

Enseigner avec
intention;
En se concentrant sur
ce qui est important
7é-8é

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Ottawa

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ordre du jour

- aujourd'hui
- Focusing on teaching intentions and big ideas

ordre du jour

- la prochaine fois
- Using concrete materials effectively
- Working with open questions

Let's do a problem

- Your car is going 21 km in 15 min.
- How far will it go, at that rate, in 20 min?

Possible solutions

- 21 km in 15 min is the same as
- 7 km in 5 min is the same as
- 28 km in 20 min

Possible solutions

- 21 km in 15 min is the same as
- 1.4 km in 1 min is the same as
- 28 km in 20 min

Possible solutions

- 21 km in 15 min is the same as
- 84 km/h is the same as
- 28 km in $\frac{1}{3}$ of an hour

What did I get out of this?

- The idea that solving a rate problem is always about getting an equivalent rate.
- The idea that "unit" rate is not always the best option, but it is always an option.
- The idea that there are always different ways to solve a rate problem.

What contenu was I addressing?

- 7^e année:

utiliser des rapports et des taux dans des situations réelles (p. ex., si une voiture roule à 100 km/h, elle pourra parcourir 400 kilomètres en 4 heures).

But I put “meat” on the content

I was addressing what I will call essential understandings.

Teachers need to borrow or learn about them so they don't just “do stuff”.

I would argue

- That just “doing” content is not enough to lead to mathematical success.
- Students need to meet ideas and not just solve random problems.

Valuable work

- Would be to look at content and think about what ideas need to be addressed.
- Teachers should be expected to be able to articulate what those ideas are.

Let's try one together

- What ideas are embedded in this content?

7^e année

établir et expliquer à l'aide de matériel concret, la relation entre les fractions, les nombres décimaux, les pourcentages et les rapports.

For me, it might be these ideas:

Every percent is automatically a fraction.

Every terminating decimal is automatically a fraction.

$$42\% = 42/100$$

$$0.125 = 125/1000$$



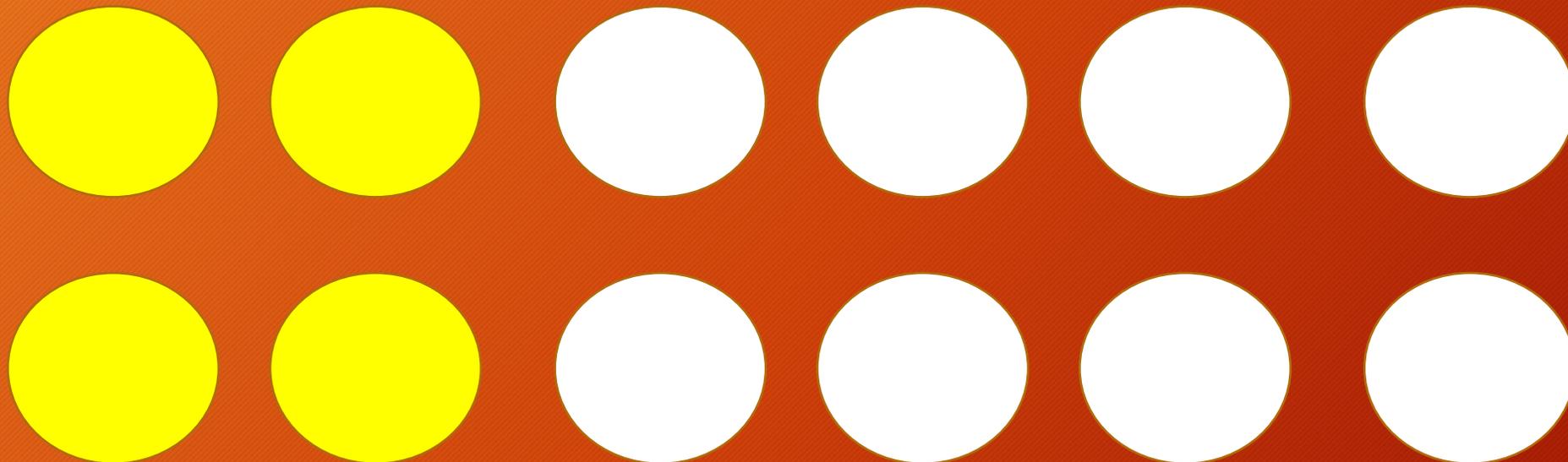
Every fraction can be written as a percent,
but not necessarily a whole number percent.

