MATHEMATICS IN GR 9 - 12

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WE WILL TALK ABOUT

- Making decisions about what might be somewhat less crucial in Gr 9 – 12 curriculum
- Inserting data management into grades 10 and 11 (a bit)
- Developing deeper thinking and more meaningful communication in Gr 9–12 math

CUTTING BACK ON CURRICULUM

- You don't really want to skip things, but you do want to cluster and to highlight some things more than others.
- We often spend a long time perfecting skills that need not be perfected; instead, those concepts could be embedded in inquiries or problem solving situations.

FOR EXAMPLE: GR 9

- You can integrate the work on equation solving with the use of measurement formulas.
- You can integrate the work with solving problems modelled by first-degree equations with the work on linear relations.

FOR EXAMPLE: GR 9

- Expectations in the section Understanding Characteristics of Linear Relations overlap those in Connecting Various Representations of Linear Relations and some in Analytic Geometry

FOR EXAMPLE: GR 9

- You can make your work with polynomials less time consuming by focusing on problem solving and not facility with skills.
- E.g. You multiply two polynomials and can model the multiplication with 35 tiles. What could the multiplication have been?

FOR EXAMPLE: GR 9

- Students have already worked with composite figures in 2-D and with optimization with area/perimeters of rectangles, so maybe those problems are less critical and can be short activities, not lessons.

FOR EXAMPLE: GR 9

- Some might minimize the geometry investigations but for some students this is so interesting--- so it's a tricky decision.

FOR EXAMPLE: GR 10

- Again, expectations in different groupings overlap
- The amount of time on factoring and expanding and solving quadratic equations has to be considered given we have digital tools for this. Instead, less detail and more on problem solving might be better.

FOR EXAMPLE

- How could this chart help you see the factors of $2x^2 + 3x + 1$?

CHART

x	$2x^2+3x+1$
0	1
1	6
2	15
3	28

CHART

x	$2x^2+3x+1$	
0	1	
1	6	$= 2 \times 3$
2	15	$= 3 \times 5$
3	28	$= 4 \times 7$

CHART

x	$2x^2+3x+1$	
0	1	
1	6	$= 2 \times 3$
2	15	$= 3 \times 5$
3	28	$= 4 \times 7$

is $(x + 1) \times (2x + 1)$

FOR EXAMPLE

- Are the solutions for $ax^2 + bx + c = 0$ usually closer to the solutions of $(a+1)x^2 + bx + c = 0$ or $ax^2 + (b+1)x + c = 0$ or $ax^2 + bx + c + 1 = 0$?
- E.g. compare solutions for $3x^2 + 2x - 5 = 0$ and $4x^2 + 2x - 5 = 0$ and $3x^2 + 3x - 5 = 0$ and $3x^2 + 2x - 4 = 0$.

SOLVING SYSTEMS

- You have to decide if it's worth the time to perfect algebraic skills for elimination, substitution or not.
- Even if you expose students to these methods, you have options for how long to spend or how to approach the situations.

YOU MIGHT SOLVE PROBLEMS LIKE

$4.77

$5.07

ANALYTIC GEOMETRY

- You have to decide the amount of time you want to give to using analytic geometry to explore geometric properties.

THE PRINCIPLES ARE ABOUT

- Clustering
- Using activities rather than lessons
- Using investigations rather than perfecting skills

SMALL DATA PROJECTS

- An idea for Grade 10 or 11
- For baseball fans
- **The Baseball Archive:** Sean Lahman's website features baseball statistics (beginning in 1871!), including player salary data.
- http://seanlahman.com

PROJECT I HAVE SEEN DONE IN ONTARIO

- Compare the Blue Jays and Rangers

Question: Compare the Wins for the Bluejays and the Rangers since 1977 (not including two shorten seasons 1981 and 1994). Who is the better/more consistent club?

Non-Mastery: Redo	Initial Mastery: 7/10	Mastery: 8.5/10	Advanced Mastery: 10/10
Unable to demonstrate the main ideas of the learning target	Demonstrates the main ideas of the learning target	Demonstrates a thorough understanding of the learning target	Understands all aspects of the learning target

OR MAYBE THIS

- http://www.eeps.com/zoo/index.html

Roller Coasters

Data an some of the world's largest roller coasters, with some smaller ones thrown in.

Attributes:
largest_drop is in feet
height is the maximum height of the coaster in feet
length is in feet as well
top_speed is in miles per hour

The main question is, what determines the **top_speed**? We can make conservation-of-energy arguments that suggest it might be related to the **largest_drop**. Do they hold up when you look at the data? How much energy seems to be lost? Do any coasters seem to gain energy?

Also: what assumptions are we making when we expect this energy calculation to match the data? Do the assumptions seem reasonable?

Oh: don't forget (as the curator did) to convert speed into, say, feet per second!

Roller coaster data from http://ultimaterollercoaster.com/coasters/
(found by Bryan Cooley, Oct 2002)

You can also find a lot of roller-coaster data at rcdb.com. Click on "census" and follow your nose.

