What should maths instruction look like in 2017?

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Most critical

- A positive classroom environment
- A caring teacher

I propose we must

- make LOTS of time for critical thinking
- make LOTS of time for creative thinking

Critical thinking

- Maths is clearly full of problem solving.
- There is often decision making in how to solve a problem.

Critical thinking

- But we should also involve decision making in interpreting problems.

For example

- Which bank account grew most last year?

	Ben’s	Laurie’s
January 1	$1000	$100
December 31	$1200	$250

Or

- A 3-D shape looks much more like a pyramid than a prism, but it is not either one.
- What might it look like? Why?

Or

- Estimate the number of crisps that all of the children in New Zealand eat in one week.
- What assumptions will you make?

Or

- You look at two different graphs.
- The line on one graph is MUCH steeper than the line on the other.
- What do you know about their slopes?

Critical thinking involves

- Review, analysis and assessment of information from different points of view.

Critical thinking involves

- There is always an element of setting criteria in order to do the analysis and assessment.

This might happen if

- I asked you to predict the result when you add a group of +1 counters to a slightly larger group of –1 counters.

This might happen if

- Children make different decisions about “slightly larger group”.

This might happen if

- I asked you to decide whether the number of centimetres in the perimeter of a rectangle is usually a greater or smaller number than the number of square centimetres in the area.

The issue is…

- What does “usually” mean?

This might happen if

- I asked you whether the square roots of consecutive numbers are always about the same distance apart.
- You experiment …

You notice

- that the square roots of two greater consecutive numbers (e.g. 1000 and 1001) are closer together than the square roots of two smaller consecutive numbers (e.g. 8 and 9).
- I ask why.

What would be good criteria?

- Just an example?
- Do we need the use of algebra?
- Could/should the argument be visual?
- What other criteria?

Example

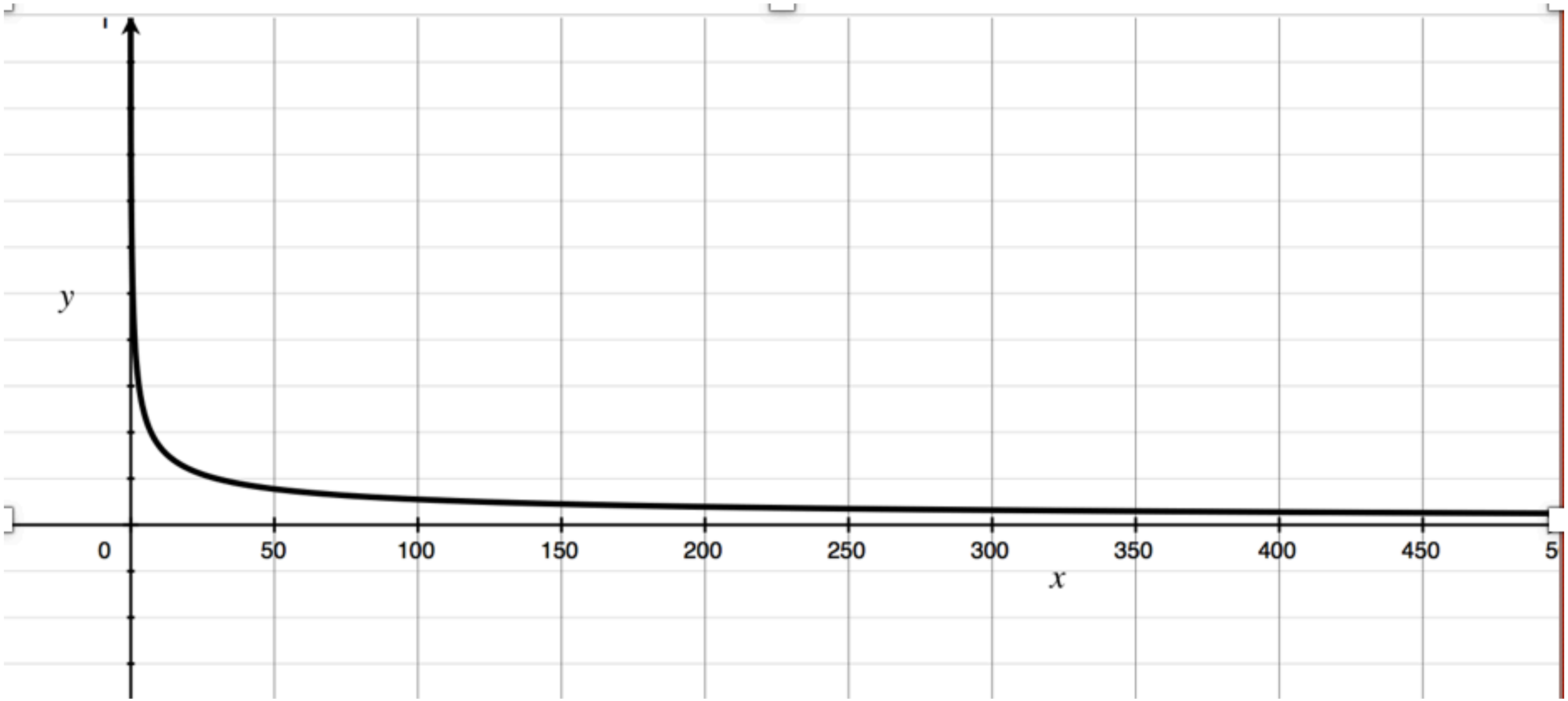
- Compare square roots of 3000 and 3001 (54.77226 and 54.78138) to square roots of 8 and 9 (2.8284 and 3)