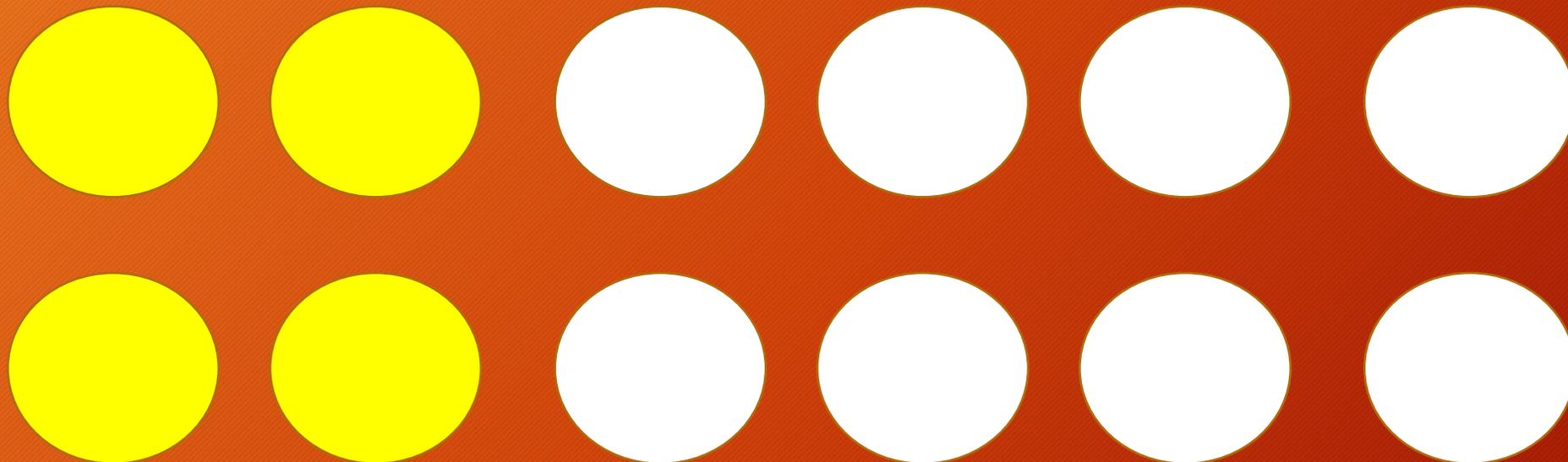
$3/4 = 75\%$ but $3/8 = 37.5\%$



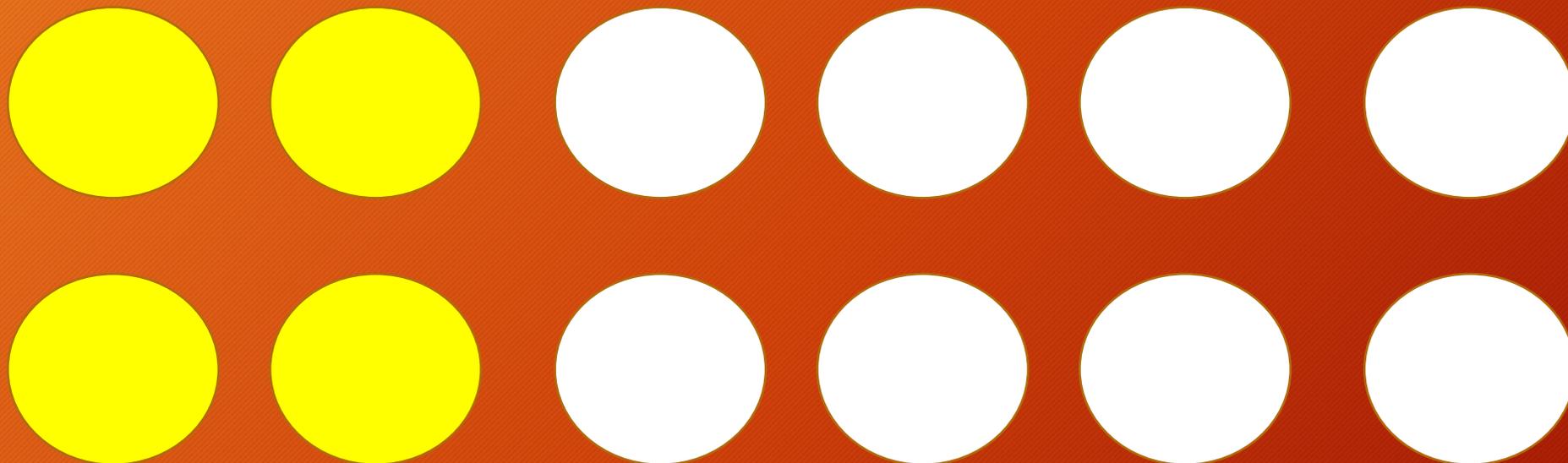
Every ratio situation can be described with fractions.

What fractions are visible here?