Name	Location	Track	Type	largest_drop	height	length	top_speed
Millennium Force	Cedar Point, Sandusky, OH	steel	hypercoaster	300	310	6595	93
Steel Dragon 2000	Nagashima Spaland, Japan	steel	hypercoaster?	306	318	8133	95
Goliath	Six Flags Magic Mountain, Valencia, CA	steel	hypercoaster	255	235	4500	85
Titan	Six Flags Over Texas, Arlington, TX	steel	hypercoaster	255	245	5280	85
Desperado	Buffalo Bill's Hotel and Casino, Primm, NV	steel	hypercoaster	225	209	5483	80
Son of Beast	Paramount's Kings Island, Kings Island, OH	wood	looping, twister	214	218	7032	78.3
Mean Streak	Cedar Point, Sandusky, OH	wood	twister	155	161	5427	65
Hercules	Dorney Park, Allentown, PA	wood	double out and back	157	157	4000	65
The Boss	Six Flags St. Louis, Eureka, MO	wood	terrain, twister	150	122	5051	66.3
Rampage	Visionland Amusement Park, Bessemer, AL	wood	twister	120	120	3500	55
Demon	Paramount's Great America, Santa Clara, CA	steel	looping corkscrew	82	95	2300	45
Grizzly	Paramount's Great America	wood	double out and back	88	90	3250	50
Project Stealth	Paramount's Great America	steel	flying dutchman	100	115	2766	51
Skyliner	Lakemont Park, Altoona, PA	wood	double out and back	45	60	2400	40
Cobra	Six Flags Marina World, Vallejo, CA	steel	family, tivoli	25	26	1164	18

<text form of the data>

MIGHT FOCUS ON

- Statistics like range, SD, median, mean
- Correlational data

OR SOMETHING INVOLVING SURVEYS/SAMPLING

- Students might do a small project where they explore the effect of sampling different sorts of or numbers of people on an issue.

DEVELOPING DEEPER THINKING

WHAT DO YOU THINK?

Is it good enough if students can answer A and not B?

A: What is x if 3/x = 14/30?

B: How do you know that x has to be more than 6 without actually solving?

WHAT DO YOU THINK?

Is it good enough if students can answer A and not B?

A: Create a table showing partial variation.

B: Describe a situation where if you double one variable, you double the other. Write an equation describing the variables.

WHAT DO YOU THINK?

Is it good enough if students can answer A and not B?

A: Simplify $\sqrt{18}$.

B: You divide $\sqrt{a}$ by $\sqrt{b}$ and the answer is a rational number. What could a and b be if they are not equal?

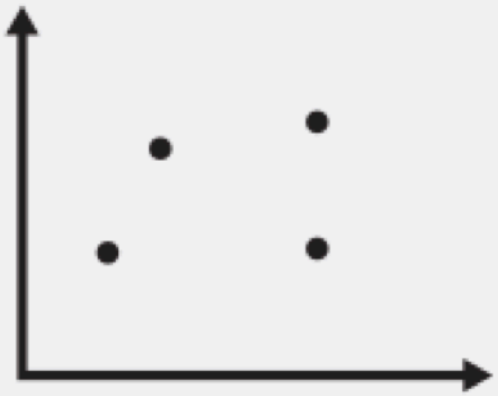
DEEPER THINKING

- It's not about just doing math; it's about having insight into it.
- It's about seeing what's going on.

FOR EXAMPLE

A line passes through two of these points:

What could the equation of the line be?



FOR EXAMPLE

- A and B are different kinds of shapes.
- A has a greater volume than B.
- B has a greater surface area than A.
- Sketch and show the dimensions.

FOR EXAMPLE

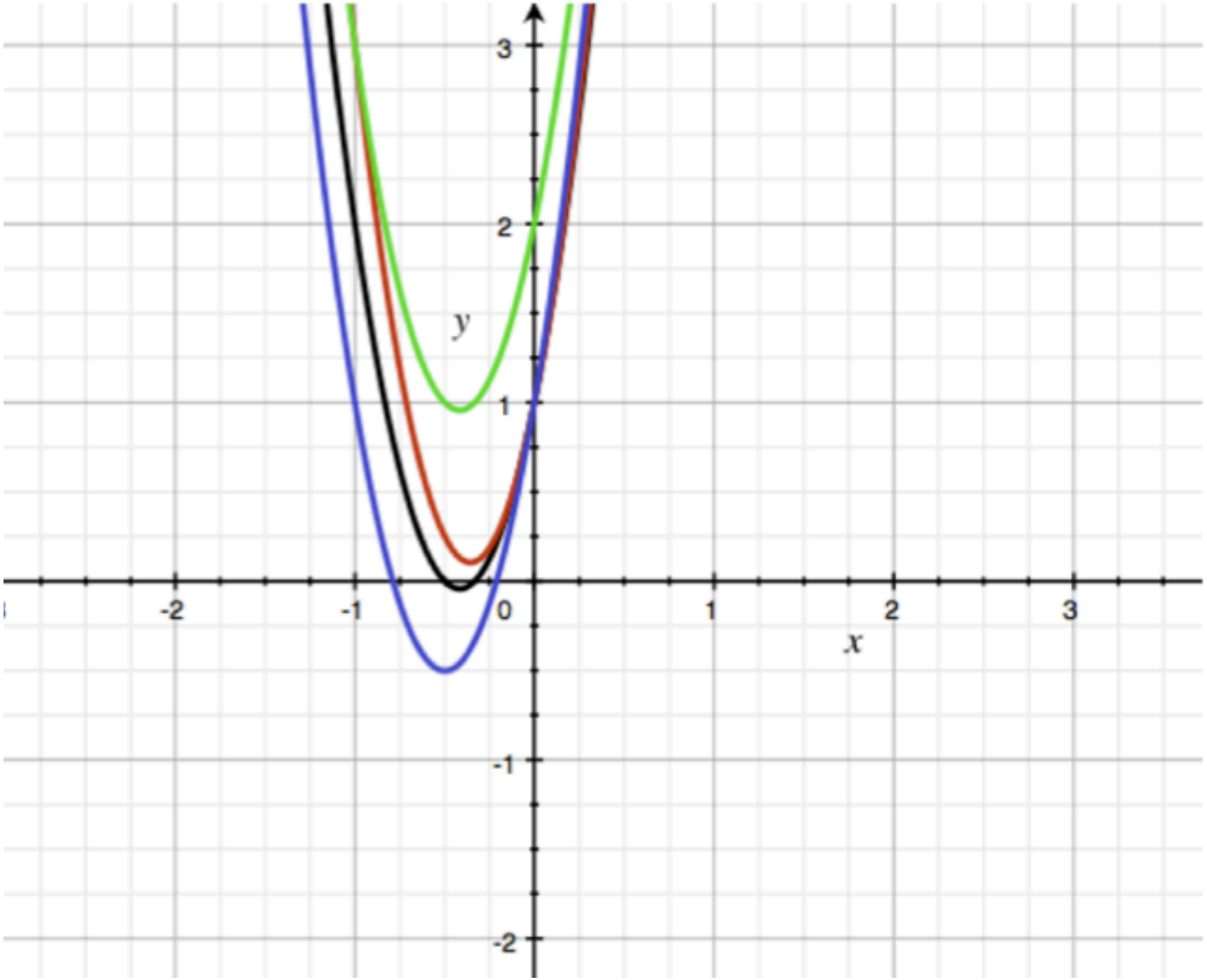
One cell phone plan is a better deal if you use 120 minutes a month but worse if you use 100 minutes a month.

