

What should K-8 math look like?

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Agenda

- Classroom environment
- The math to focus on
- The 3-part lesson
- Critical thinking
- Differentiation of instruction

The classroom

- Lots of student conversation /debate about mathematics
- Kids working in pairs/groups
- Even as a supply teacher, you could set up debates.
- e.g., I might ask:

The classroom

- Jason says that you can't get a little answer when you subtract two big numbers.
- Lia says you can.
- With whom do you agree? Why?

OR

The classroom

- A 3-D figure can have more vertices than edges, sometimes.

OR

The classroom

- Kyle says that when the numerator and a denominator of one fraction are closer together than the numerator and denominator of another, it is greater.
- Lia says that this might not be true.
- With whom do you agree? Why?

OR

The classroom

2 truths and a lie— which is the lie?

1. A number that takes 4 words to say can be greater than a number that takes 7 words to say.
2. A number with more digits is always greater.
3. There are only 90 numbers between 100 and 999 that are of the form ____hundreds + ____ones if single digit whole numbers go in the blanks.

You try

- What might be a fun math debate for you to lead?
- OR
- What might be a good math 2 truths and a lie to try?

The classroom

- A safe environment- mistakes not a big deal
- Lots of use of thinking tools, whether manipulatives or technology

Most valuable tools for number work

- Counters
- Ten-frames
- Rekenreks (or beaded number lines)
- Cuisenaire rods
- Linking cubes
- Hundred charts
- Base ten blocks
- Hundredth grids
- Thousandths grids
- Fraction pieces
- Pattern blocks
- Integer chips

Counters

- A task might be:
- Take 20 counters.
- How can you arrange them in 3 piles so that the piles are close to equal in size?
- How can you arrange them in 2 piles so that one pile is about double another pile?

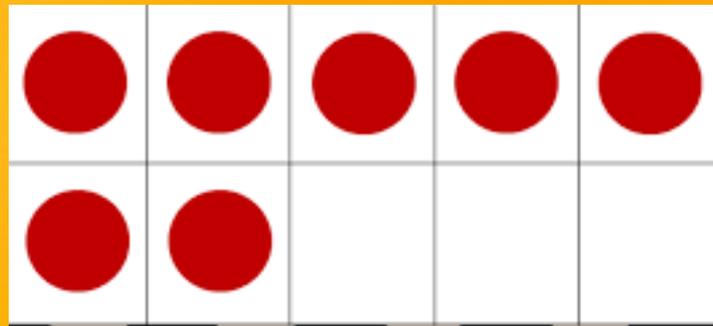
Counters

- Take 20 counters.
- How can you arrange them in 3 piles so that there is 1 really big pile and 2 small ones?
- Can you arrange them into 4 piles so that two piles are equal and small and two are equal and big?
- Make up your own counters challenge.

Ten-frames

You might ask:

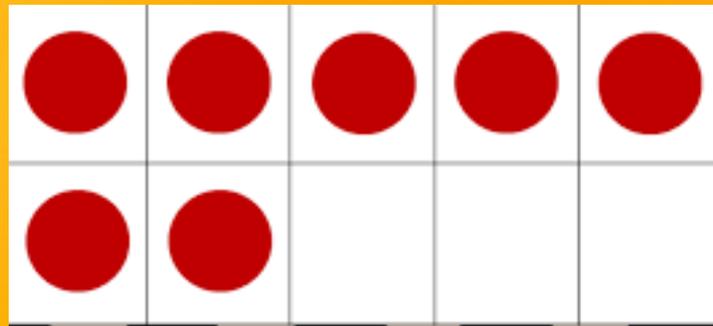
- How could you use 10-frames to help you figure out pairs of numbers that add to 10?
- How could you use 10-frames to show why $8 + 9 = 17$?
- What numbers could you show if you fill 3 ten-frames and part of a fourth one?