Algebra

- If a = $\sqrt{x+1}$ and b = $\sqrt{x}$, then
- $a^2 - b^2 = 1$ and a – b = $\frac{1}{a+b}$.
- But as x increases, so do a, b and a + b, so

$\frac{1}{a+b}$ decreases, which means a– b decreases.

Graph



A different visual

For a younger crowd

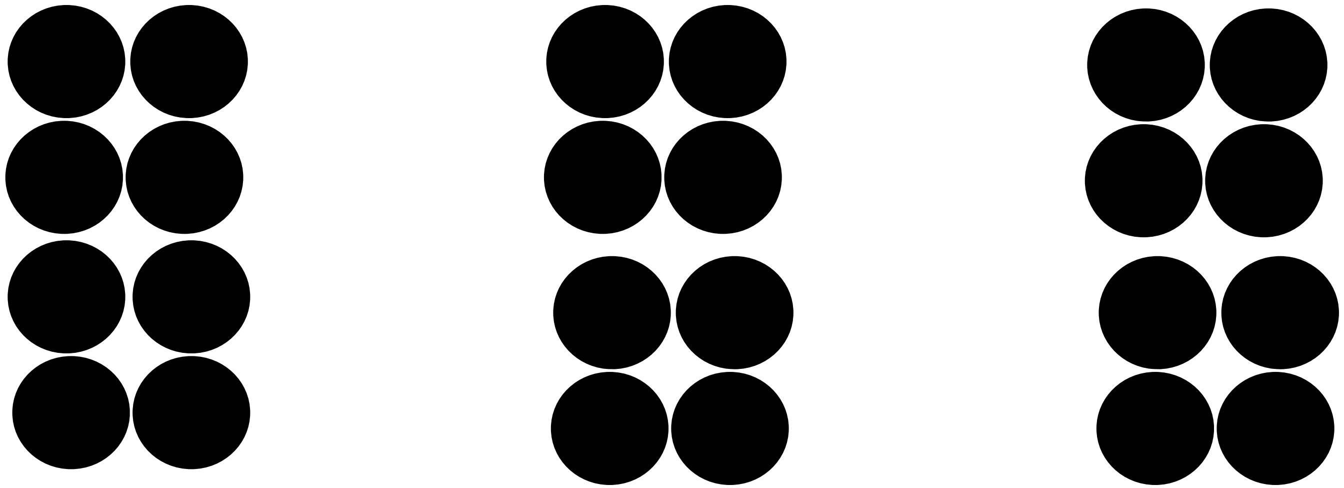
What would be good criteria?

- What would be the criteria to convince someone that every multiple of 8 has to be a multiple of 4?

