

Developing Critical Thinkers & Problem Solvers

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Critical thinking

Goes hand in hand with problem solving and
decision making



Critical thinking

Math is clearly full of problem solving
There is often decision making in how to
proceed on a problem.

We can, and should, bring in decision making
into interpretation too.



10-frame

For example...

Jamie filled two ten-frames and most of another one.

How many counters do you think Jamie used?

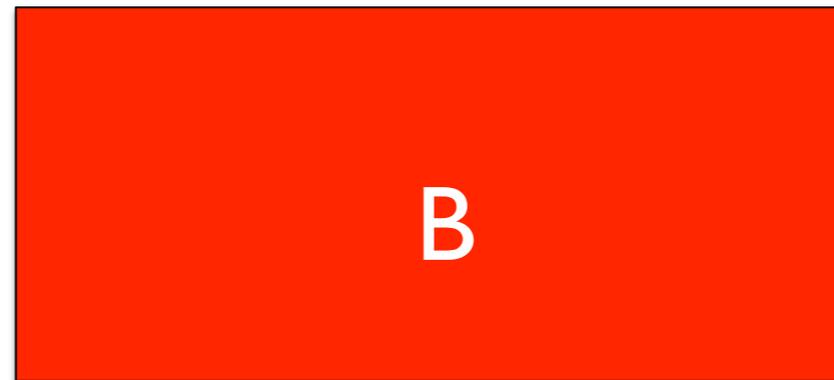
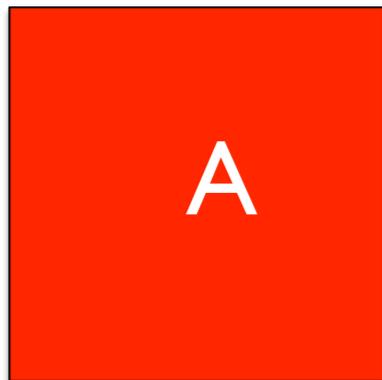


Or...

Kayla had just a few more pencils than Levi.

Together they had 20 pencils.

How many do you think each one had?



Or...

A shape is more like shape A than shape B.
What might it be? Why?



Or...

What could you measure about an apple?
How would you do it?



Critical thinking involves

Review, analysis and assessment of information
from different points of view

There is always an element of setting criteria in
order to do the analysis and assessment.



This might happen if...

I want you to make an argument as to why when you add any two even numbers the answer is always even.

We will then listen to the arguments and decide which are most convincing and why.

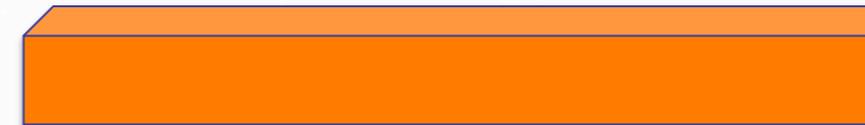


Or I could have asked...

When you subtract a number from another one, how will you know if the result is more or less than the number you subtract?



Or I could have asked...



You are going to measure the same length
twice.

First time you use the green rod.

Second time you will use the orange rod.

BUT first predict how many oranges you will
need.

Then test your thinking.



Critical thinking involves

Reflection on your own and others' thinking
and reasoning

Confidence as a problem solver

Flexibility in approaches to solutions



Asking the right questions

This is the heart of the issue.

We need to ask questions that encourage or even demand critical thinking behaviours.

You could make it the “normal” way you teach.