Ten-frames

You might ask:

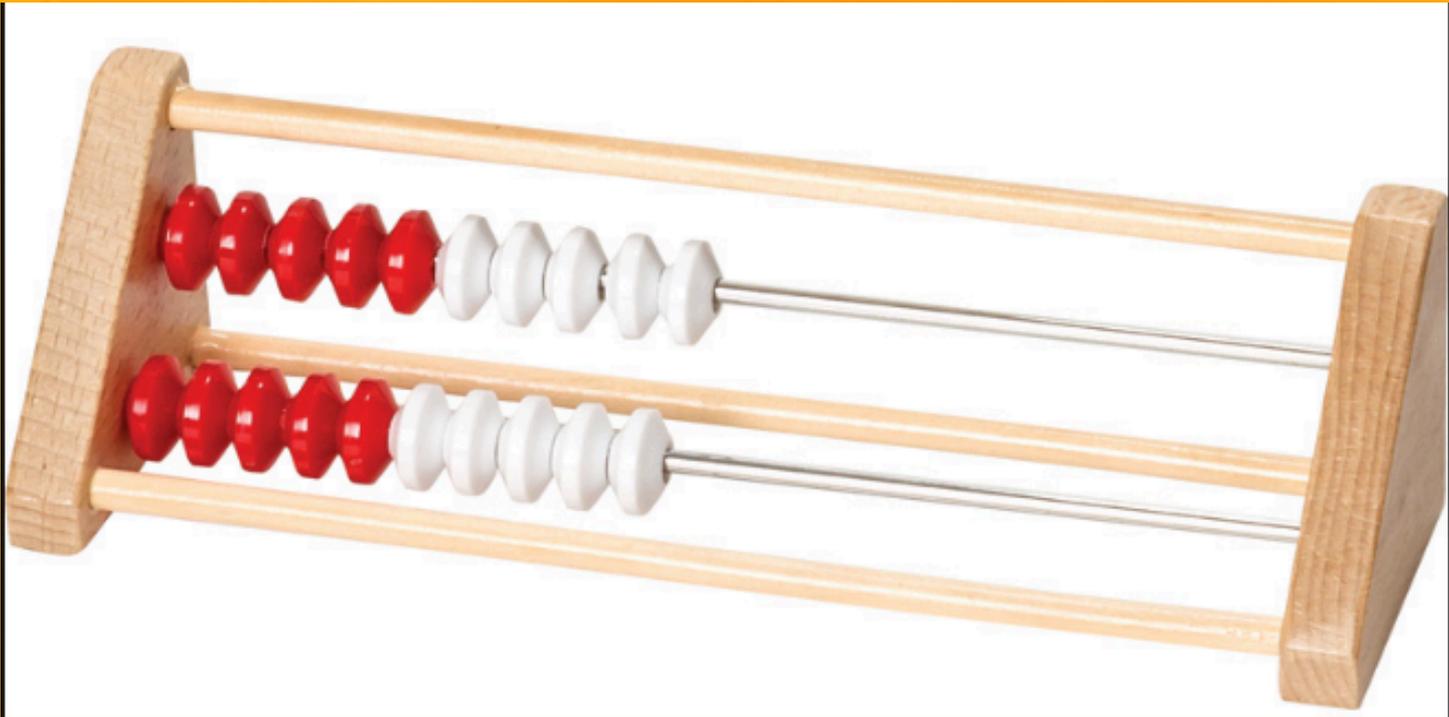
- What ten-frame challenge can you think of?



Rekenrek

You might ask:

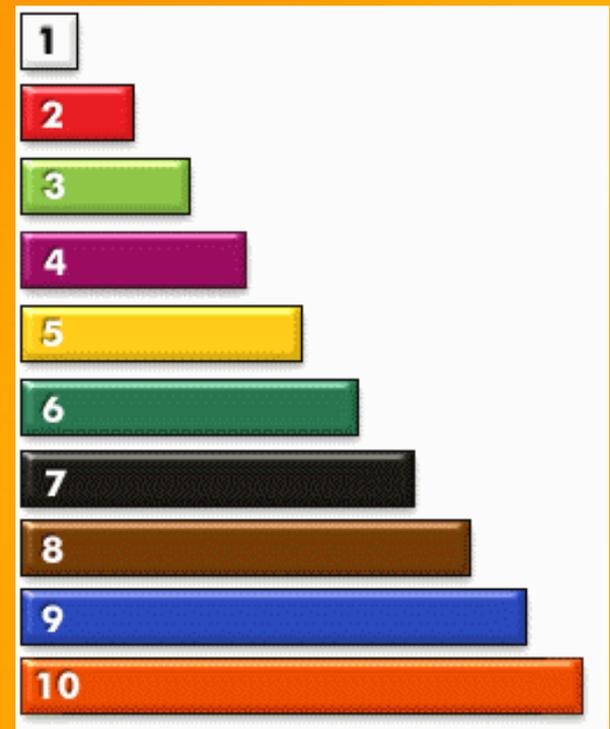
- How would you count out 17?
- How could you subtract $17 - 8$?