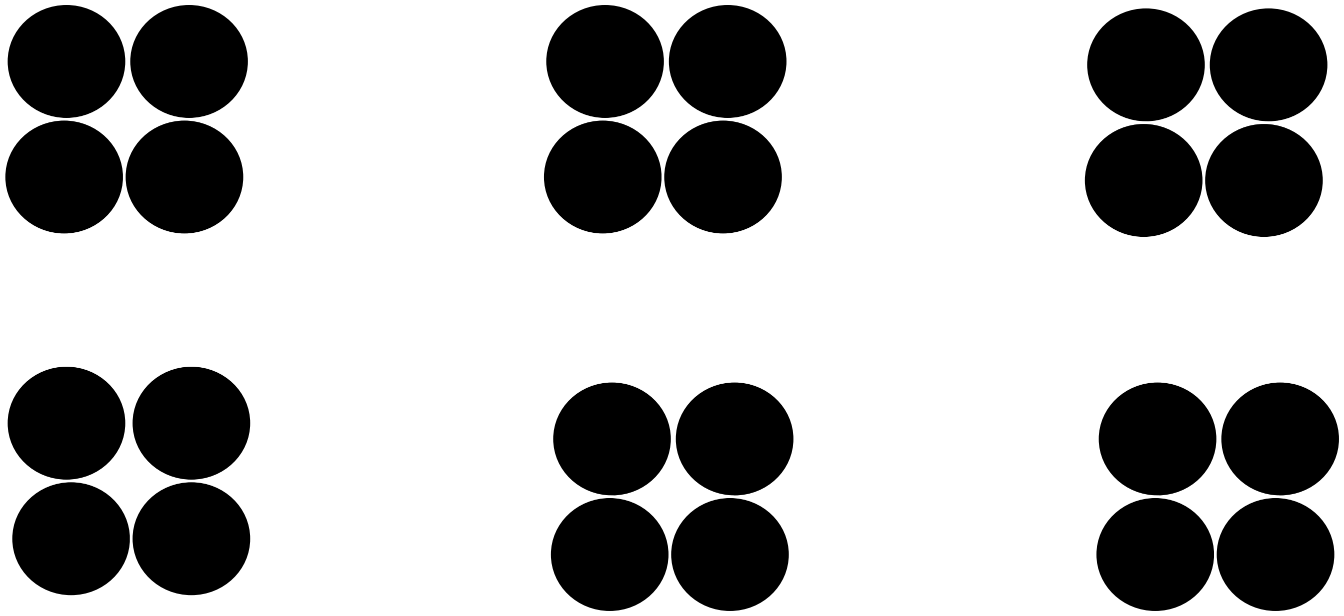
Multiple of 8 vs. multiple of 4

- Can you just say because $8 = 2 \times 4$?
- Do you need to write $8n = 4 \times (2n)$?
- Can you draw a picture like this one?

Multiple of 8 vs. multiple of 4



Multiple of 8 vs. multiple of 4



Critical thinking is said to involve

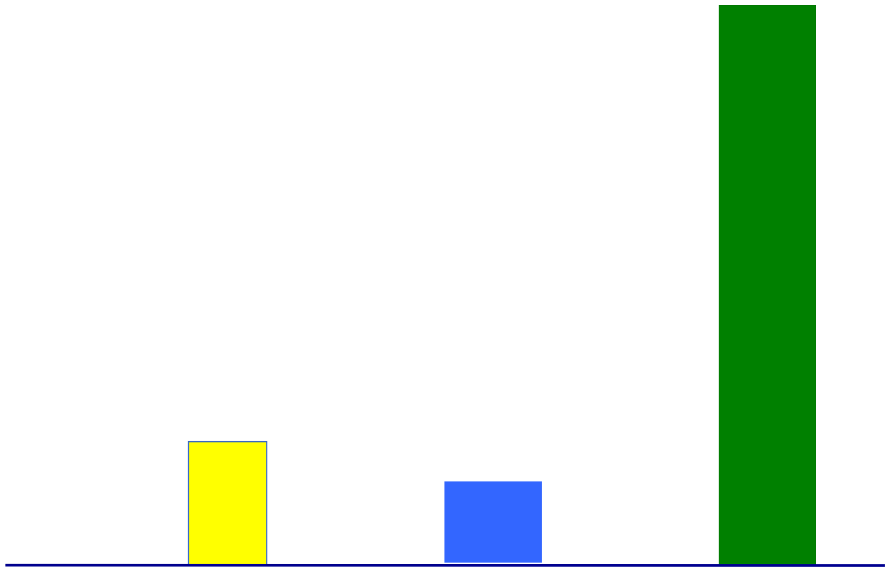
- Analysis
- Evaluation
- Conceptualization
- Recognition of assumptions

I could ask

- When you divide one number by another one, can the quotient be close to the divisor?
- How can you predict before you divide?