For example

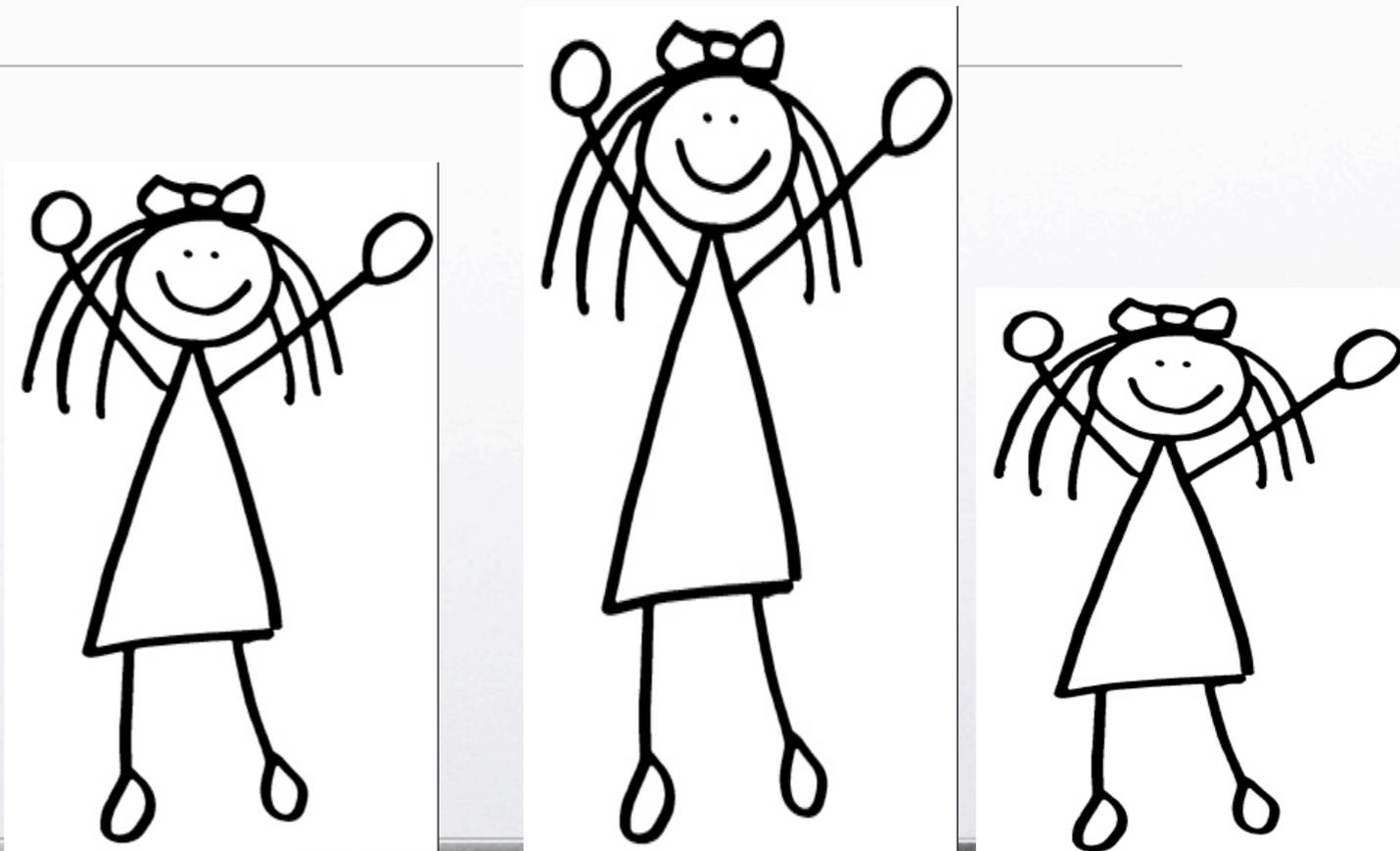
We might ask how old *Sindy* is if she is 6 years older than *Amy*, who is 4.

OR

We might ask whether *Sindy* can be both 6 years older than *Amy* and also twice as old.

