Exploring HS Mathematics Instruction

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Agenda

- We will talk about how things went when you played around with open questions or filling in gaps with suggested materials.
- We will focus on backwards planning in terms of big ideas.
- We will also explore understanding and thinking questions (vs. knowledge questions).

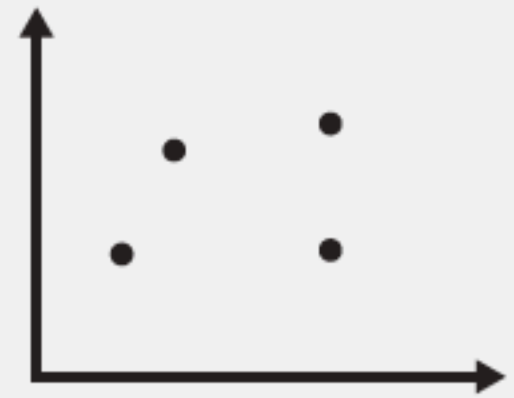
Who tried

- Filling gaps?

Who tried?

A line passes through two of these points:

What could the equation of the line be?



When you add a polynomial that can be represented with 3 algebra tiles to one that can be represented with 5 algebra tiles, how many tiles does the sum require?

Who tried?

You combine a shape with a **surface area** of 22 cm^2 with a shape with a surface area of 24 cm^2. What could the surface area of the combined shape be?

Who tried?

- Your own open questions?

What do you think?

Is it good enough if students can answer A and not B?

A: What is x if $3/x = 14/30$?

B: Before solving, how do you know that x has to be more than 6?

It would be great

- To focus our teaching more on ideas that underlie the math curriculum than performances

Bigger ideas are important

- To make the curriculum manageable
- To make the curriculum more meaningful

We want

- Questions and tasks that allow for richer discussion and not just a quick check on performance

This is true

- No matter what the topic

For example

- Right angle trig

I still want

- You can calculate sines, cosines, tangents and know what they represent a problem.
- Solving for angles of elevation/depression, etc.

BUT ALSO

I want

- An understanding why if you know sine, cosine or tangent, you know the others.
- An understanding that if you can use sine, you could have used cosine of the other acute angle

I want

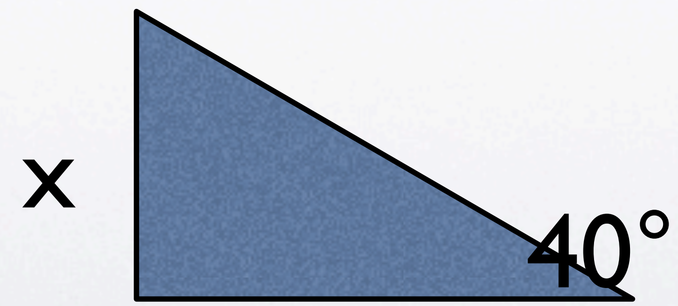
- An understanding of when each ratio is big or small and how big or small it can be
- An understanding that if you don't know one side length in a triangle, you can't solve for others.

So I might ask

- Suppose you know that the sine of an angle is 0.8.
- Can you figure out cosine and tangent without knowing the side lengths?
- How or why not?

So I might ask

- You want to figure out the length of x in this triangle. What choices do you have?

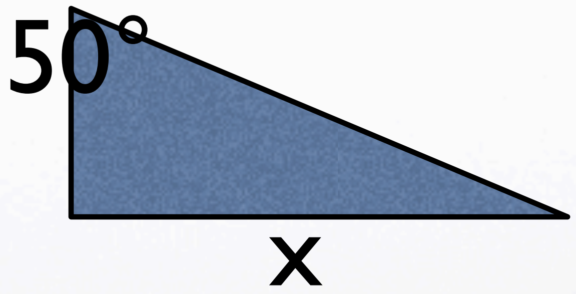


So I might ask

- The cosine of an angle is really big.
- How big might it be?
- What might the angle be?
- How big would the sine and tangent be?