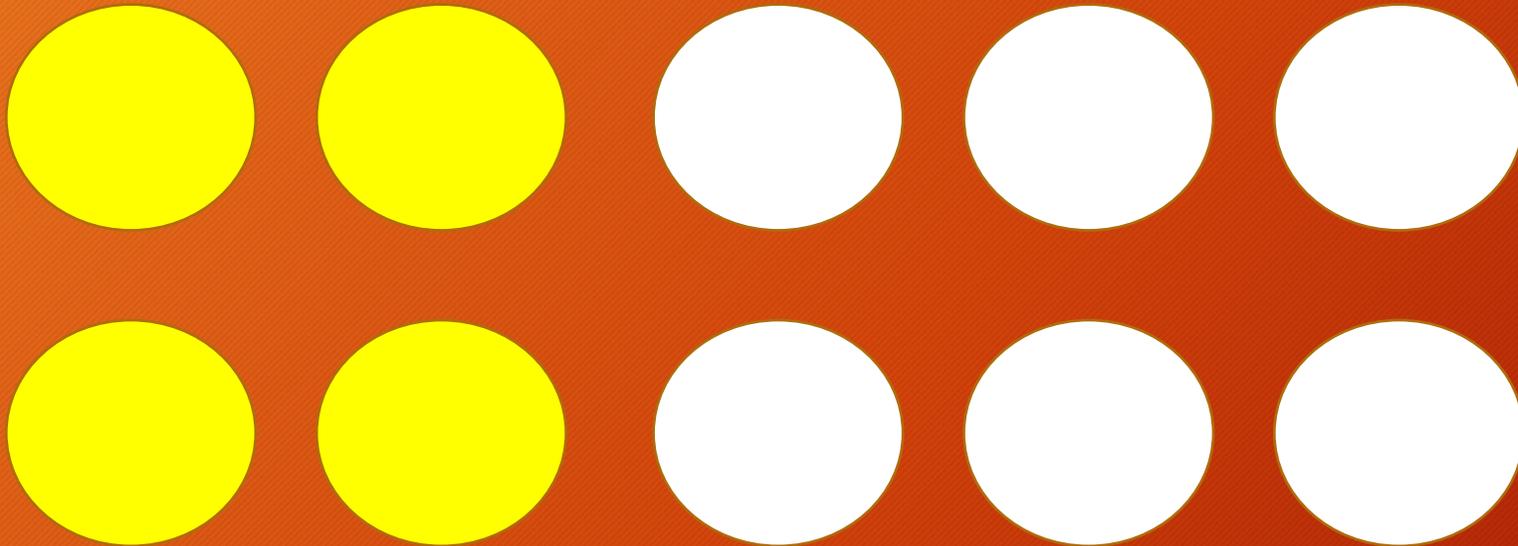




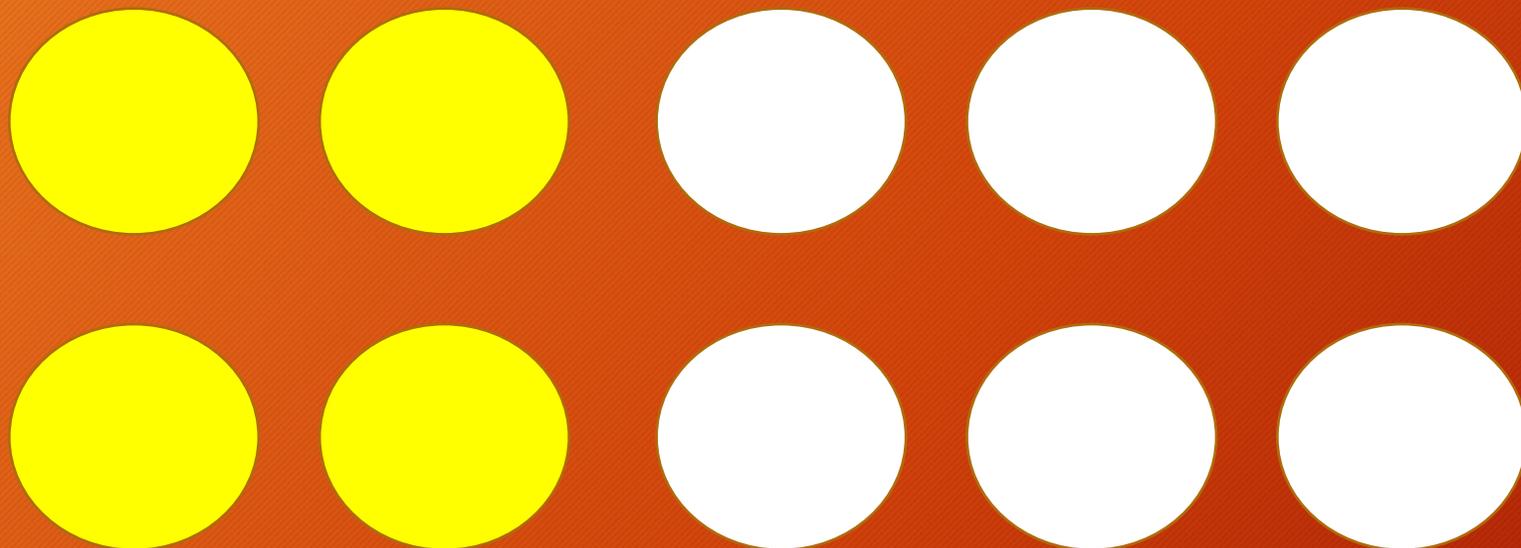
Where is $\frac{1}{3}$? $\frac{2}{3}$? $\frac{4}{12}$? $\frac{8}{12}$?



