Teaching Math through Big Ideas: What Does it Look Like

Marian Small

Niagara on the Lake
August, 2017

Consider measurement

- Some measurements of an object are independent of other measurements, but some are not.

For example

- You know that the perimeter of a regular hexagon is 15 cm.
- What other measurements of the hexagon are you sure of?

Or

- You know that the volume of a cylinder is 200 cm^2.
- What one other measurement could I tell you that would tell you its surface area? (or is there one?)

Or

- The hypotenuse of a right triangle is 10 cm. What other measures of the triangle are you sure of?

Or

- You know the surface area of a triangular prism and the perimeter of its base.
- Are there other measures that you are sure of?

Or

- You know the surface area of a triangular prism and the perimeter of its base.
- Is there one other measurement that would automatically give you other measures? Explain.

Or

- The sine of an angle in a right triangle is half the value of the cosine.
- What other measurements are you sure of?

So during instruction

- Instead of focusing on the actual values, you focus on what information automatically tells you other information and/or
- What you automatically know if you know some measurements.

In terms of assessment

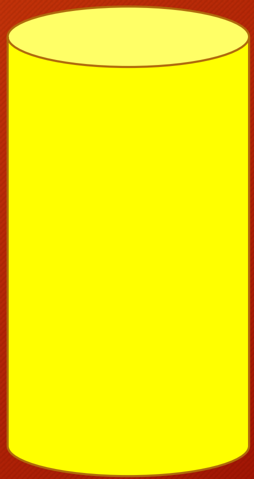
- You might ask something like:
- How come you know the surface area of a sphere if you know its volume, but not the surface area of a rectangular prism if you know its volume?

Or it could be

- Knowing the measurements of one shape can sometimes give you information about the measures of another shape.

For example

- What are some other shapes that have to have four times the volume of this one?



Or

- One pyramid has 12 times the volume of another.
- What could the two pyramids be?

Or

- You know that the legs of a right triangle are a and b.
- How does the hypotenuse of that triangle compare to the one with a triangle with legs of ½a and 1/3b?

In terms of instruction

- You don’t necessarily do one shape at a time, but you focus on the relationship idea.
- For example, what can I do to each shape to double the volume or surface area?

In terms of assessment

- Give an example where knowing the surface area of one figure helps you figure out the surface area of another.

Let’s think about algebra

- Interpreting equations or expressions involving unknowns is based on using properties of numbers and meanings of operations.

For example

- Choose a value that you want $Ax^2 + Bx + C$ to be worth when x is about 10.
- Give different possibilities for A,B and C.

Or

- Choose a value that you want A^{Bx} to be worth when x is about 10.
- Give different possibilities for A and B

Or

- An inequality involving the variable m is true when m = 4, but false when m= 8. List some possible inequalities that are linear and some that are not.

Or

- Create an equation involving integer exponents that you know is true. Tell why it is true.

In terms of instruction

- The focus is on how what you know about the operations and numbers helped you figure these things out.

In terms of assessment

- You might ask

- What do you know about division that would help you interpret:
- $(4x - 2)/(2x - 1) = y$?

Let’s think about algebra

- Variables can be used to efficiently describe relationships.

You might ask

- You chose two numbers.
- When you added them, the answer was double what you got when you subtracted them.
- What could they be?

Or

- How can you more efficiently say this?
- Double a number.
- Triple the number that is one more than your first number.
- Add 2 to the sum

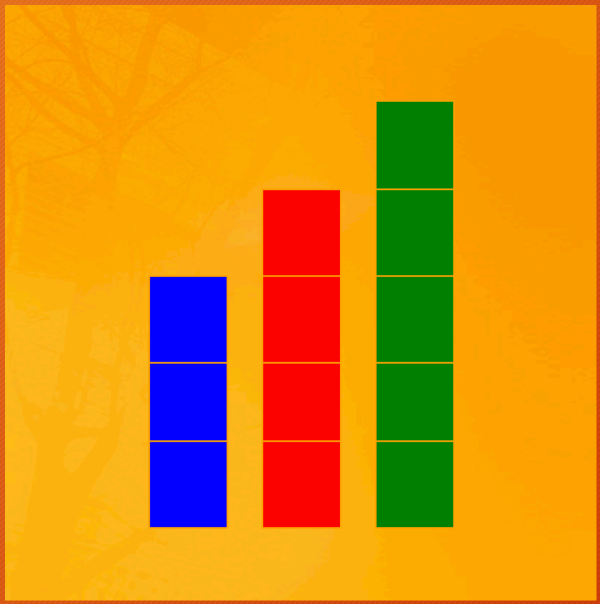
You might ask

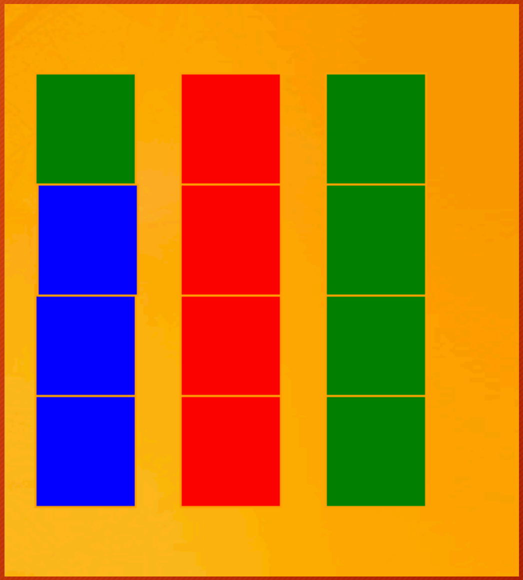
- Choose three consecutive whole numbers. Add them.
- Repeat with three other consecutive whole numbers.
- What do you notice?
- How could you describe this algebraically?