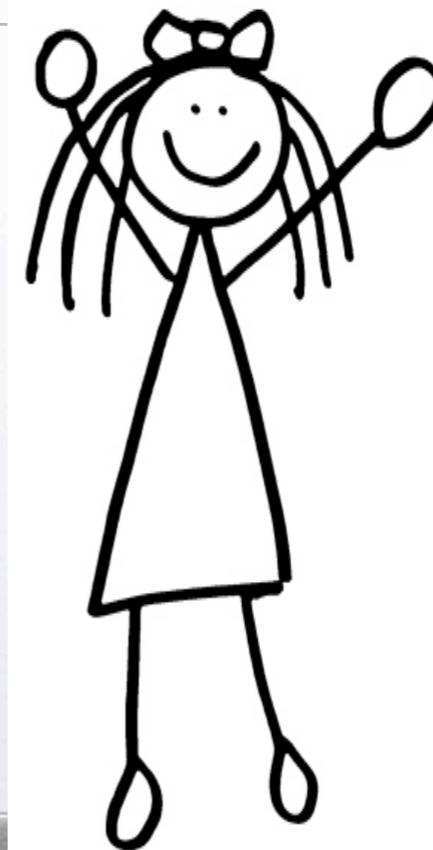
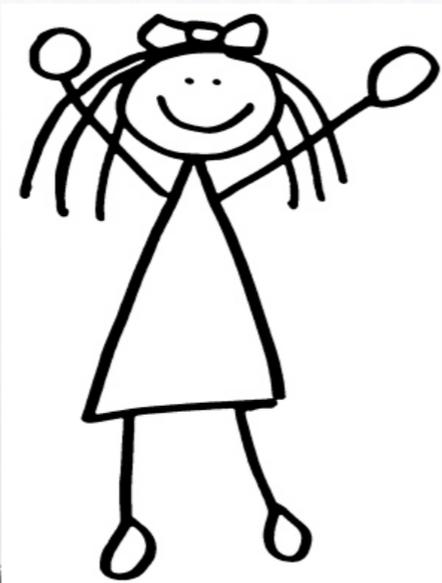
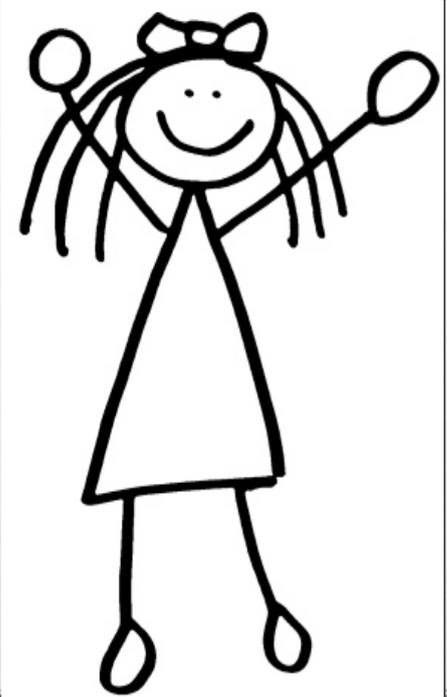


Tara is 9 cm taller than Brianna.
Rachel is 4 cm taller than Brianna.
Which is Brianna? How do you know?





Tara is 9 cm taller than Brianna.
Rachel is 4 cm taller than Brianna.
Which is Brianna? How do you know?





Or

Alison has 12 cookies.

Simone has 15 cookies.

Both girls have to divide up their cookies into
equal shares.

Who do you think will have an easier job?



Asking the right questions

It could be an appropriate “puzzle”.

For example---

Pick a number.

Add 3.

Subtract 2.

Add 4.

Tell me your answer and I'll tell you your number.



Asking the right questions

Or

What numbers can you make by using only 2s, 5s, +s and -s?

$$\text{e.g. } 3 = 5 - 2$$

$$10 = 5 + 5$$

$$9 = 5 + 2 + 2$$



Asking the right questions

It is often useful to ask “provocative”
questions. e.g.



Asking the right questions

Is it more useful to know how to add or how to subtract?

What would be the criteria for “useful”?



Asking the right questions

Is a shape's area or perimeter a more important aspect of the shape?
What are the criteria for "important"?



We always start with curriculum

Let's look at some standards and see if we can
develop critical thinking approaches.

We will do some together and some in small
groups.