Where is $1/2$? $2/1$? $3/1$? $6/2$?



Where is $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{10}$? $\frac{6}{10}$?



Where is $2/3$? $4/6$? $3/2$? $6/4$? $10/4$?

Every fraction's size is about the ratio between its numerator and denominator.

All fractions equivalent to $\frac{2}{3}$ have a numerator: denominator ratio of 2:3.

Sometimes using a fraction is easier than using its decimal or percent equivalent, but sometimes it is not.

What is 25% of 484?

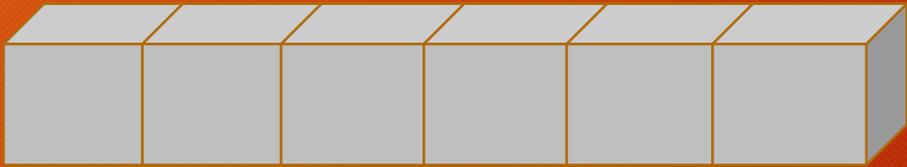
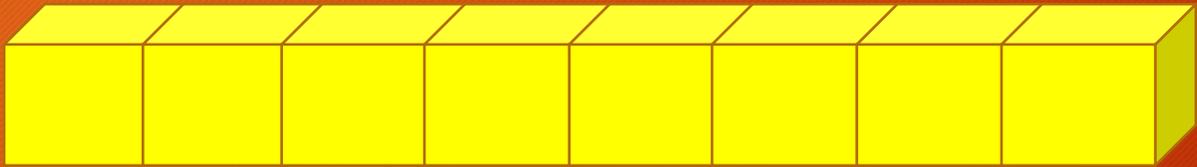
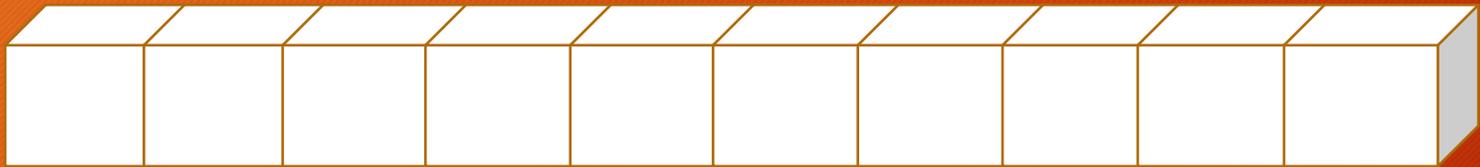
What is 40% of 25?

Another contenu

utiliser diverses techniques pour déterminer la moyenne d'un ensemble de données (p. ex., répartition en parts égales, dessin, tour de cubes emboîtables).

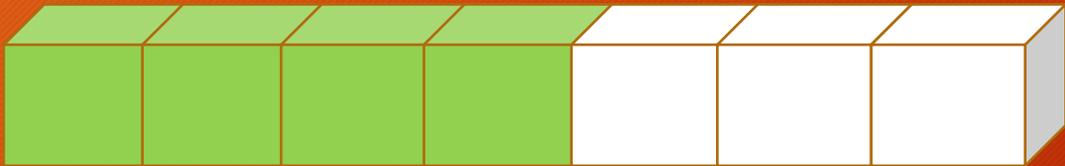
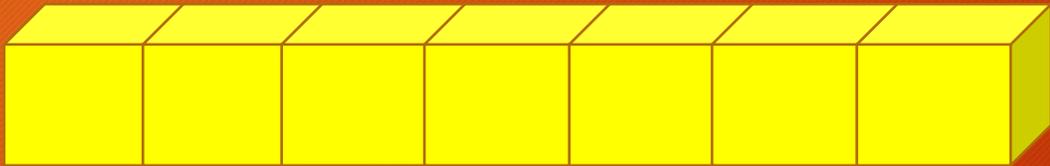
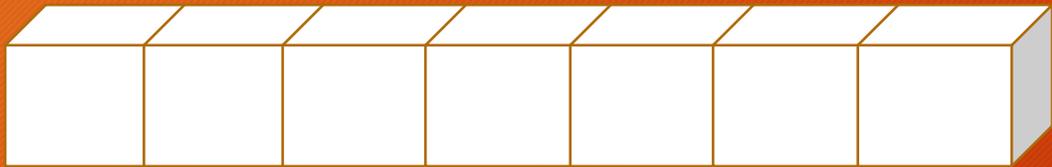
Relating the definition of mean to sharing all the data equally

Let's see the mean of 10, 8, 4, 6





Thinking about the mean as a balance point



Recognizing that the mean is a “summary” of a bunch of data in one number.

