Developing Critical Thinkers and Problem Solvers

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October 2015

Critical thinking

Goes hand in hand with problem solving and decision making

Critical thinking

Math is clearly full of problem solving
There is often decision making in how to proceed on a problem.
We can, and should, bring in decision making into interpretation too.

$5.99 to $2.99

$46 945.00 to $44 999.00

Which price changed the most?

Or…

A car goes 280 km in 3 hours.
Which would be easiest for you to figure out?
How far it goes in

9 hours? 1 hour? 1.5 hours?

Or…

A shape does not have much area but it has lots of perimeter.
What might it look like?

Or...

Estimate the number of grains of rice that all of the kids in SIngapore eat in one week.

Critical thinking involves

Review, analysis and assessment of information from different points of view

There is always an element of setting criteria in order to do the analysis and assessment.

This might happen if...

I want you to make an argument as to why you can multiply by 4 by doubling and then doubling again.

We will then listen to the arguments and decide which are most convincing and why.

Involves

It is often said that critical thinking makes use of:
application,
analysis,
evaluation and
conceptualization,
including recognition of assumptions
Which did we just use?

Or I could have asked…

When you divide one number by another one, how will you know if the result is more or less than the number you divided by?

Or I could have asked…

You are going to spin a spinner.
You are twice as likely to get red as blue.
You are half as likely to get blue as green.
What could the spinner look like?

Critical thinking involves

Reflection on your own and others' thinking and reasoning

Confidence as a problem solver

Flexibility in approaches to solutions

Asking the right questions

This is the heart of the issue.
We need to ask questions that encourage or even demand critical thinking behaviours.
You could make it the “normal” way you teach.

For example

Amy is 12. Sindy is twice as old. How old is Sindy?

OR

Can Sindy can be 7 times as old as Amy one year, 4 times as old the next year, and 3 times as old the following year?

One height is $\frac{7}{8}$ of another.

One height is $1\frac{1}{3}$ times another.

Which is which?

Asking the right questions

It could be an appropriate "puzzle".
For example---
Pick a number.
Double it.
Subtract 2.
Add 4.
Take half
Tell me your answer and I'll tell you your number.

Asking the right questions

Or

What numbers can you make by using only 3s, 7s, +s, -s, and xs?

e.g. $21 = 3 \times 7$

$13 = 3 + 3 + 7$

$11 = 7 + 7 - 3$

Maybe

$1 = 7 - 3 - 3$

$2 = 7 + 7 - 3 - 3 - 3$

$3 = 3$

$4 = 7 - 3$

$5 = 3 + 3 + 3 + 3 - 7$

$6 = 3 + 3$

$7 = 7$

Asking the right questions

It is often useful to ask “provocative” questions. e.g.

Asking the right questions

Is it more useful to know how to multiply or how to add?
What would be the criteria for "useful"?

Asking the right questions

Is a shape's area or perimeter a more important aspect of the shape?

What are the criteria for "important"?

We always start with curriculum

Let's look at some standards and see if we can develop critical thinking approaches.

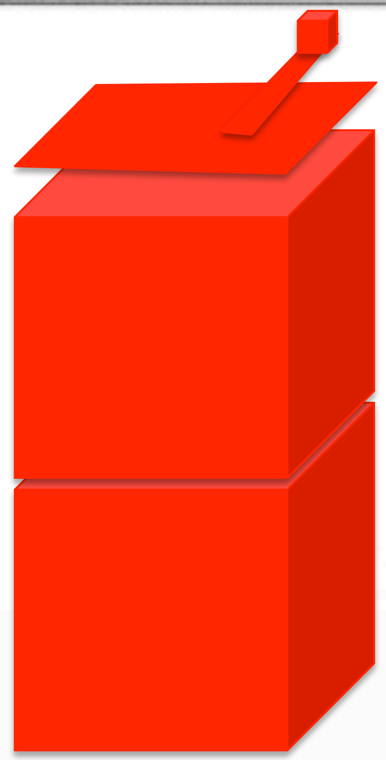
We will do some together and some in small groups.

Grade 4 standard

Compare and order numbers to 1 000 000

One idea

When is 4[]2 []29 > []91 221?
When is it less?



OR

You build base ten block towers by putting large blocks on bottom, then flats, then sticks, then units.