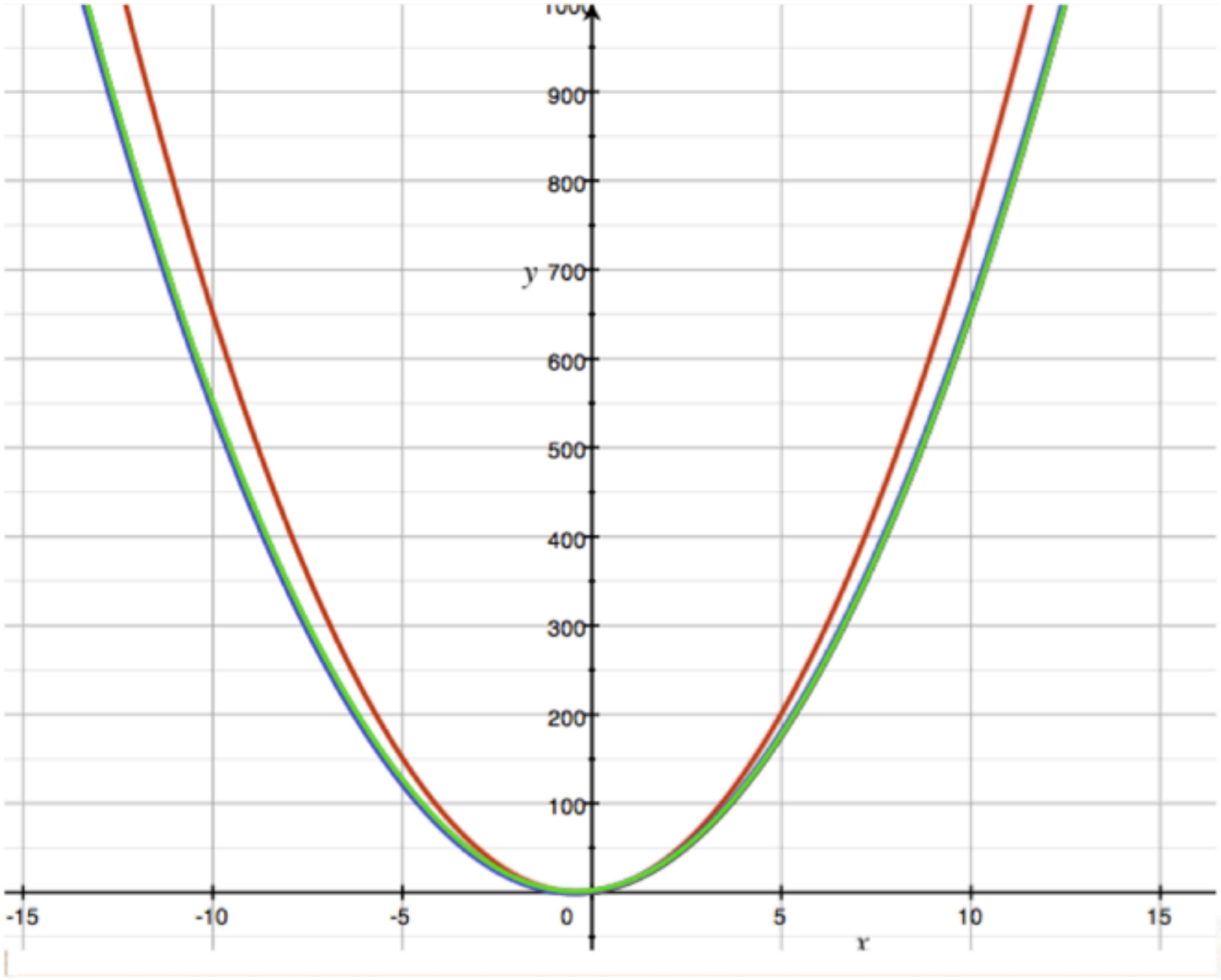
What could the plan be?

FOR EXAMPLE

Which of the following is most like $y = 6x^2 + 5x + 1$?