8^e année

estimer et calculer l'aire de cercles.

- Areas of circles can be really big or really little.
- If a circle has a bigger radius, it has a bigger area.

- The number of centimetres in the circumference can be more or less than the number of square centimetres in the area.
- You only need one other measurement of a circle to figure out an area, but there are choices of what it could be.

The area of a circle is about 75% of the area of a square with side length the diameter of the circle.

Or this one

évaluer des expressions algébriques et des équations simples en substituant des nombres entiers, des fractions positives et des nombres décimaux.

Ability to estimate the value

Ability to predict whether the values increase or decrease when the value of the substituted value increases or decreases

Ability to predict whether the resulting value from one expression will be more or less than the resulting value from a similar one

e.g. $3x - 4$ vs $2x - 4$ OR $3x - 4$ vs $3x + 2$

Now you choose a contenu in one domain and do the same thing that I've been doing, i.e. telling what really matters

So is there a list of essential understandings?

Yes and no.

I have various lists in different books I have written or am writing, but they are not everybody's.

How does it play for me when teaching?

The first place is in setting learning goals for the lesson.

The second is in choosing activities to lead me to those goals.

How does it play for me when teaching?

The third is in consolidation.

The fourth is in assessment of learning.

Setting Learning Goals

Here are examples of learning goals I might set to fit 7^e année contenus.

Setting Learning Goals

évaluer des puissances ayant un nombre naturel comme base et comme exposant.

Setting Learning Goals

- My goal might be that increasing the exponent a little usually has a much bigger effect than increasing the base a little.

Setting Learning Goals

additionner et soustraire dans divers contextes des fractions positives en utilisant une variété de stratégies (p. ex., matériel concret, dessins, tableau).

My learning goal might be that the student can explain how adding and subtracting fractions is a lot like adding and subtracting whole numbers.

8^e année

décomposer des nombres naturels
inférieurs à 144 en produits de facteurs
premiers
(p. ex., $36 = 4 \times 9 = 2 \times 2 \times 3 \times 3$).

1/ 1 1 1

8^e année

My learning goal might be that when you decompose numbers using multiplication, you get the most “pieces” when you decompose into primes.

8^e année

déterminer, par estimation et à l'aide du théorème de Pythagore, la mesure manquante d'un des côtés d'un triangle rectangle.

8^e année

My learning goal might be that for some triangles, when you know two sides, the third is automatic, but not for all triangles.

Your turn

Each team needs to choose one or two contents that you might address by being much more clear about a learning goal which is about an IDEA (not skill) that you want students to learn.

Choosing activities to lead to a learning goal

Once I have a goal, it should make it easier to choose an appropriate activity.

For example...

My learning goal in Grade 7 might be that the student can list lots of pairs of fractions that result in the same difference and notice a relationship.

For example...

My activity might be to suggest that two fractions were subtracted and the result was $3/10$.

For example...

My question is what those fractions might have been.

Plot them on a number line using different colours for different pairs.

How do you know your answers are right?

You can get some answers now

$$4/10 - 1/10$$

$$12/10 - 9/10$$

$$1/2 - 1/5$$

$$3/4 - 9/20$$

You can get some answers now

$$19/10 - 8/5$$

$$2/3 - 11/30$$

On the number line



I might check by...

Adding $3/10$ to the result to see if I get the greater number.

Then I consolidate..

- Look at your pairs of numbers.
Could they have both been less than 1?
Could they have both been greater than 1?
Could one have been greater and one less?

Then I consolidate..

- How far apart were the pairs that you got on the number line?
- Why did that happen?

- How would that help you figure out other answers?

Then I extend...

- I probably ask them to get more pairs.

Or my learning goal could have been

- If you know the difference between two fractions, then you can sometimes make good guesses about the denominators of the fractions you subtracted.

Then I consolidate..

- So this time I ask:
- What pairs of denominators did you use?
- Why did it make more sense that denominators were 10 and 5 than, e.g. 8 and 5?

Then I extend...

- What if the difference was $\frac{1}{2}$?

What fractions might you have subtracted?

Is there anything true about all of the pairs of denominators you get?