OR

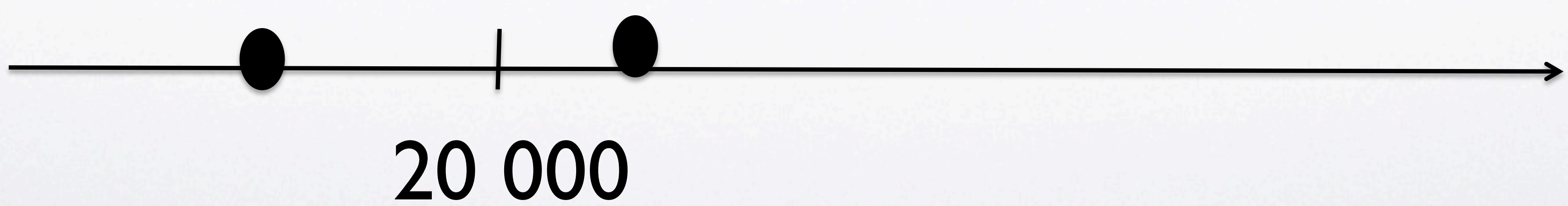
You build base ten block towers by putting large blocks on bottom, then flats, then sticks, then units.

Does a higher tower always represent a greater number? When does it or does it not?

What might be reasonable values for the dots on the number line?

What are unreasonable values?

OR



Grade 4 standard

Solve problems involving… division….

One possible question

Is it true that every division question can be solved by using multiplication instead?

Let's listen to each other's arguments and evaluate them.

Another possible question

When you divide 412 by 4, you can write 412 = 400 + 12 and divide each part by 4 and add.

But you can't write 4 = 2 + 2 and divide 412 by 2 and then 2 and then add.

How come, in general?

Or

We write 24 ÷ 4 to ask for the size of a group when 24 is divided into 4 equal groups.
We also write 24 ÷ 4 to ask for how many groups we can make if 24 is divided into groups of 4.
Do you think we should use a different symbol in the two situations?

Grade 5 standard

Compare and order fractions….

An idea

Compare and contrast the strategies you would use to compare

2/3 to 7/3

8/3 to 8/5

2/3 to 12/10

2/3 to 4/5

Another idea

Is it true that fractions are greater if their numerator and denominator are closer together?
(e.g. 2/3 is more than 1/9 since 2 and 3 are closer than 1 and 9)

Another idea

Is it true that when you compare two fractions, if you use an in-between numerator and an in-between denominator, you get an in-between fraction?

(e.g. Between 4/5 and 8/9 is 6/7)

Grade 5 standard

Calculate the area of parallelograms and triangles

One idea

The area of a certain parallelogram is 2/3 of the area of a certain triangle.
What could their dimensions be?