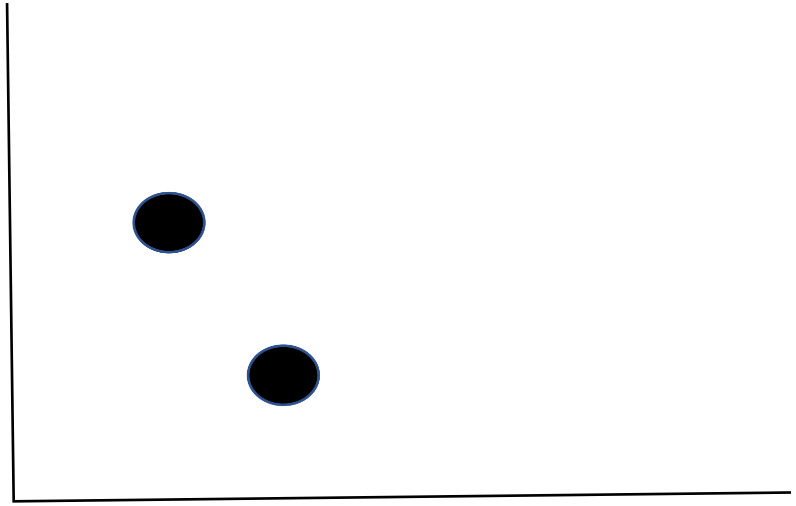
Or I could ask

- What sort of data might this graph be about? Why?



Or I could ask

- for both reasonable and unreasonable possible equations for the line connecting these two points
-



Critical thinking involves

- Reflection on your own and others' thinking and reasoning

For example

- I might ask other students which argument they find stronger to explain why the sum of three consecutive whole numbers is a multiple of 3.

Argument 1

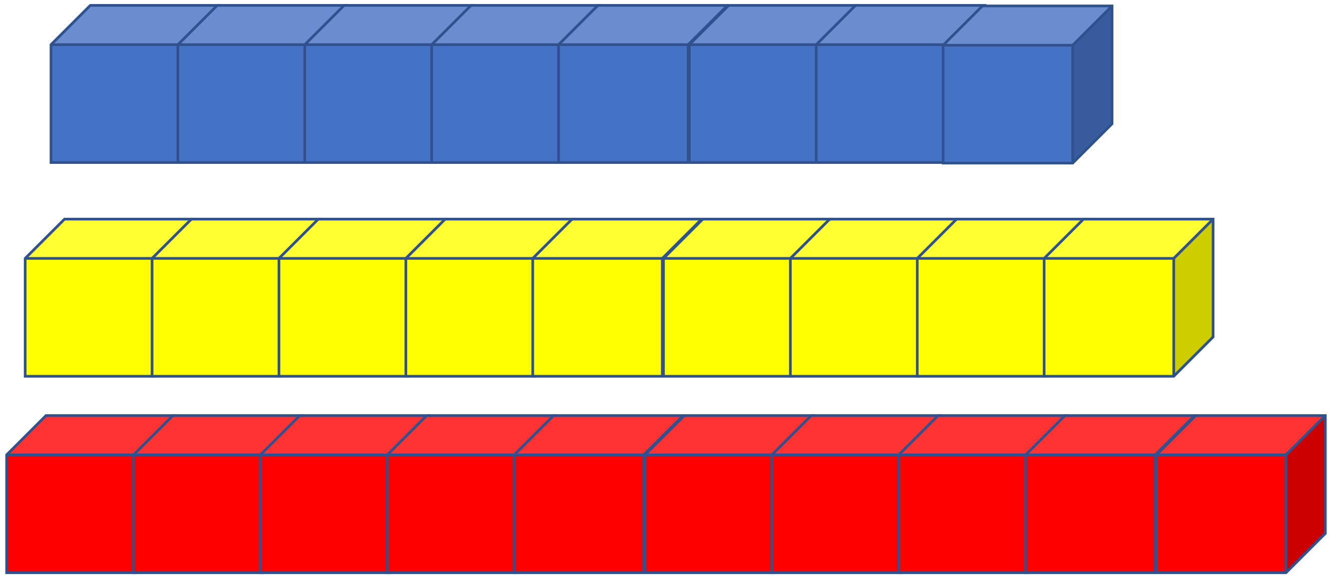
- Let n be the first number.
- $n + (n + 1) + (n + 2) = 3n + 3$
- $3n + 3 = 3(n + 1)$
- Since n is a whole number, so is $n + 1$ and so $3(n + 1)$ is a multiple of 3.