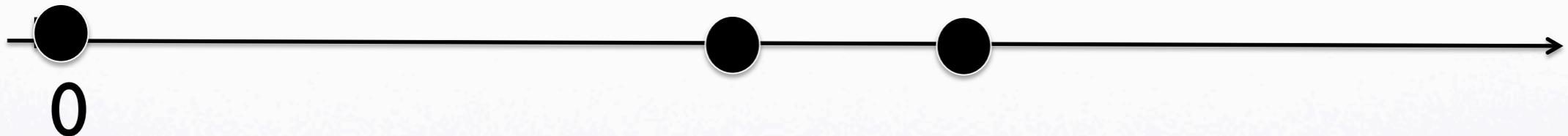


Grade 1 standard

Count, recognize, model, read, write and order numbers to 120...



One possible question



What numbers might belong at the dots
marked on the number line?
What numbers are bad choices?



Another idea

How would you arrange 15 counters to show as many things as you can about 15?

Be ready to tell what you are showing about 15.



Another idea

~~You partition a number into three parts.~~

One part is small.

One part is about twice as much as another
part.

What number could it be and how is it
partitioned?



For example

$$10 = 1 + 3 + 6$$

$$25 = 4 + 8 + 13$$

$$30 = 3 + 9 + 18$$



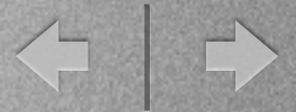
Grade 2 standard

Recall and use addition and subtraction
strategies to 100



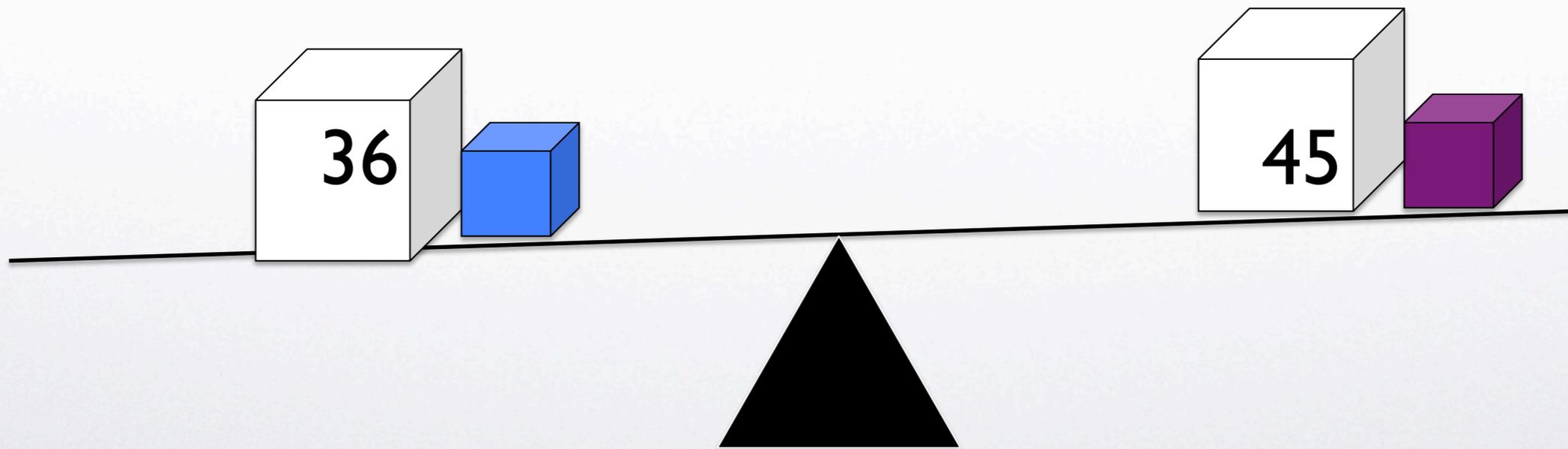
One idea

How does knowing that $50 + 50 = 100$ help you know other addition answers? Which ones?



Each box holds marbles. What do you know about the blue and purple amounts?

Another idea





Another idea

Make up a trick involving at least 5 steps so that after following your steps, the result is always 10.



One possibility

Start with a number.

Add 1.

Add 8.

Subtract original number.

Add 1.



Another idea

Is it possible to solve every possible subtraction question using addition?