Grade 6 standard

… express one quantity as a percentage of another…

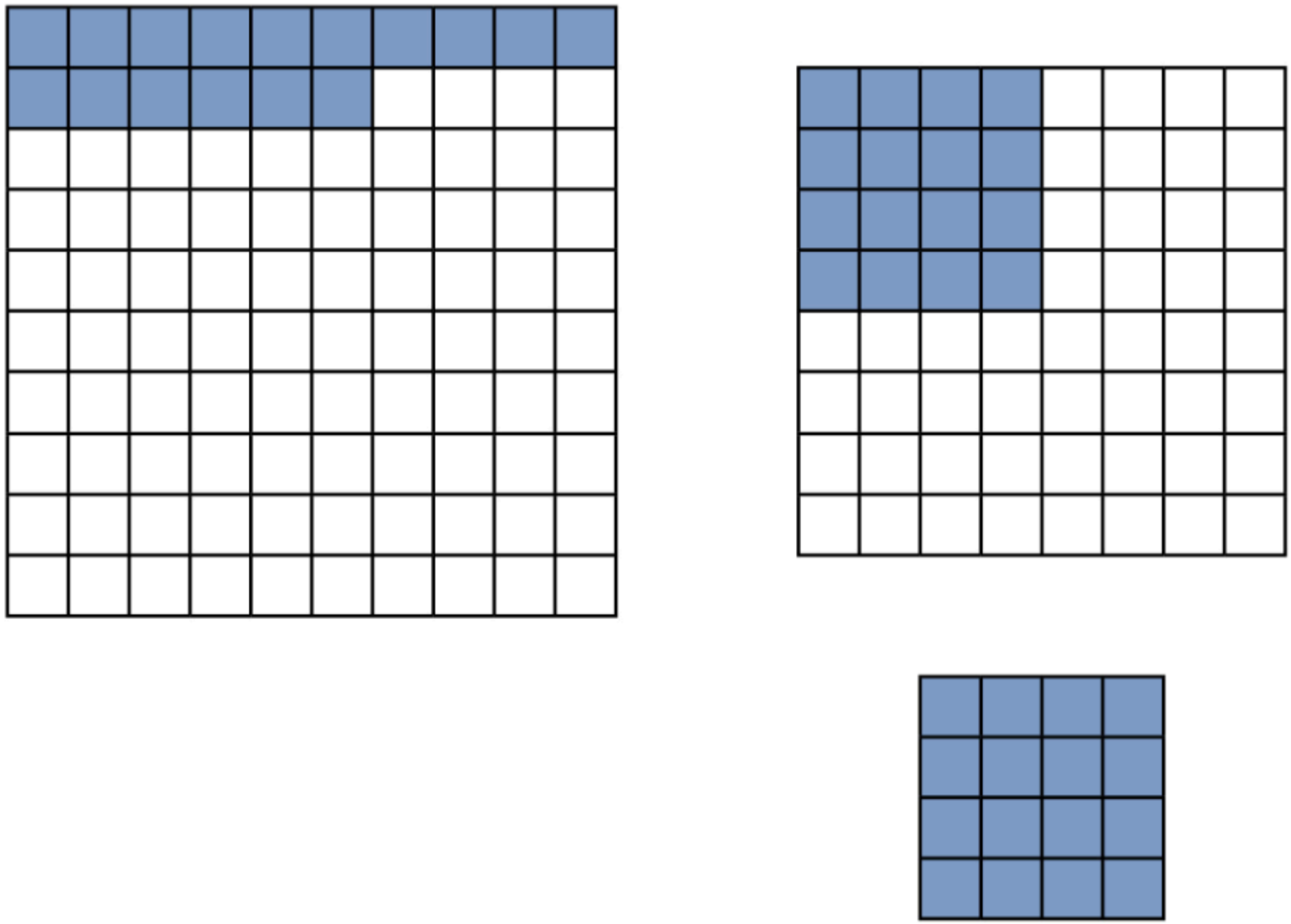
One idea

Is 10% a lot or not?

Another idea

If you know 20% of an amount, what other percents of it do you also know? Which don't you know?

Which grid or grids show percent?



Or..

Percents are out of 100.

What if there were something called pervigs (out of 20)?

How would you calculate 10 pervig of a number? 30 pervig of a number?

What pervigs of a number would be easiest to calculate?

Grade 6 standard

Calculate and solve problems involving the perimeters of 2-D shapes…

One idea

You have a rectangle made of square tiles. You split it in half.

Is the perimeter halved?

Could it be 2/3?

Could it be 3/4?

Another idea

The perimeter of a rectangle and hexagon are equal.
How are their areas related?

A few more examples

Which pattern gets to 1000 first? How do you know?

15, 25, 35, 45, 55, 65,…
500, 502, 504, 506, 508,…

A few more examples

You multiply a number by another that is two greater.
How does the answer relate to the number between them?

A few more examples

Do you think squares or rhombuses are better shapes to use as floor tiles? Why?

A few more examples

Which fraction doesn't belong? Why?