Argument 2

- Let n be the middle number.
- $(n - 1) + n + (n + 1) = 3n$
- Since n is a whole number, 3n is a multiple of 3.

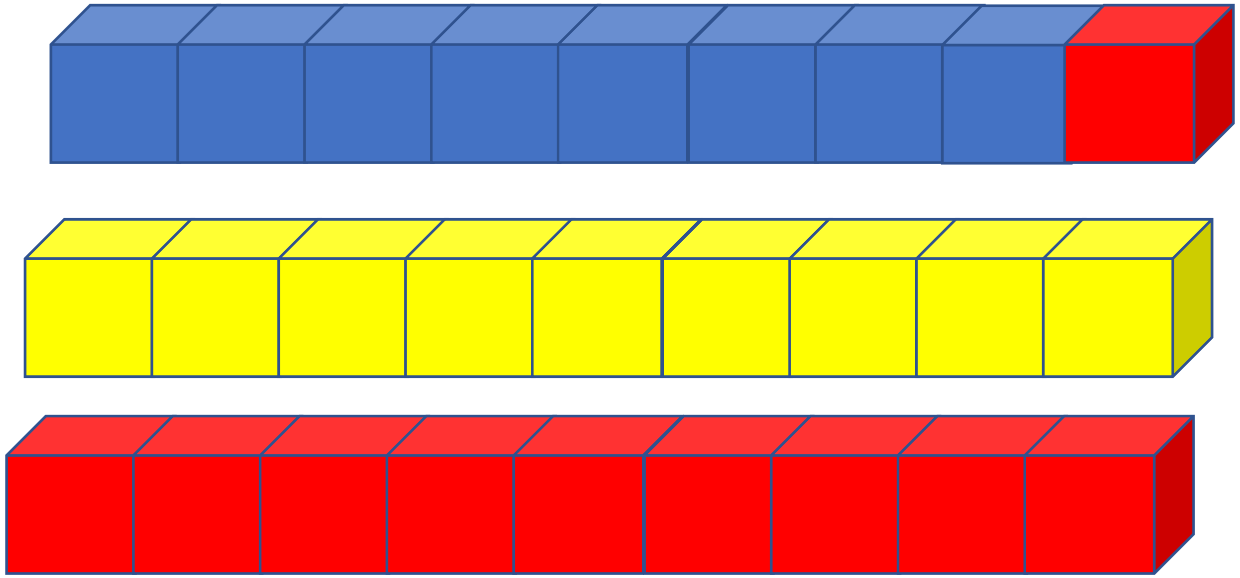
Argument 3

- I can show 3 consecutive numbers with linking cube towers that are 1 apart in height.



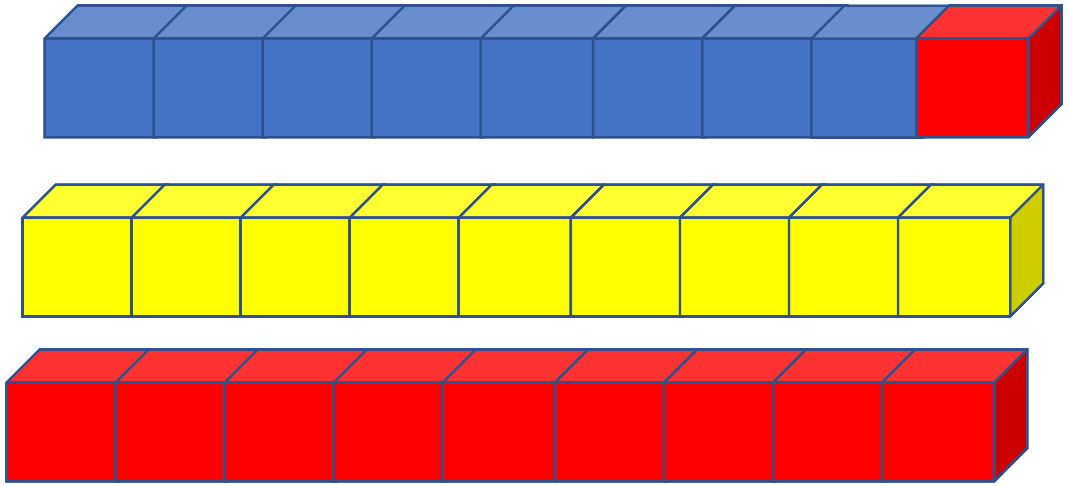
Argument 3

- Move a red cube to the brown ones.
- There are 3 equal towers, so it's a multiple of 3.