Cuisenaire rods

You might ask:

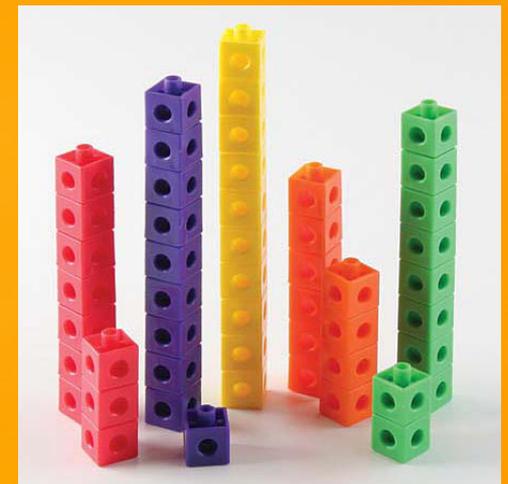
- Which pairs of rods show $\frac{1}{2}$?
- If light green is worth 9, what should the other rods be worth?
- How would you show $14 - 8$?
- How would you show 4×5 ?
- How would you show $32 \div 4$?
- What challenge can you create?



Linking cubes

You might ask:

- How could you show 43?
- How could you show that $12 - 4 = 13 - 5$?
- How could you show that the least common multiple of 3 and 4 is 12?
- What lengths can you have if you use 3 colours and consecutive amounts for the colours?



Hundred chart

You might ask:

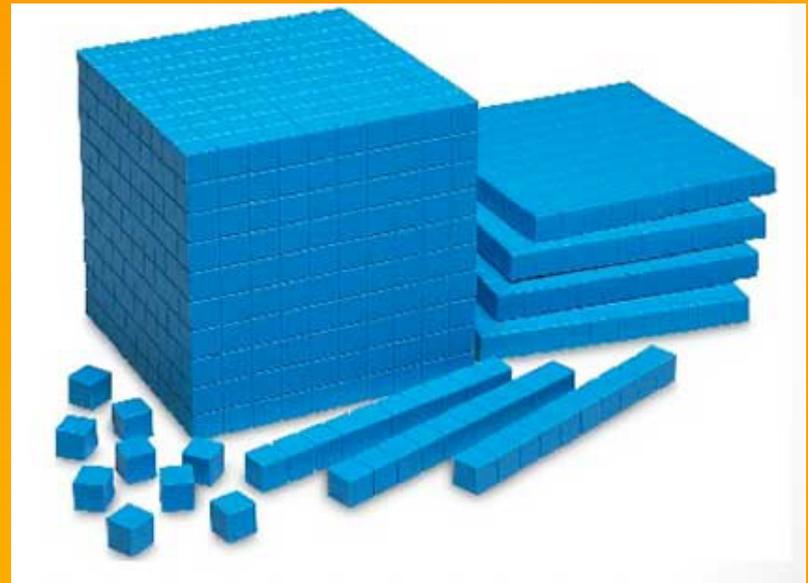
- What number might be in the bottom left quarter of the chart?
- What numbers is 34 near?
- How could you use the chart to add $43 + 25$?
- What would happen if you coloured all numbers including the digit 2?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Base ten blocks

You might ask:

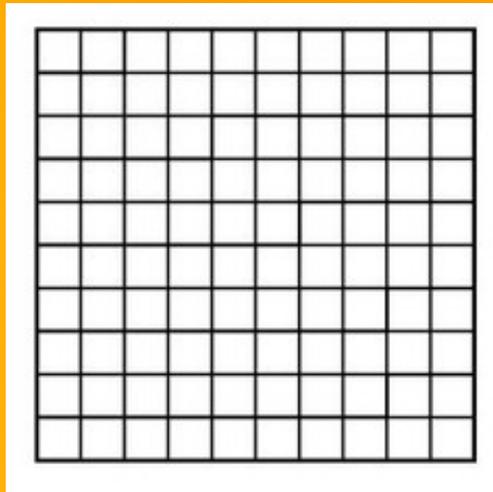
- What numbers can you represent with 12 blocks?
- What numbers can you represent with a lot of sticks and only a few small cubes?
- How might you represent 10 000?