3/4 3/5 1/3 5/7

A few more examples

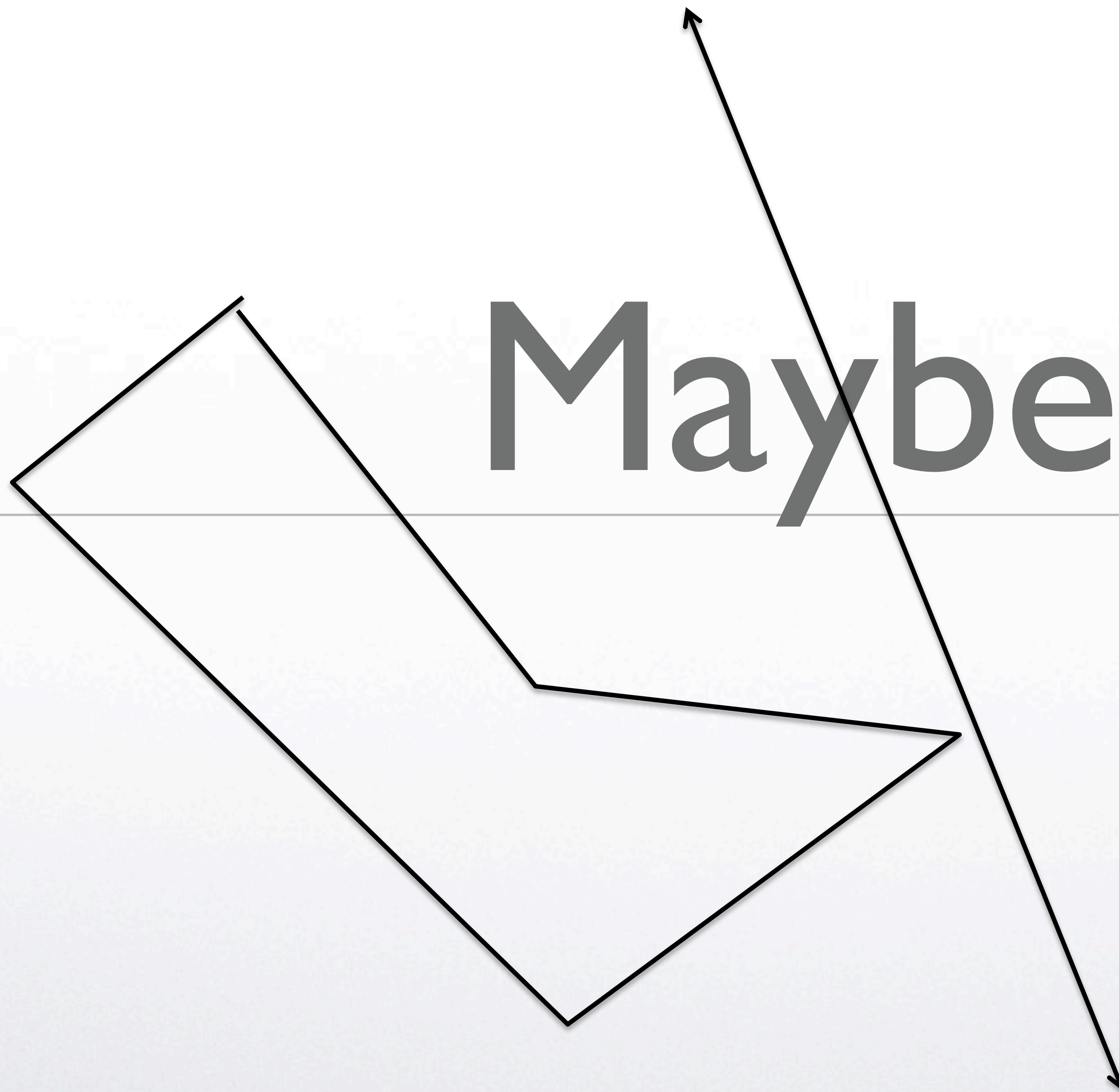
Which two patterns are most alike? Why?

7, 13, 19, 25, 31,…
10, 16, 22, 28, 34,…
7, 9, 11, 13, 15, 17,..

Or

You perform a reflection and some points on the original shape move A LOT and some not much at all.
Every point moves some.
What could the initial position and final position look like?