Maybe

- $1 - \frac{1}{2}$
 $\frac{3}{2} - 1$
 $\frac{8}{2} - \frac{7}{2}$

But it could be

$$\frac{7}{7} - \frac{2}{4}$$

Notice

My consolidation focused on my learning goal.

It could be 8^e année

My learning goal might be that students understand that an effective way to divide fractions is to get common denominators and divide the numerators, e.g. $\frac{1}{2} \div \frac{2}{5} = \frac{5}{10} \div \frac{4}{10} = \frac{5}{4}$

My activity might be to start by asking students to show me using Cuisenaire rods what $15 \div 5$ means.



We would discuss how one meaning of dividing is figuring out how many of one thing fits into another.

Then...

- I next ask students to use a fraction tower and ask these questions.

How many times does:

- $\frac{1}{4}$ fit into $\frac{1}{2}$?
- $\frac{1}{4}$ fit into $\frac{3}{4}$?

Then...

- does

$1/3$ fit into $6/9$?

$2/5$ fit into $7/10$?

$1/5$ fit into $1/2$?

$2/5$ fit into $4/5$?

$2/5$ fit into $5/5$?

Consolidation

- I might ask:
 - Why is it easy to figure out how many $1/10$ s fit in $[\]/10$?
 - Why is it easy to figure out how many $1/6$ s fit into $[\]/6$?

Consolidation

- I might ask:

- How do these answers relate:

How many $\frac{1}{5}$ in $\frac{4}{5}$ and how many $\frac{2}{5}$ in $\frac{4}{5}$?

How many $\frac{1}{10}$ in $\frac{6}{10}$ and how many $\frac{2}{10}$ in $\frac{6}{10}$?

Consolidation

- I might ask:

- How do these answers relate:

How many $1/10$ in $9/10$ and how many $3/10$ in $9/10$?

How many $1/10$ in $8/10$ and how many $4/10$ in $8/10$?

Consolidation

- Why does it make sense that all of the questions we answered were division questions?
- So what is $4/10 \div 1/10$?
- How about $4/8 \div 1/8$?
- How about $4/20 \div 1/20$?

Consolidation

- How about $4/[] \div 1/[]$?
- How about $4/[] \div 2/[]$?
- What do you predict $5/3 \div 2/3$ will be?

Consolidation

- What do you predict $6/10 \div 4/10$?
- Why might it be convenient to get equivalent fractions with the same denominator to divide fractions?

Your turn

- You come up with the questions for this activity.
- My learning goal is that the student realizes WHY when a pattern goes up by, e.g. 3, the formula includes the term $3n$.

Your turn

- For example, the rule for
- 4, 7, 10, 13,.... Is $3n + 1$

- The rule for
- 11, 14, 17, 20,.... Is $3n + 8$

Your turn

- What is the pattern rule for
- 1, 4, 7, 10,....

- Why is that the rule? (You can't just say that's just how it works.)

Your turn

- You come up with the consolidation questions for this activity.

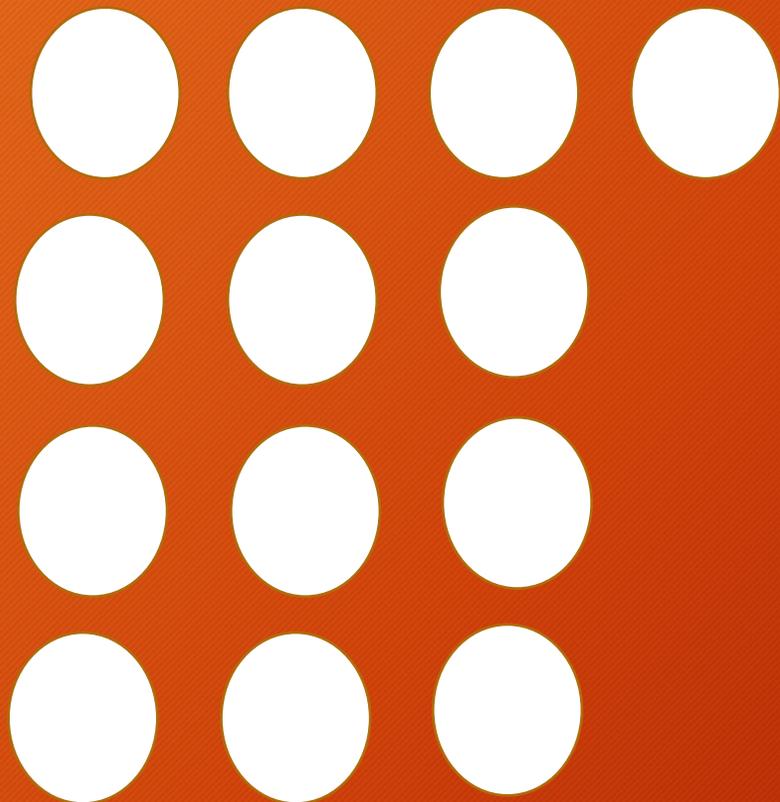
Some ideas I had

- Why is the pattern rule for 3, 6, 9,... $3n$?
- How does the pattern 4, 7, 10, 13, Relate to 3, 6, 9, 12,....?
- So what would the pattern rule have to be?