- $y = 7x^2 + 5x + 1$
- $y = 6x^2 + 6x + 1$
- $y = 6x^2 + 5x + 2$


3
2
1
-1
-2
-2
-1
0
1
2
3
y
x


900
800
y 700
600
500
400
300
200
100
-15
-10
-5
0
5
10
15
x

FOR EXAMPLE

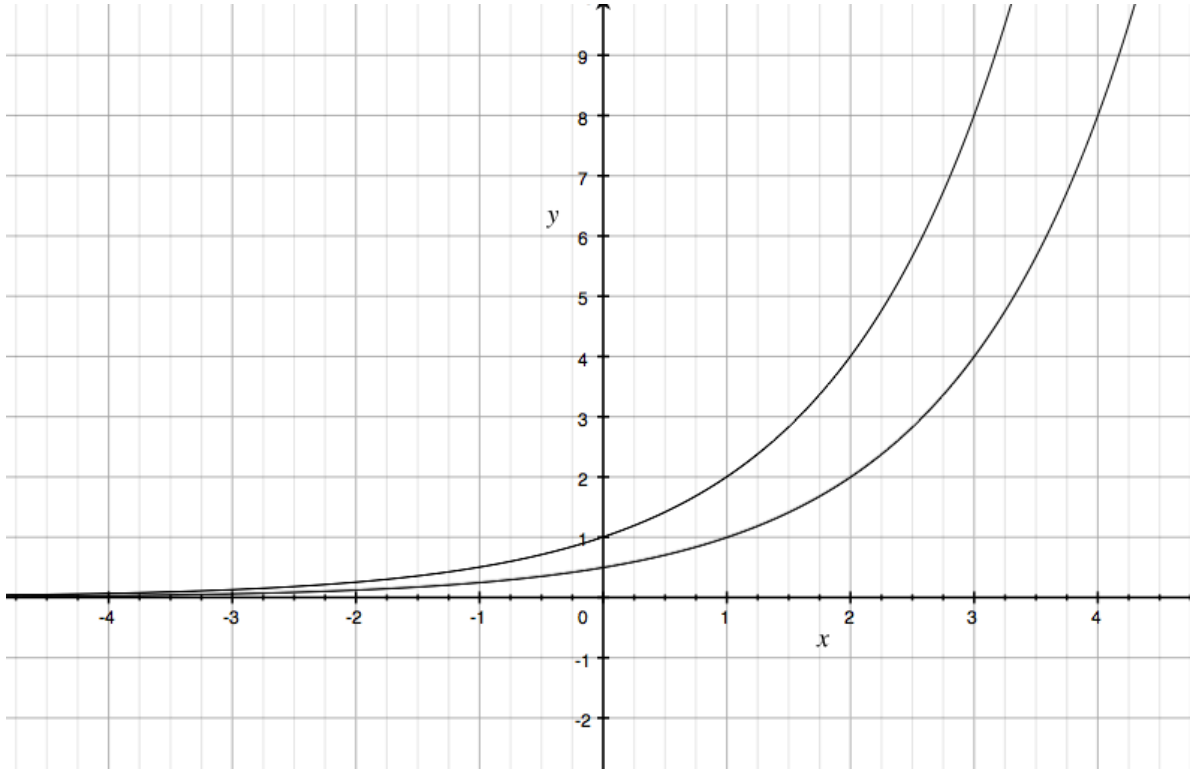
- One of the trig ratios of an angle in a right triangle is really close to 1.
- What could the angle be? Why?

FOR EXAMPLE

- An exponential function is a LOT like $y = 2^x$.
- What might it be and how are their graphs related?