Maybe

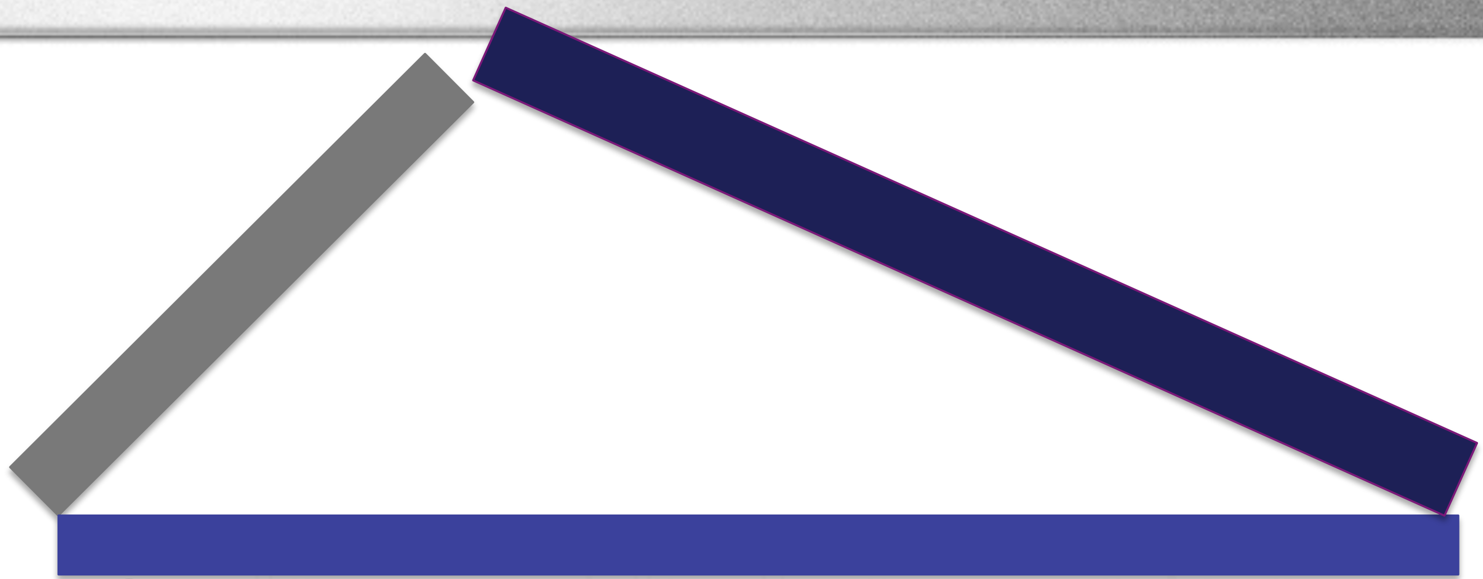


Another example

You roll two dice and form a two-digit number with the results.

Which is more likely?

That the number is a multiple of 3 or that it is a multiple of 4? Why?



Or

You pull 3 Cuisenaire rods at random from a bag.

Is it more likely that you can or cannot form a triangle?

For example

1-1-10	NO
3-4-5	YES
2-4-8	NO
3-3-5	YES

Another example

Sometimes we write $P = 2l + 2w$ and sometimes we write $P = 2(l + w)$ to determine the perimeter of a rectangle.

Why do these really say the same thing?

Which is more useful when?

Another example

Determine 3 patterns in the multiplication table
that you can explain.
Explain them.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Your work

I would like you to work with at least one partner at your grade level.
Choose 3 standards.
Create tasks related to those outcomes that foster critical thinking.
We will share thinking.

Teaching Meaningfully

Let’s model calculations with materials

22×23

$$423 \div 3$$

$$123 \div 3$$

$$2/3 + 1/5$$

$2/3 - 1/5$

2/3 x 1/5

$2/3 \div 1/5$

Factors of 24

LCM of 4 and 6

Problem solving

One strategy…

Is the use of more open-ended problems.

For example

You multiply two numbers and the product is ALMOST 400. What could the numbers have been?

For example

A **length is four times the width** of a rectangle.
What do you notice about the relationship between perimeter and width?

For example

You add 2 fractions.
The sum is []/15.
What fractions might you have added?

For example

You add two fractions less than 1.
The answer is a little less than 5/4.
What might the fractions have been?

For example

A sweater was on sale, 40% off.
A pair of pants was on sale, 20% off.
The sale prices were the same.
How did the original prices compare?

For example

A number includes the digit 7 twice and the digit 4 twice.

One 7 is worth 100 times the other.

One 4 is worth 10 times the other.

What could they be?

For example

A measurement described in yards is about 10 more than if it were described in metres. What could it be?

For example

The median of a set of data is half the mean.
What could the data set be?

For example

One angle in a triangle is double another.
What kind of triangle can it be?

Let’s practice

Starting with “exercises” and turning them into problems.

For example

Round 19 456 to the nearest thousand.

Could become

You round a number to the nearest thousand and it’s less than rounding to the nearest ten thousand. What could it be?

For example

What are the common factors of 12 and 18?

Could become

Two numbers have exactly four common factors. What could they be?

For example

What is the 30^{th} term of 4, 7, 10, 13,…? Could become a slowly increasing arithmetic sequence has a 30^{th} term of 61. Wht could the sequence be?

Now you try

Start with standard exercises and create problems

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- www.onetwoinfinity.ca
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