Hundredth grid

You might ask:

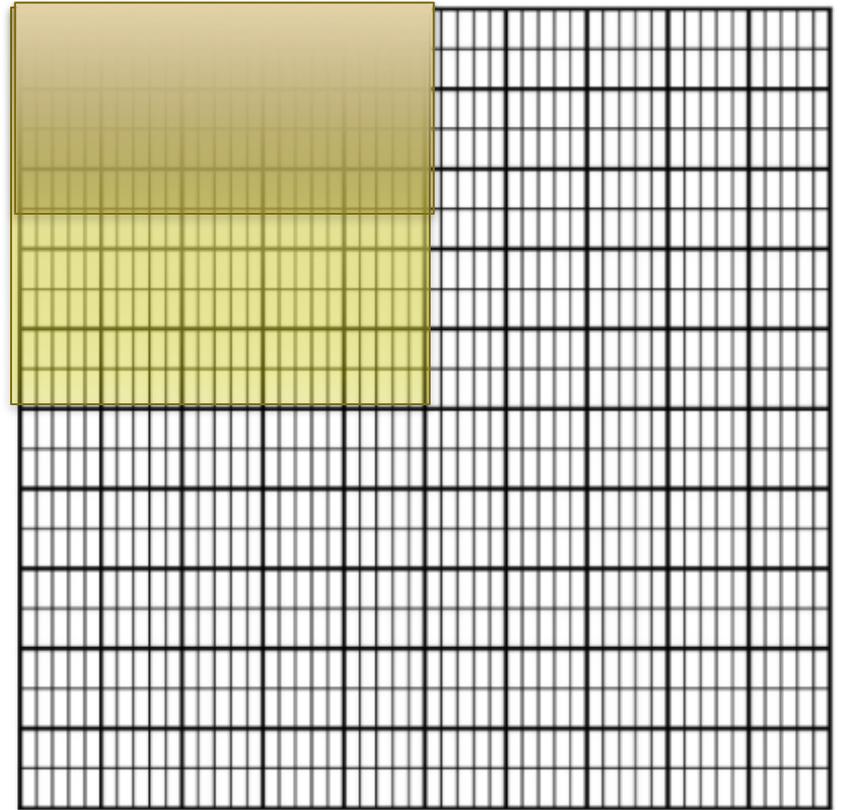
- What decimal is the same as 0.4?
- What decimal is just slightly less than $\frac{1}{2}$?
- What decimal is close to $\frac{1}{4}$?
- What other challenges might be interesting to kids?



Thousandth grid

You might ask:

- What fraction is 0.125?
- Why does $0.100 = 0.1$?
- What decimal is about $\frac{2}{3}$?



Pattern blocks

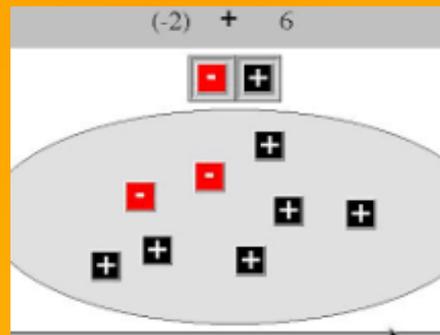
- You might ask:
- If red is 1, what fraction is blue?
- If yellow is worth 6, make a design worth 20.
- What blocks could you put together to make your initial?
How else could you make your initial?



Integer chips

You might ask:

- If you have a few more -1 chips than $+1$ chips, what value might your collection be?
- How would you model: $4 + (-2)$, $(-4) - (-2)$, $(-4) - (3)$?
- If you add a number you can show with 8 chips to one you can show with 6 chips, how many chips might you need to show the sum?



The classroom

- Teachers leave the “work” to the students.
- High expectations

The classroom

Curiosity is piqued.

- How long is long hair?
- How many text messages does a typical 12 year old send in a year?
- How much recycling does your school do?
- Come up with another mathematical curiosity to investigate.

The classroom

- Accessible tasks with high ceilings

I might ask:

- Choose two numbers so that when you add them, the answer is double when you subtract them.
- Come up with at least a few answers.

- What if the sum is triple the difference?