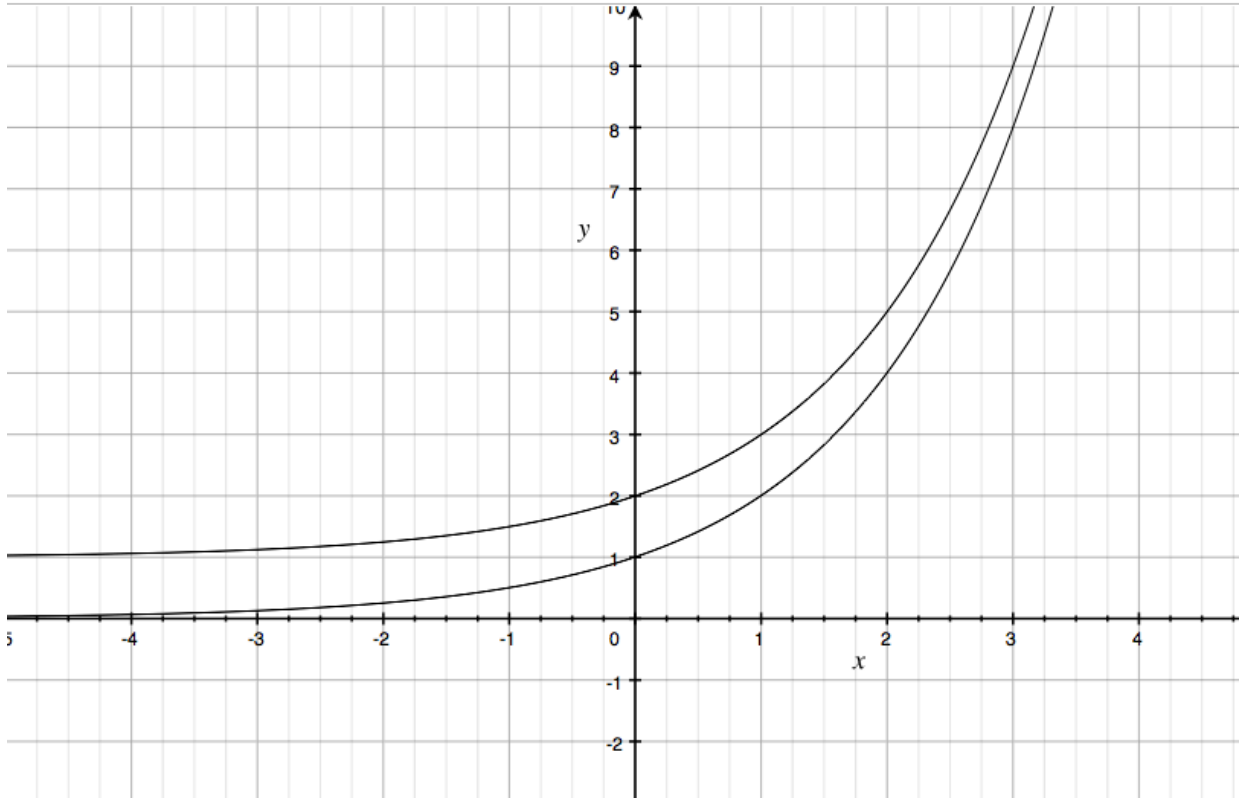
EXPONENTIALS

- $y=2^x$ vs $y=2^{x-1}$



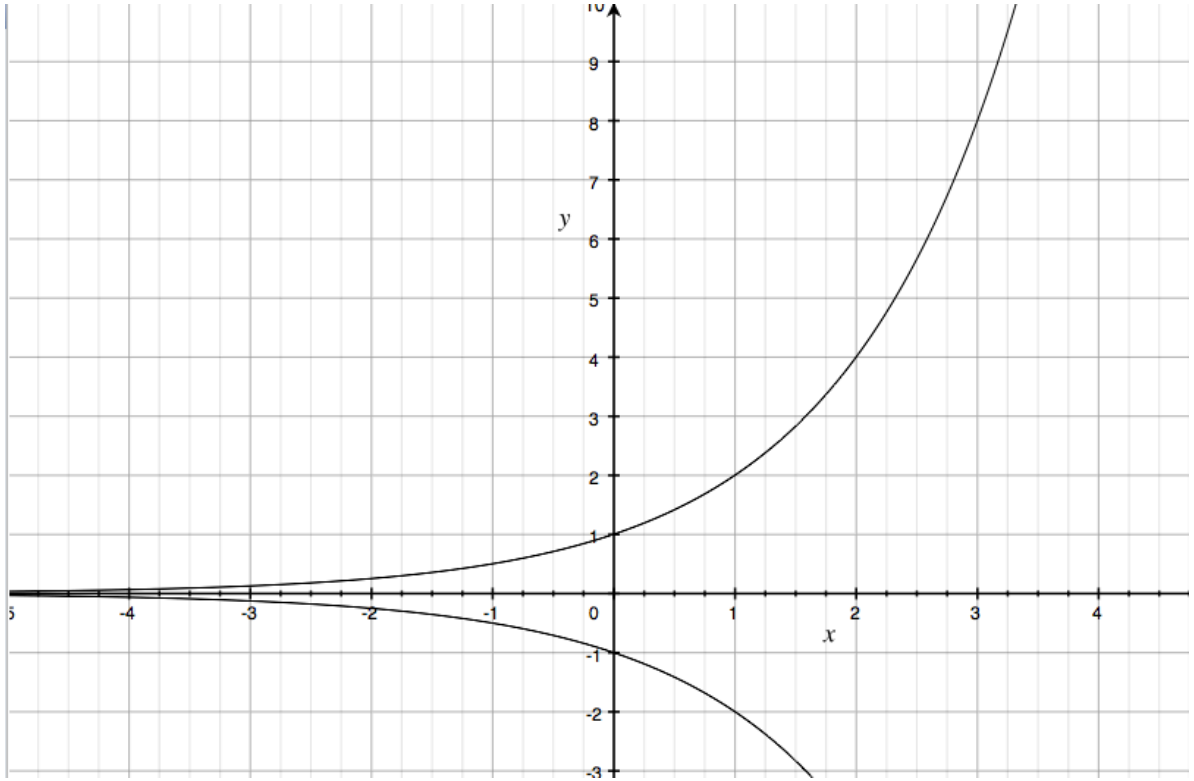
EXPONENTIALS

- $y=2^x$ vs $y= 1 +2^x$



EXPONENTIALS

- $y=2^x$ vs $y= -2^x$



DEEPER THINKING

- Would like to propose we "adopt" from the U.S. their standard for mathematical practice:
- Construct viable arguments and critique the reasoning of others
- I think the heart of math really is reasoning, so this makes sense to me.

SO WHAT MIGHT THIS LOOK LIKE?

- Sam: It is not possible to multiply two fractions and get a product with a smaller denominator than the ones you started with.
- Kevin: Yes, you can.

- With whom do you agree?

SO WHAT MIGHT THIS LOOK LIKE?

- Julie: Every function of the form $y = a \sin(x - h) + b$ can be renamed as a function of the form $y = c \cos(x - k) + d$.
- Xavier: Some can, but not every.

- With whom do you agree?

2 TRUTHS AND A LIE

- Suppose A = 30% of B. Which is the lie?
- 1. A = 60% of 2B.
- 2. 3A = 45% of 2B
- 3. A/2 = 15% of B.

2 TRUTHS AND A LIE

- 1. A cone and a pyramid cannot have EXACTLY the same volume.
- 2. The volume of a pyramid can be 2/3 the volume of a prism.
- 3. The volume of a pyramid can be 3 times the volume of a hexagonal prism.