Convince us.



A grade 3 standard

Recognize and use factor pairs and commutativity in mental calculations



One idea

Find three patterns in the multiplication table
that you can explain.
Explain them.



x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

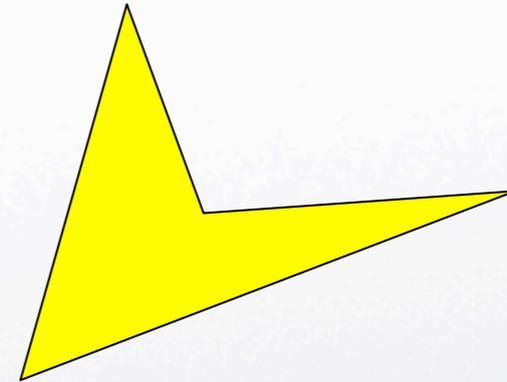
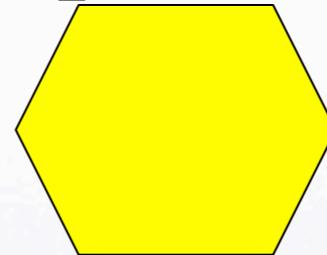
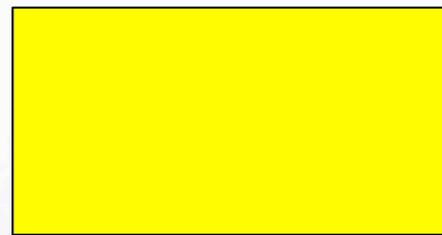
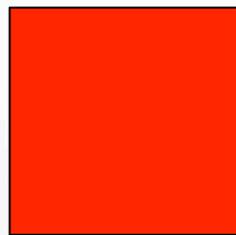


Another idea

Do you agree or disagree? Why?
When you multiply numbers, you are more likely to get an even answer than an odd one.



A few more examples



Which shape doesn't belong? Why?



A few more examples

Which number doesn't belong? Why?

2

5

12

20



A few more examples

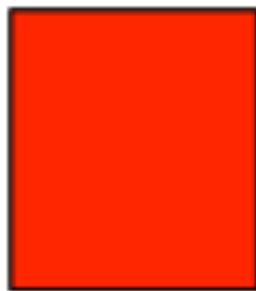
What other numbers go with the given ones?

4, 8, 12

5, 9, 3

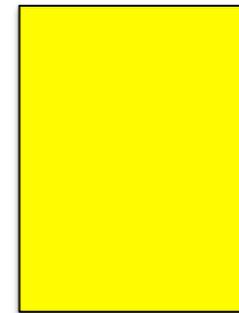
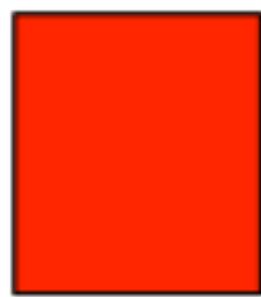
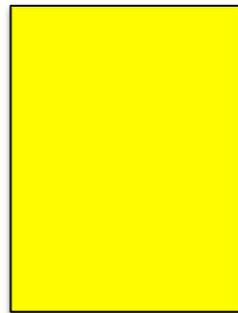
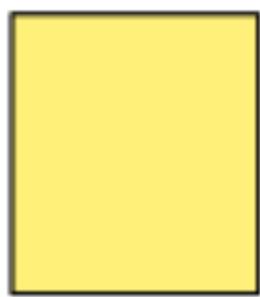