Argument 3

- And I could add as many cubes as I want to the left end of all three towers and nothing changes, so number size does not



Asking the right questions

- This is the heart of the issue.
- We need to ask questions that encourage or even demand critical thinking behaviours.
- You could make it the “normal” way you teach.

Compare

- Read this number: 4023

- Do greater numbers always/usually/sometimes or never take more words to say than lesser numbers?

Compare

- What are the sine and cosine of 60°?
- Is sine more like cosine or more like tangent? Why?

Compare

- What are 10% of 300 and 90% of 7?
- Which is more: a little percent of a big number or a big percent of a little one?

Compare

- What is $\frac{2}{3} \div \frac{3}{4}$?
- Could you use a ¼ cup measure to measure out thirds of a cup? How or why not?

Compare

- What is the solution to
- $\frac{3}{4}$ x – 2 = $\frac{5}{8}$ x + 9?

- Do equations with fractions in them usually have whole number solutions or fraction solutions?

There are appropriate questions at ALL levels.

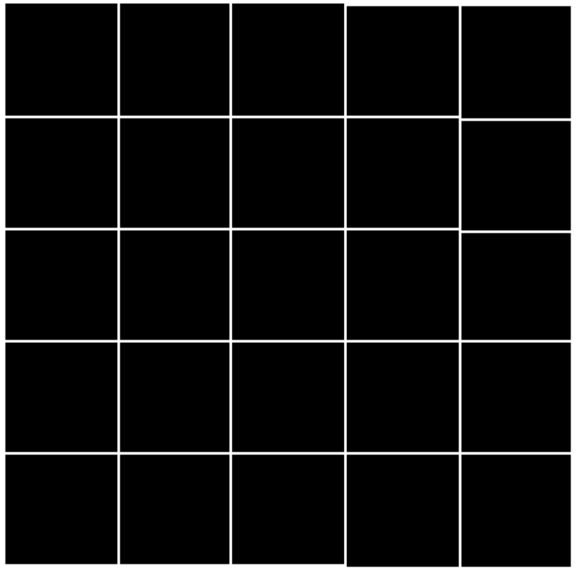
- What could you measure about an apple? How would you

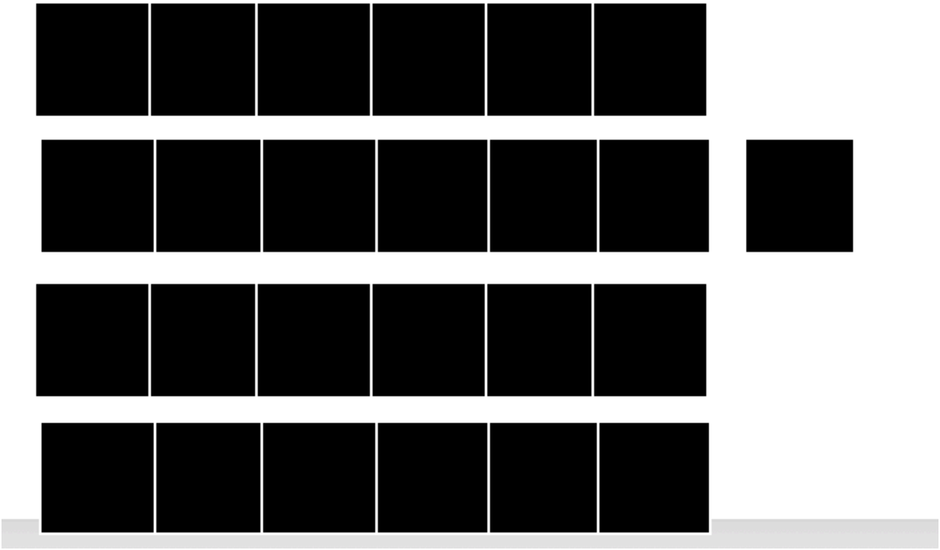
Or

- There are a lot of people at a rugby game.
- How many might that be?

Or

- What do you learn about 25 from each of these representations?