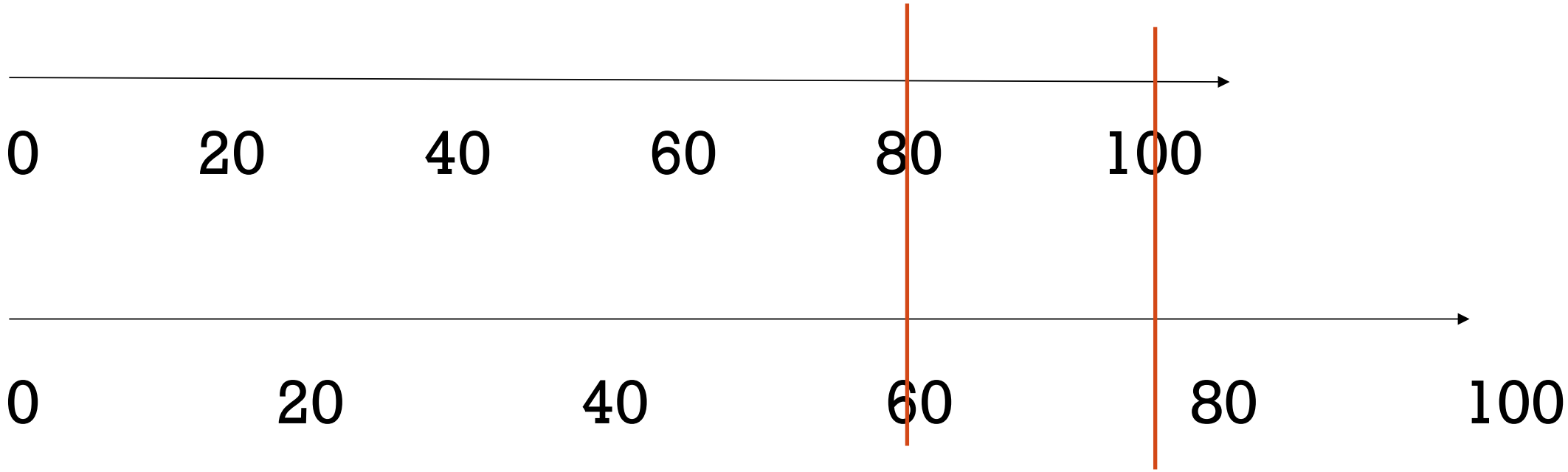
2 TRUTHS AND A LIE

- 1. The cosine of an angle close to 45° is very close to the sine of that angle.
- 2. The tangent of an angle close to 90° is very close to the cosine of that angle.
- 3. The tangent of an angle close to 0° is very close to the sine of that angle.

PROBLEMS THAT GENERALIZE, E.G.

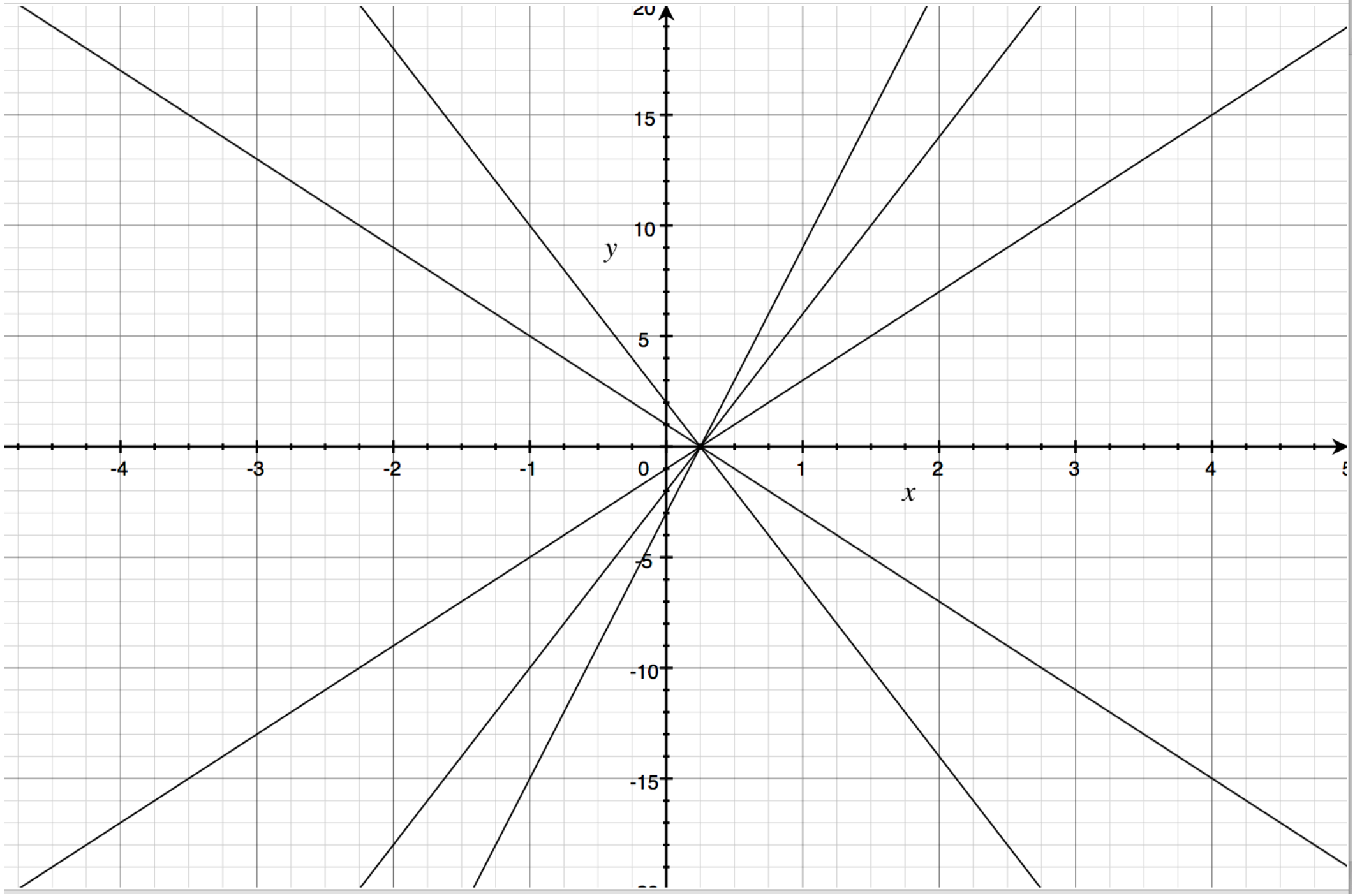
- An item that was 40% off cost the same as an item that was 20% off.
- How were the original prices related?