A few more examples



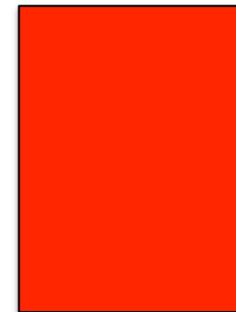
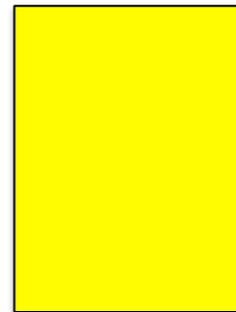
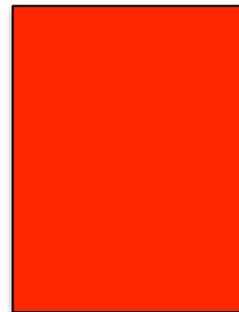
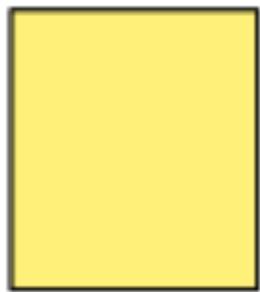


Maybe





Maybe





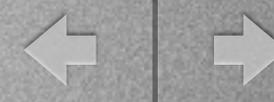
A few more examples

What ideas about numbers are easier to see on
the 10×10 hundreds chart?

Which on the 9×9 hundreds chart?



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81
82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99



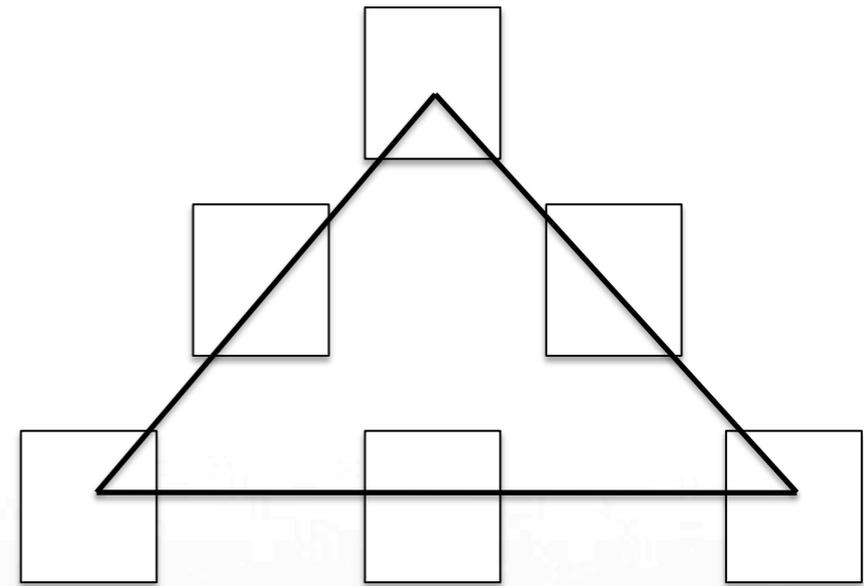
Or

How are these alike and how are they
different?