30 – 5

100 ÷ 4

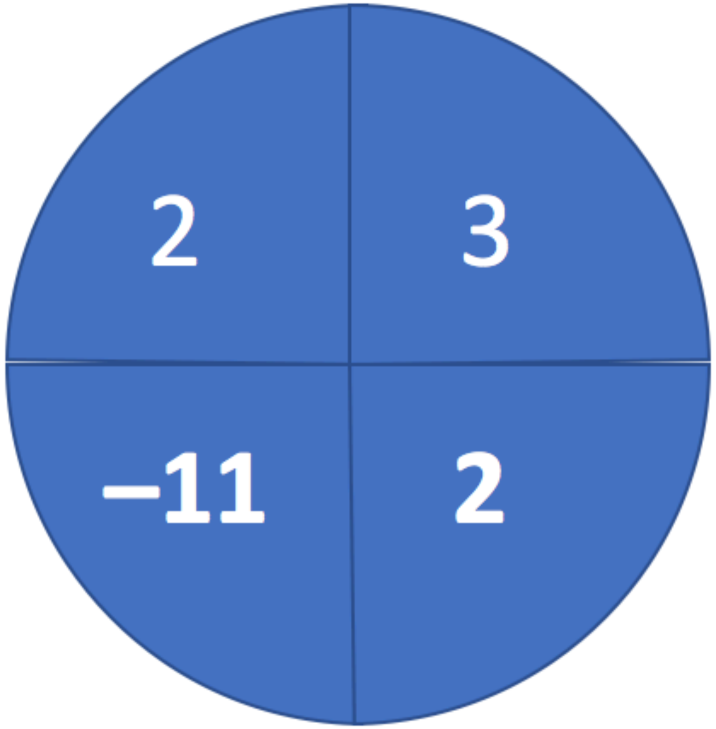
Or

- An algebraic equation is a lot MORE like $10 = 3x – 2$ than it's like $5x + 4 = 20$.
- What might it be? Why?

Or

- You spin a spinner.
- The points you win or lose are indicated on the spinner.
- What might be a good example of what the spinner could look like if your expected value is a loss of 1?

Might be



This is a good example because it's easier to make sure the expected value is right when the section sizes are equal.

Or

Is it a good idea to call a fraction a part of a whole?
Why or why not?

Or

Would you use the same strategy or different strategies to decide which number in each pair is greater? Explain.

Pair 1: $\frac{3\pi}{2}$ and $\frac{5\pi}{8}$

Pair 2: $3\sqrt{20}$ and $\sqrt{50}$

Or

A fraction is made up of **a LOT of small pieces**. What might it be? Is there a fraction it cannot be?

Or

I subtracted an integer from another one and the answer was less than but close to what I subtracted. What could they have been?

Creative thinking

Not terribly different from critical thinking since we have seen some overlap, but…

We want

Fluency
Flexibility
Novelty

What would you say?

You are asked how the numbers 4.1 and 5.23 are alike and different.
What are some “unusual” things you might say?

What would you say?

Give a reason why each computation does not belong with the others.

What would you say?

8×9

23×11

12×4

$42 + 68$

What would you say?

You have to create two very different shapes that are both sort of like a triangular prism, but also sort of like a triangular pyramid.
What might they look like?

What would you do?

What might be a unique way to show the number $\frac{2}{3}$?

What would you do?

What might be an UNUSUAL way to continue this pattern: 4, 9, ….?

What would you do?

Create an UNUSUAL design that looks the same if you turn it 90°.

What would you do?

Draw a picture that has no clock in it, but would help me guess the time of day it is.

What would you do?

To solve a certain problem, you need to know about negative reciprocal slopes for perpendicular lines.
What might the problem be if I don’t just ask you directly for the slopes?

What would you do?

You have to create a problem that might be solved using a quadratic function, but it’s not so obvious that is what you’d have to do.
What might the problem be?

What would you do?

You have to create a geometric diagram where you are only given one or two angle sizes and you can figure out a LOT of other angle sizes.

What might it look like?

What would you do?

Tell an interesting story that this equation might describe.

$$4x - 10 = 54$$

What would you do?

Draw an unusual graph and tell a story about it.

What would you do?

Draw a picture that would help you figure out a set of data to satisfy these criteria: The mean is 1 less than triple the median.

What would you do?

Draw a picture that would help show why when you multiply []5 x []5, you write []x([]+1) followed by 25.

Do you notice?

Math is not divorced from creativity even though many in the general public see it this way.
The math thinking itself can be creative; it doesn’t have to be about a poem, writing a play or music.

Do you notice?

Thinking is more inviting.
Asking students to think tells them that you have faith in their abilities.
Thinking leads to sense making, and that is always a good thing.

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