The classroom

- Feedback – more questions, less marks

The math to focus on

- The seven processes require attention no matter the grade level.
- Students need opportunities to:
- Problem solve
- Communicate
- Reason
- Reflect
- Connect
- Represent
- Select tools and strategies

The math to focus on

- Each expectation requires “interpretation”.
- Teachers have to decide they are focusing on what is important.
- For example... Grade 1:
- Estimate, measure and record lengths, heights and distances
- What are the most important markers for you that the student has been successful on this expectation?

It might be

- How reasonable are their estimates? (i.e. can they relate two measurements to each other)
- Do they use unit size as a factor?
- Do they recognize bad measuring vs good measuring?

It might be

- Do they have a good sense of which attributes matter and which don't when they measure?
- Do they realize units need to be the same size and why?
- Do they choose appropriate units?

It might be

- Do they know what to do if they don't have enough units?
- Do they know what to do if the fit is not perfect?

So a task might be

- Maybe we provide some paper clips—maybe 4 of one size and 5 of another.
- Ask them to predict something that will be about 20 paper clips long.
- Explain their prediction.
- Test it.
- Then ask how the answer would change with this paper clip? (show a giant one)

Or it might be Grade 4

- Describe relationships that involve simple whole number multiplication
- What are the important markers?

It might be

- That some numbers are whole number bunches of other numbers, but some are not.
- That one number can be, e.g. twice another and A LOT more than it or NOT A LOT more than it.
- That visually saying one number is twice or three times or four times another is saying that if the first number of counters is in each box, I have 2 or 3 or 4 of those boxes.

So my task might be

- You add a number to its triple.
- List a bunch of possible sums- some big and some small.
- List a bunch of numbers that could not be the sums- some big and some small.
- Why do these answers make sense?

Or Grade 7

- Make predictions about linear growing patterns, through investigation with concrete materials

I might consider

- Maybe that they recognize how multiplication is useful in predicting future terms in patterns
- Maybe it's relating this pattern to an easier one to help predict future terms in the pattern
- Maybe it's recognizing that prediction might be enhanced by visual representations

So I might ask

- I created a pattern by adding the same number of items to the figure each time to get the next one.
- One figure used 35 items.
- Another used 59 items.
- What could the pattern have looked like?
What could the increase have been? NOT been?
- How could you predict the 100th term?

The 3-part lesson

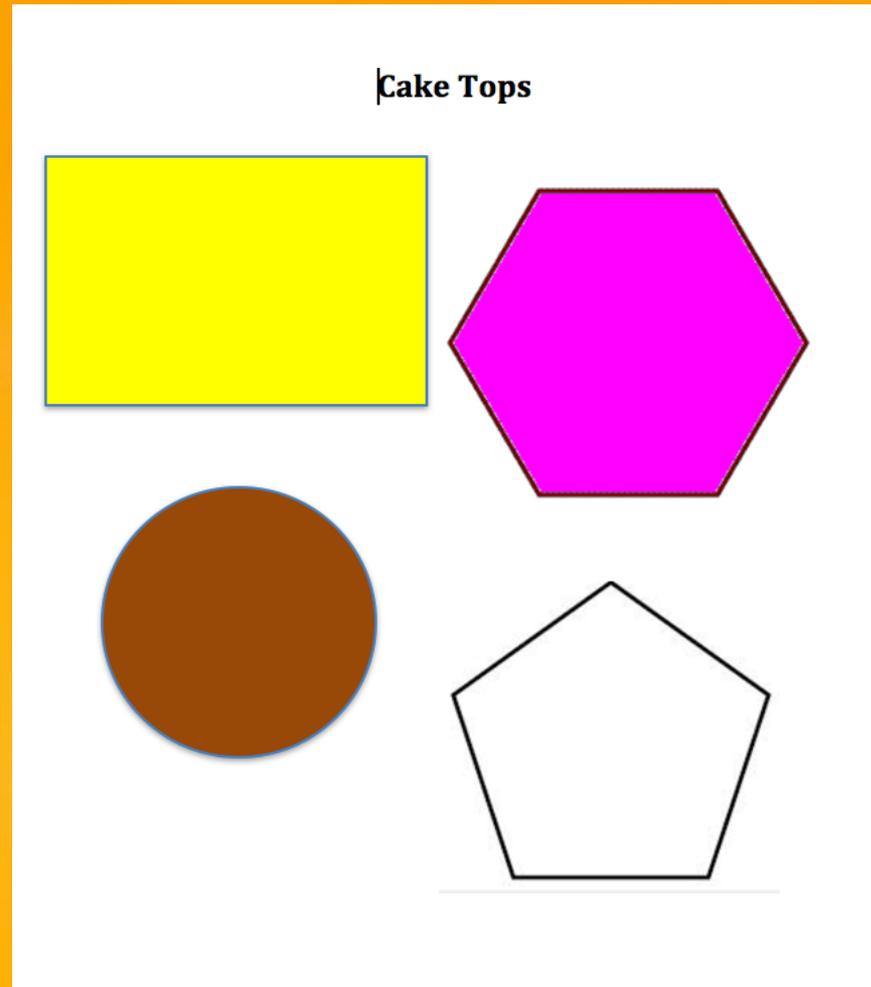
- Getting started
 - Action
 - Consolidation
-
- The action is a problem that is focused on a meaningful learning goal.
 - The consolidation is planned to bring out the big ideas based on the work on the action but about the learning goal.
 - The getting started is planned last to provide a way into the action.