VISUAL



PROBLEMS THAT GENERALIZE, E.G.

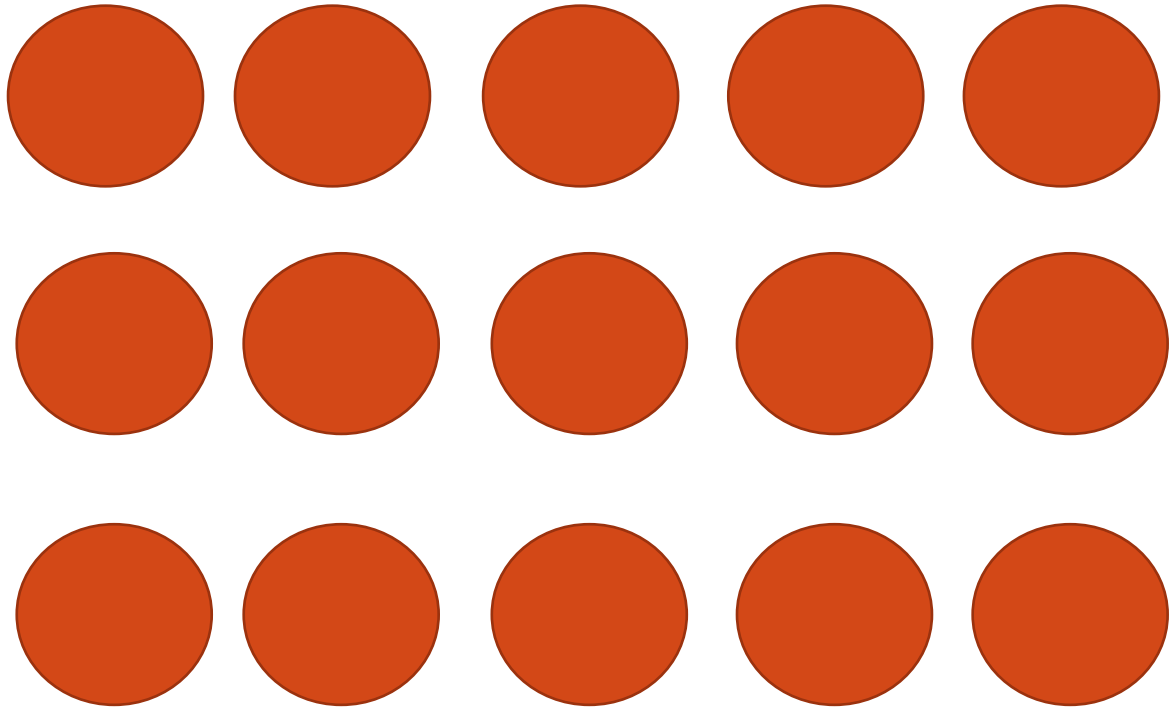
- Graph $y = mx + b$ if $m/b = -4$.


20
15
10
5
0
-5
-10
-15
y
-4
-3
-2
-1
1
2
3
4
x

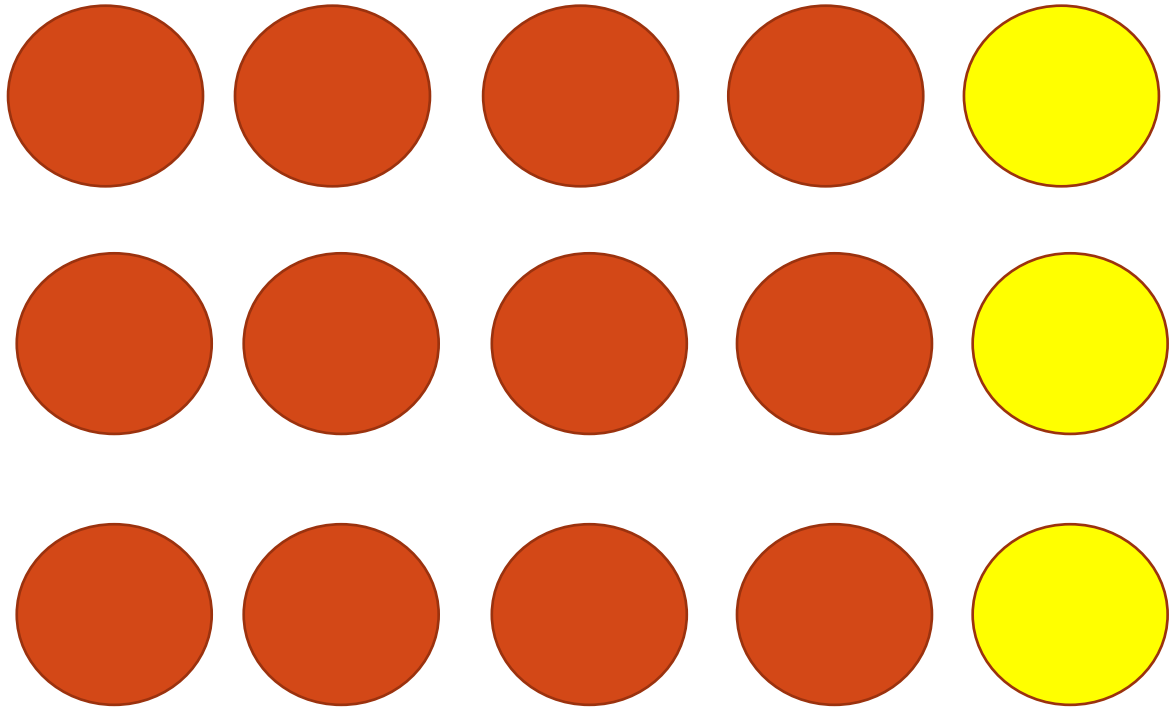
PROBLEMS THAT GENERALIZE, E.G.