Algebra:

- $n + (n+1) + (n+2) = 3(n+1)$ OR
- $(n - 1) + n + (n + 1) = 3n$

Visually





Or

- What does $a^2 + b^2 = c^2$ mean?

In terms of instruction

- You focus on how long it would take you to work out things or say things if you didn't use variables.

In terms of assessment

- Do these say the same thing or not? If not, why not?
- If yes, which description do you prefer? Why?

- Choose a number.
- Multiply it by 2 more.
- Compare the result to the square of the number between them.
- OR $(n-1)(n+1) = n^2 - 1.$

Let’s think about algebra

- The same algebraic expression or equation can be related to different real-world istuations, and different algebraic expressions or equations can describe the same situation.

You might ask

- The equation $4x - 5 = 15$ describes two VERY DIFFERENT situations.
- What might they be?

Or

- Choose a quadratic function.
- Describe two different situations it might be describing.

In terms of instruction

- You focus on any situation and ask students to write more than one equation to describe it.

- You also regularly ask for different situations for any equation.

In terms of assessment

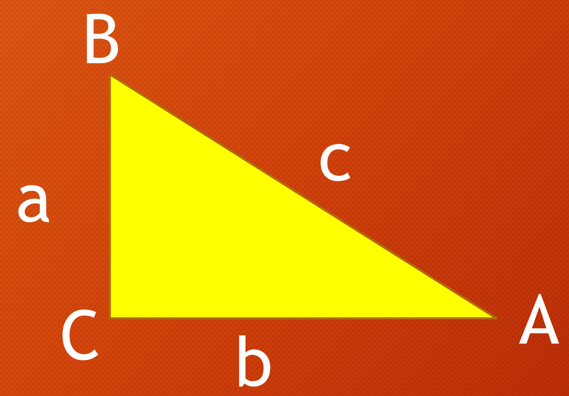
- Sarah says that ANY equation can be written in more than one way.
- Do you agree or not?
- Convince me your answer makes sense.

Let’s think about algebra

- Many equivalent representations can describe the same situation or generalization. Each may give somewhat different insight into certain characteristics of the situation or generalization.

Or

- Use different equations to describe what you see here.



It might be

- Choose a linear relation.
- Show it as a graph, a table of values and an equation.
- Describe what each shows best about the relation.

It might be

- Consider the function $f(x) = (6-x)^2$.
- How could you represent it as:
- A sum, a difference, a product, a quotient, and a composition of two functions?

In terms of instruction

- You focus on what representations are convenient? Help you see other relationships? etc.

In terms of assessment

- It could be:
- Consider vertex form, general form and factored form for a quadratic.
- Which is most useful when?

Let’s think about algebra

- Comparing mathematical relationships either algebraically or graphically helps us see classes of relationships with common characteristics and help us describe each member of the class.

You could ask

- Create a relation that behaves a lot like y – 3x, but not exactly. How are they alike and different?

You could ask

- Are $y = 3(x - 2)^2 + 5$ and $y = 8(x - 2)^2 + 5$ more alike when x is between 0 and 5 or when x is closer to 100? Why?

It could be

- Which two of these are most alike and why?
- $Y = 3x - 4$
- $Y = 3x + 8$
- $Y = -3x - 4$

It could be

- Describe four ways that a quadratic and linear relationship are different.

It could be

- A bunch of polynomials have a root of 3.
- What else do they have in common?

It could be

- A bunch of right triangles have a sine between 0.5 and 0.6.
- How else are they alike?

It could be

- Draw a graph of a parabola that grows quickly as x increases from 10 to 20 and the graph of a parabola that grows slowly in that domain. What are their equations?

It could be

- You use algebra tiles to factor a quadratic expression.
- There were 3 rows of 5 tiles.
- What might the quadratic and factors have been?

In terms of instruction

- The focus is on noticing similarities and commonalities, as well as differences, in a variety of algebraic situations (or measurement or geometric ones).

In terms of assessment

- You might ask:
- Think of some ways that a bunch of quadratics could be alike- but different from other ones.
- How would their algebraic descriptions make that clear?

Let’s think about algebra

- The transformations that are fundamental in determining the relationships between shapes can be described algebraically.

It could be

- The function $f(x) = \sin x$ is transformed by translating to the right and then vertically stretching.
- Draw the new function. What is its equation?

It could be

- An exponential function is a LOT like $y = 2^x$.
- What might it be and how are the graphs similar?

It could be

- You graph a sinusoidal function with an amplitude of 3 that stays above the x-axis.
- What might the function be?

In terms of instruction

- You focus less on the “rules” and more on the “signals” that various transformations happen.

In terms of assessment

- The graph of a parabola is far to the right, up from and much narrower than the graph of another.
- What could the two equations be?

Let's think about algebra

- Limited information about a mathematical relationship can sometimes, but not always, allow us to predict information about that relationship.

It could be

- You know that $3x + 4 = 10$.
- Without solving the equation, tell what else you know about x.

It could be

- WITHOUT SOLVING, tell why the solution to $100x + 6 = 87x + 2$ HAS TO be negative.

Or

- What if $3/x = 14/30$?
- What do you know about x WITHOUT SOLVING?

Or

- You know that a line goes through (4,2) and slants up and to the right.
- Name at least one other thing that you are SURE is NOT true about that line.

Or

- An angle in a right triangle has a sine of 0.2. What else do you know about the triangle?

In terms of instruction

- The focus is how in math it is almost always about taking what you know and deducing other things form it.
- You repeatedly ask; What else do you know now?

In terms of assessment

- You know that when you multiply two linear binomials, the resulting expression has a linear term with a negative coefficient.
- What else do you know?

- Questions?

Download

- Go to www.onetwoinfinity.ca
- Go to Recent Presentations
- UGSec