An example

- **Getting Started:**
- <We provide kids with some fraction materials, e.g. four colour square tiles, fraction strips, fraction circles; pattern blocks; a number line from 0 to 1>
-
- Ask: What are some things you know about the fraction two thirds?
- Show what two thirds might look like.

An example

- **Main Task:**



An example

- Cut each cake into a DIFFERENT NUMBER of slices.
- For each cake, decide on how many slices to keep and how many to give away, but:
- You have to give away MOST of the cake.
- You have to give away a DIFFERENT NUMBER of slices for each cake.
- Tell what fraction of the cake you gave away and what fraction you kept. And how you know those fractions are correct.

An example

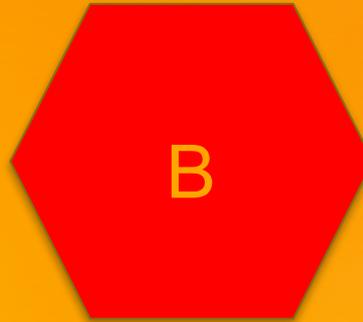
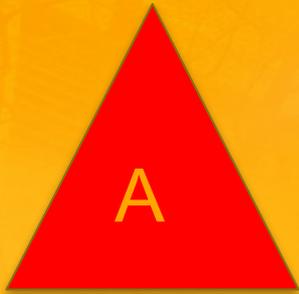
- Did the shape of your cake make any difference in deciding how many slices to cut it into? Explain.
- Look at the fractions you wrote.
- What does the denominator of each fraction tell?
- What does the numerator tell?
- Were the numerators and denominators closer together for the slices you kept or the slices you gave away? Why?

An example

- Look at the pairs of fractions you wrote for each shape.
- What do you notice about the numerators in each pair?
- Suppose you cut the cakes into a different number of equal slices. Would your fractions change? How?
- What fractions would you use if you gave away the whole cake?

Critical thinking

- A third shape is more like shape A than shape B.
- What might it look like?



Critical thinking

- There is a plate with more than 200, but less than 300, cookies.
- You think it is VERY EASY to share them equally onto either 3 plates or 5 plates.
- How many cookies might there have been?

Critical thinking

- You show a number with a LOT more ten rods than one cubes.
- What could the number be?

