 4 x 5 = 20
- 3 x 7 = 21
- 8 x 8 = 64
- 9 x 3 = 27

Some possible games

ACTIVITY 9.11

Encourage students to play the game Metre Race.

- Each player chooses a coloured counter and then picks 2 number cards, labelled from 1 to 6.
- The first card tells how many jumps to make, and the second card tells how far to jump each time along the metre stick.
- The first player to get to 100 cm wins.

Some possible games

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

1	2	3	4	5	6	7	8	9

Some possible games

- Roll two dice. Multiply the values. The person with the greater ones digit wins a point. The first person with 10 points wins the game.
- Use the 2 to 9 cards from a deck of cards. Pick four cards. Arrange them into two pairs. Multiply the values in the pairs and then add the two products. The person closest to 50 wins a point. The first person with 10 points wins the game.

Some created practice

- Choose 4 multiplication facts that are “tricky” and show that you know them.

One source

- The resource Making Math Meaningful to Canadian Students, K – 8, might provide valuable information about computation.

Download

www.onetwoinfinity.ca

Recent presentations

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