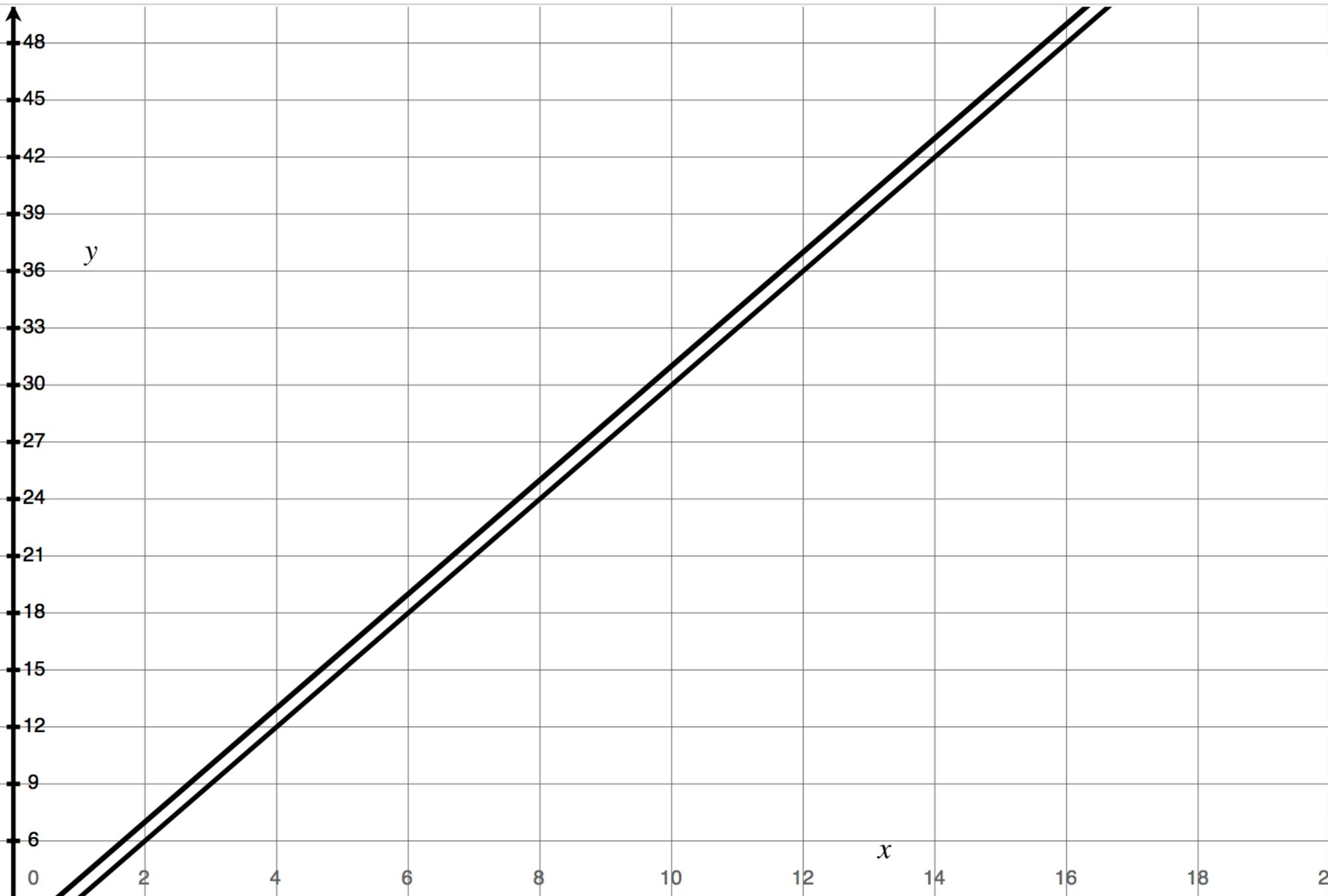
Some ideas I had

- What kind of picture could you draw to make sense of the rule for 4, 7, 10,....?

What about this one?



Wh



Assessment

Assessment needs to match instruction.

Teaching with intention means assessing with intention.

Assessment

If your goal for your students in math is just “doing it”, you will have mostly knowledge items with some application thrown in.

Assessment

But if your goal is to build math thinkers, you will focus much more on understanding and thinking questions.