- Use counters to model as arrays:
- 3 x 5
- 4 x 6
- 5 x 7
- Each time move the rightmost column to the bottom. What do you notice? How could you describe it algebraically?

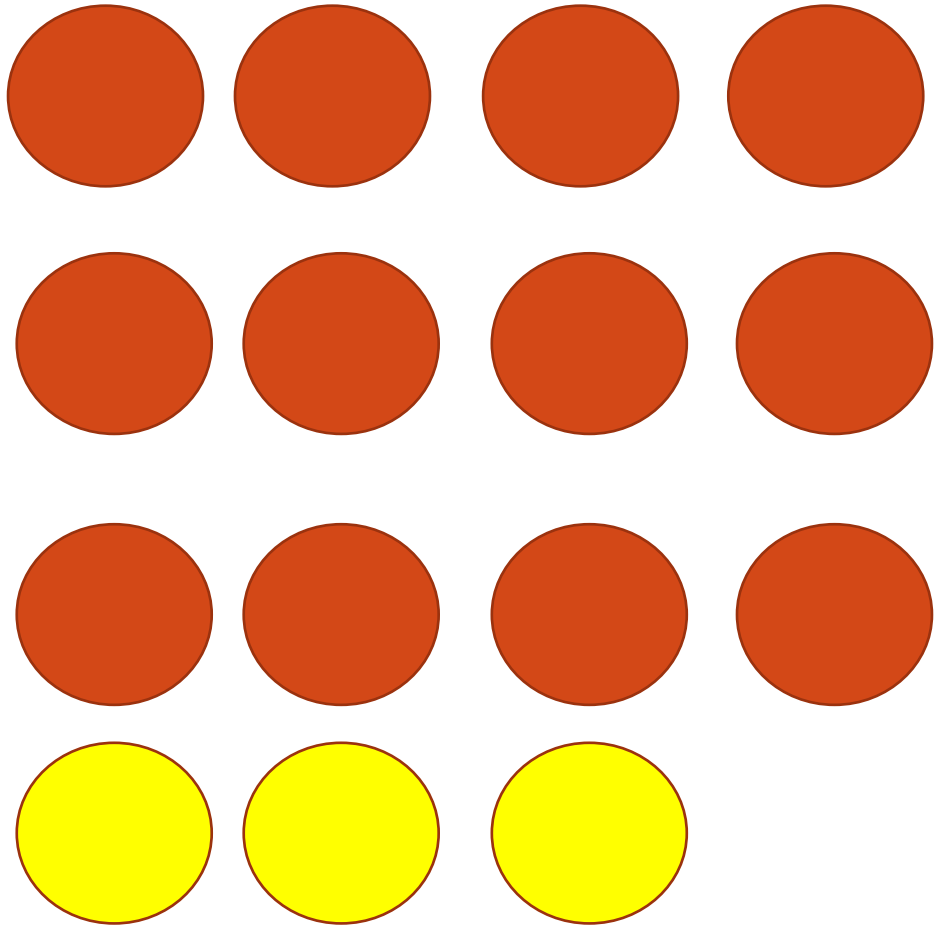
MAYBE



MAYBE



MAYBE



USING MORE UNDERSTANDING RATHER THAN KNOWLEDGE QUESTIONS

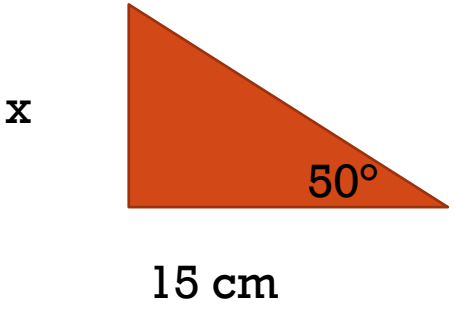
- **Knowledge:** What is –42 – (–17)?
- **Understanding:** You subtract a negative number from a negative number? Could you get a positive answer? How or why? OR
- Draw a **picture** that shows **why** –8 – (–3) has to be –5?

USING MORE UNDERSTANDING RATHER THAN KNOWLEDGE QUESTIONS

- **Knowledge:** What is the equation of a line with a y-intercept of 5 that goes through (2,5) and (3, 6)?
- **Understanding:** A line goes through Quadrants II, III and IV. What could its equation NOT be? Why? OR
- Describe a real-life situation that can be represented by the equation $y = 50x + 400$.

USING MORE UNDERSTANDING RATHER THAN KNOWLEDGE QUESTIONS

- **Knowledge:** What is the length of x?



- **Understanding:** For what angle is the tangent really close to the sine? Why does that happen? OR
- Suppose you did not know that the base in the triangle (above) was 15 cm. Could you find the length of x? Why or why not?

USING MORE UNDERSTANDING RATHER THAN KNOWLEDGE QUESTIONS

- KNOWLEDGE: What is the slope of the line that goes through (3,2) and (4, 5)?
- UNDERSTANDING: Two lines go through (3,2).
- One has a greater slope than the other.
- How do the intercepts relate?

USING MORE UNDERSTANDING RATHER THAN KNOWLEDGE QUESTIONS

- KNOWLEDGE: Solve $100x + 6 = 87x + 2$.
- UNDERSTANDING: WITHOUT SOLVING, tell why the solution to $100x + 6 = 87x + 2$ HAS TO be negative.

USING MORE UNDERSTANDING RATHER THAN KNOWLEDGE QUESTIONS

- KNOWLEDGE: What is $(3x + 2) - (2x^2 - 8)$?
- UNDERSTANDING: When you subtract two binomials, show that your answer could be a monomial, a binomial, or a trinomial.

SO LET'S TRY

- Take something straightforward you do.
- Let's work together to "deepen" it.

DOWNLOAD

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