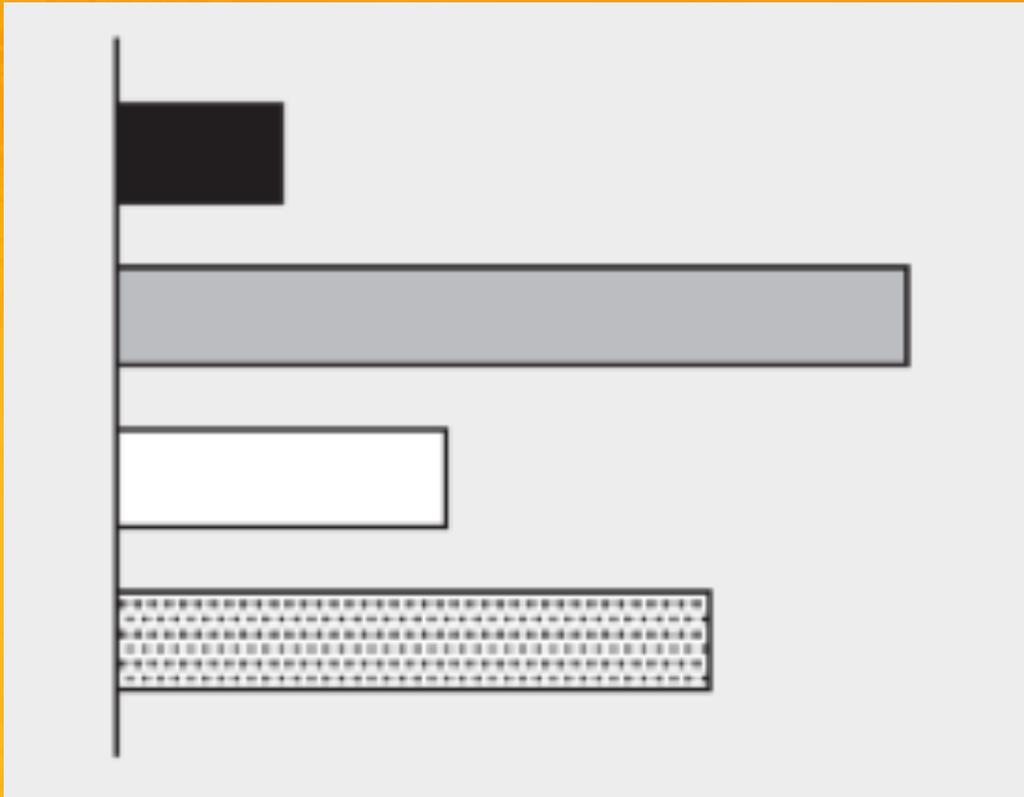
Critical thinking

- An amount you can show with 6 coins is added to an amount you can show with 3 coins.
- How many coins might you NEED to show the sum?

Critical thinking

- Which is a better description of how far away your birthday is?
- Is it 50 days?
- Is it about 7 weeks?
- Is it almost 2 months?
- Why is it better?

Critical thinking



Critical thinking

- How could 1000 be a lot?
- How could it be a little?

Critical thinking

- Who doesn't belong?
- 11
- 14
- 23
- 17

Differentiation

- The strategy I count on the most is the use of open questions.
- I also use parallel tasks.

Open questions

- The answer is 100.
- What might the question have been?

Maybe

- How old is really old?
- What is the first 3-digit number?
- What is a number with a digit sum of 1?
- What is 10×10 ?
- What comes after 99?
- What is a perfect score on a test?
- What is an easy number to multiply by?

More examples

- There are a LOT of people in a car. How many might that be?
- There are NOT MANY students in a school. How many might there be?

More examples

- The 10th shape in a pattern is a red triangle.
- What could the pattern look like?

More examples

- Choose a number for the red mark. Then choose an appropriate number for the blue mark.



More examples

- A number is just A LITTLE more than $\frac{1}{2}$.
- What might it be?

More examples

- Two decimal numbers are not that far apart.
- Their sum is 95.1.
- What could they be?

More examples

- You buy an item and give the clerk one bill. Your change is one bill and 6 coins.
- What might the price have been?

Parallel tasks

- Very similar tasks that differ only in complexity, but address the same ideas.
- They can be debriefed using common questions.
- For example...

Choice 1: Choose a number to subtract from 20. Draw two different pictures that would help you figure out the result.

Choice 2: Choose a number to subtract from 5. Draw two different pictures that would help you figure out the result.

- *How did you decide what number to choose?*
- *Could you have predicted about how much your answer would be?*
- *How do the pictures show the subtraction?*
- *How are your pictures alike? How are they different?*

Choice 1: Choose two 3-digit numbers to add for a total close to 400. How could you use base ten blocks to help you add?

Choice 2: Choose two 2-digit numbers to add for a total close to 40. How could you use base ten blocks to help you add?

- *How would you model the total you were trying to get close to?*
- *What would you do to show a number close to that number?*
- *How did you select the two numbers to make that total?*
- *How did using the blocks make the problem easier to solve?*

Lisa's dad was driving 16 miles every 15 minutes.

Choice 1: How far would he drive in 20 minutes?

Choice 2: How far would he drive in 2.5 hours?

- *Why was it more helpful to know the distance for 15 minutes than for, say, 17 minutes?*
- *What other numbers of minutes would it be very easy to figure out the distance for? Why those amounts of time?*
- *Is it possible to figure out the distance for any number of minutes or only certain numbers?*

Any questions

- Do you have additional questions you would like to discuss?

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