For example

Instead of: Find pairs of numbers that multiply to 125.

How could you represent 125 to make it easy for you to see what a factor of it is?

For example

Instead of: What is $4 - (-2)$? ,

I could ask:

Why does it make sense (without getting the answer) that $4 - (-2)$ is more than $4 - (-1)$?

For example

Instead of: What is 30% of 58?

You would ask: How do you know, without getting an answer, that 30% of 58 has to be close to 20?

For example

Instead of: Figure out the area of this triangle.

You would ask: Suppose a triangle and parallelogram have the same area. What has to be true about them?

For example

Instead of: What is $(-12) \div (-4)$?

You would ask: Draw a picture to show why $(-12) \div (-4)$ is 3.

For example

Instead of: Use the order of operations to figure out $3 \times 4 + 8 \times 5$.

You would ask: Write an expression where you get the same answer using the order of operations as going left to right and another where you don't.

For example

Instead of: Solve $3x - 4 = -10$.

You would ask: Without solving the equation, how do you know that the solution has to be negative?

Your turn

Choose 2 or 3 contenus.

Think of understanding questions instead of knowledge questions you could ask.

Since we have time

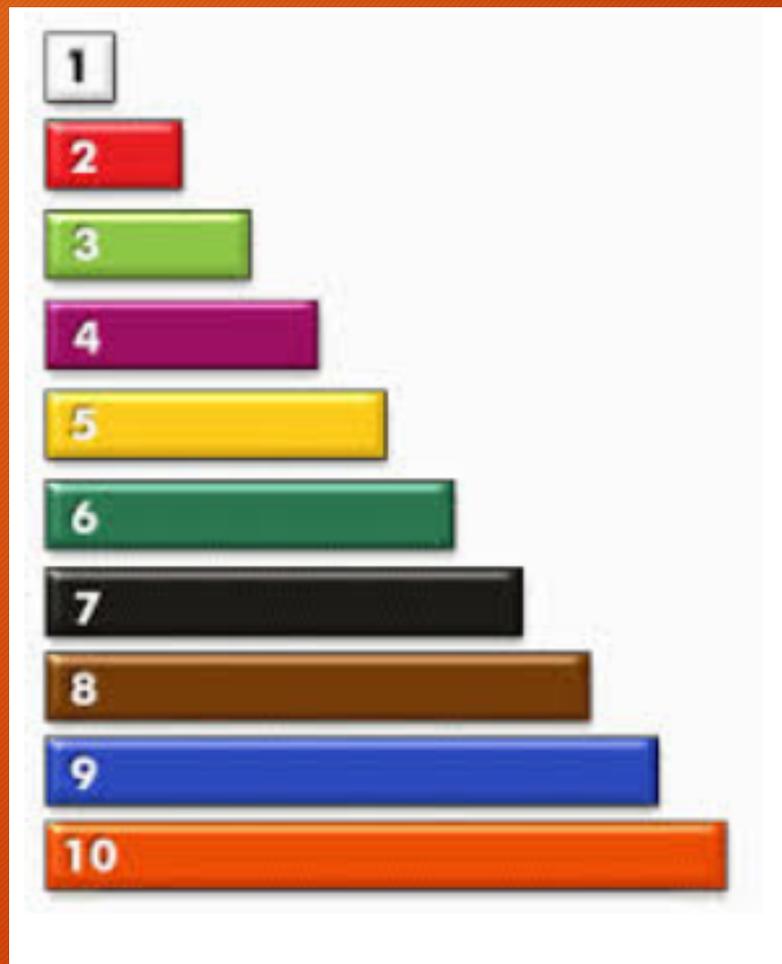
Let's look at a few tools.

We will have time for more next time we meet.

Cuisenaire rods

- How might they be helpful in Grades 7 and 8?

Cuisenaire rods



For fractions

- Use pairs of your rods to show the fraction $\frac{1}{2}$. Which rods could you use?

For fractions

- Use pairs of your rods to show the fraction $\frac{2}{5}$. Which rods could you use?

For proportional relationships

- Suppose the light green rod is worth 12. What are the other rods worth?
- Suppose the light green rod is worth 10. What are the other rods worth?

For square numbers

- Use your rods to show that 9, 16 and 25 are square numbers.

For factors and multiples

- Use your rods to show that 4 is a factor of 24.
- Use your rods to show that 36 is a multiple of 9.

For factors and multiples

- Use your rods to show that 4 is a common factor of 8 and 12.
- Use your rods to show that 24 is a common multiple of 3 and 8.

For equations

- Use your rods to find the solution to $4x + 5 = 29$.

Today

Was a lot of hard thinking.

You can't change overnight, but maybe choose something you heard today that you are ready to work on.

Download

- You can download this presentation at www.onetwoinfinity.ca

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