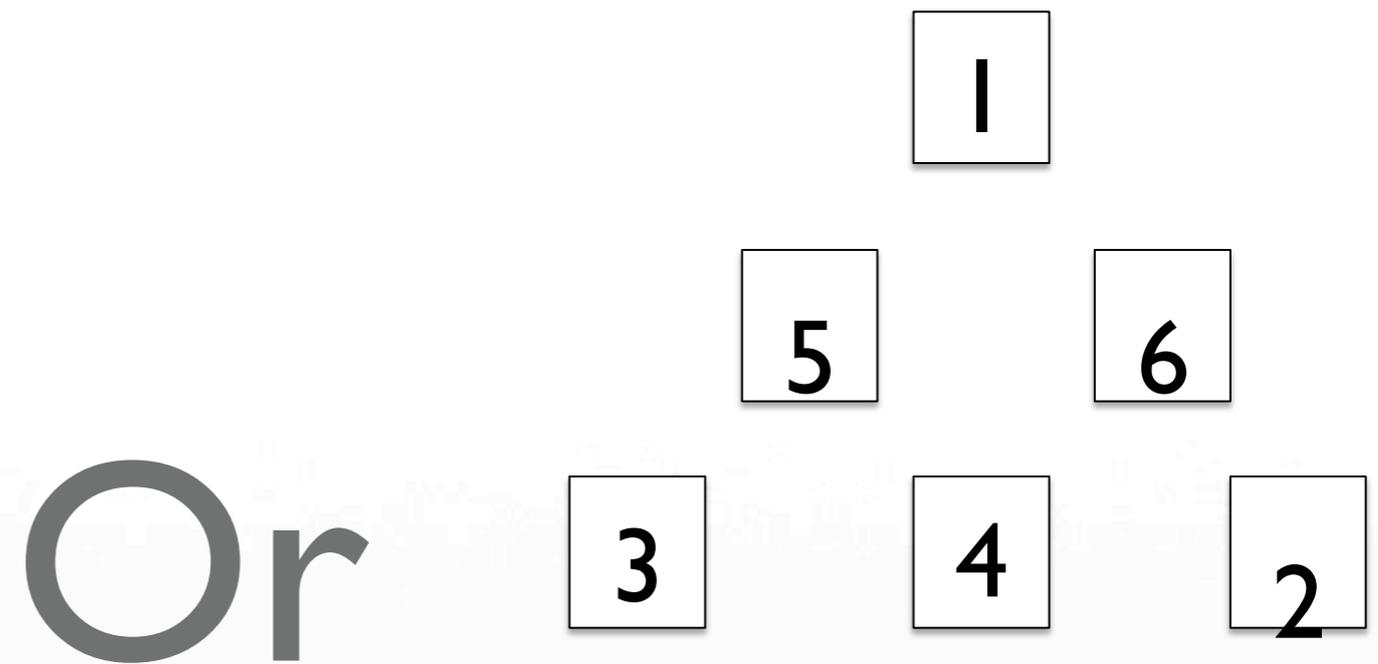
- Multiplication and division
- Counting by 2s and counting by 5s
 - A square and a circle
- Adding 29 and adding 32



Or



The numbers from 1 to 6 go in the boxes.
All 3 lines add up to the same amount.



The numbers from 1 to 6 go in the boxes.
All 3 lines add up to the same amount.



Your work

I would like you to work with at least one partner at your grade level.

Choose 3 standards.

Create tasks related to those outcomes that foster critical thinking.

We will share thinking.



Teaching meaningfully

Let's model calculations with materials



$$38 + 29$$



43 – 17



10 x 3



$$9 + 2 = 10 + 1$$



$$2 \times 6 = 4 \times 3$$



$$12 \div 2 = 24 \div 4$$



Problem solving



What sorts of problems...

Promote deeper thinking and deeper
understanding?



One strategy...

Is the use of more open-ended problems.



For example..

The answer is 100.

What might the question have been?



For example..

The 10th shape in a pattern is a red triangle.
What could the pattern be?



For example..

A three digit number is represented with twice as many flats as rods and twice as many rods as ones.

What could it be?



For example..

You can model a number with 15 base ten blocks. What could it be?



For example..

5 _____ s is less than 2 _____ s.



For example..

You add two numbers. The sum is double the difference.

What could they be?



For example..

Use square tiles. Make a design that is almost half red and almost one third blue.



For example..

The perimeter of a rectangle is triple its length.
What could the length and width be?



For example..

[]9 and 4[] are about the same distance apart as
1[] and []5.

What could go in the blanks?



For example..

You add a number more than 10 that you can represent with 5 base ten blocks to a number you can represent with 10 blocks. How many blocks will you need to show the answer?



For example

You have some counters.

When you make groups of 3, there is 1 left out.

When you make groups of 4, there are 3 left
out.

How many counters might there be?



For example

More than 3 people are equally sharing 3 sandwiches. Choose how many people.
Tell how much each gets.



Let's practice

Starting with “exercises” and turning them into problems.



For example...

What is $53 + 28$? Turns into:

You add two 2-digit numbers. The ones digit of one of them is the tens digit of the sum.

What could the numbers be?



For example...

What is the perimeter of a rectangle with a length of 10 cm and width of 4 cm? turns into
The length of a rectangle is 6 more cm than the width. What could the perimeter be?



For example..

How do you get from point X to point Y on a grid? To

You go from X to Y to Z and back to X. You move 32 spaces altogether. Where could the points be?



Now you try

Start with standard exercises and create problems.



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