So I might ask

- What else do you need to know to figure out the length of x? What choices are there?



- Why couldn't you just know the other angles?

For example

- Quadratics

While we still want

- Representing and interpreting quadratics in various forms
- Determining max/min values
- Factoring quadratics

WE ALSO NEED

Quadratics

- Different forms of a quadratic are useful for different reasons
- How quadratics differ from other relationships

Quadratics

- Recognizing why quadratics might have maxima or minima, but not both
- Associating quadratics with “projectile” motion, but also certain measurements, typically area

Quadratics

- How factoring polynomials is like factoring numbers
- How factoring polynomials is about forming rectangles

Quadratics

- An ability to predict the relationship between the parameters in a quadratic expression and the effects on a graph

So I might ask

- You have to describe a quadratic relationship.
- When might you rather have the table of values? the graph? The equation?
- Factored form? standard form?

So I might ask

- Describe four ways that make a quadratic different from a linear relationship.

So I might ask

- Create a situation involving a quadratic relationship where the maximum value is 22 when the independent variable has a value of 4.
- Tell one other fact about the situation.

So I might ask

- Describe a situation that is quadratic that has a maximum.
- Why is there no minimum?

So I might ask

- Describe three quadratic relationships involving measurements that are quadratic. Tell why.
- Then describe two that are not quadratic and tell why.

So I might ask

- How is factoring $x^2 - 5x + 6$ like factoring numbers? How is it different?

So I might ask

- How might this table help you factor $x^2 + 2x + 1$?

So I might ask

x	$x^2 + 2x + 1$
0	1
1	4
2	9
3	16

So I might ask

- How can building a rectangle of tiles help you factor 56?
- How can building a rectangle of algebra tiles help you factor $x^2 + 5x + 6$?

So I might ask

- Choose a quadratic, but not $y = x^2$, that you would find easy to predict the graph of. Draw to check. Why was it easy? OR
- Predict how the graph of $y = 2(2x - 2)^2 + 2$ is different from the graph of $y = \frac{1}{2}(x/2 - \frac{1}{2})^2 + \frac{1}{2}$?

Consider measurement

I want students to realize

- That formulas are a way to make difficult to take measurements easier to calculate
- That the variables in a formula tell me which measurements are related to which and how

I want students to realize

- That the units I use play a big role in the sizes I use to describe measurements
- That some measures are related to others and so alternative formulas are possible
- That if some measures of two figures are related, some, but may not all, measures of those figures are related

So I might ask

- Would it be hard to figure out the volume of a cylinder if you didn't know the formula? Explain.

So I might ask

- I am figuring out the volume of a cone.
- What measurements do I need to know? Which don't I care about?

So I might ask

- I know $\text{Volume}_{cone} = 1/3\ \pi\ r^2h$.
- $\text{Volume}_{sphere} = 4/3\ \pi r^3$.
- How many measurements of the cone do I need to figure out its volume? The sphere?
- Why does that make sense?

So I might ask

- How could the volume of a sphere be both 7.8 cubic units and 7800 cubic units?
- Can every measurement be made to sound bigger? Smaller?

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So I might ask

- One formula for the volume of a cone is $V = 1/3\pi r^2 h$.
- What are some alternate formulas that don't involve the radius? Why are they also legitimate?

So I might ask

- There are two cones.
- One has twice the height of the other.
- Do you know how their volumes compare?
- What if the one with the double height has half the radius?

Algebraic equations/ expressions

- I want students to know

that

- Any equation or expression can describe many situations
- Any situation that can be modelled algebraically can be modelled in multiple ways.
- Algebra is often used to describe numerical generalizations.

So I might ask

- Describe three different situations that the expression $3x + 2y - 8$ could describe.

Perhaps

- I filled 3 boxes with the same number of chocolates and 2 other boxes with the same number of chocolates and then ate 8 of them. How many chocolates are left?

So I might ask

- How can you describe this situation algebraically two different ways.
- The perimeter of a rectangle whose length is three times the width is 58.

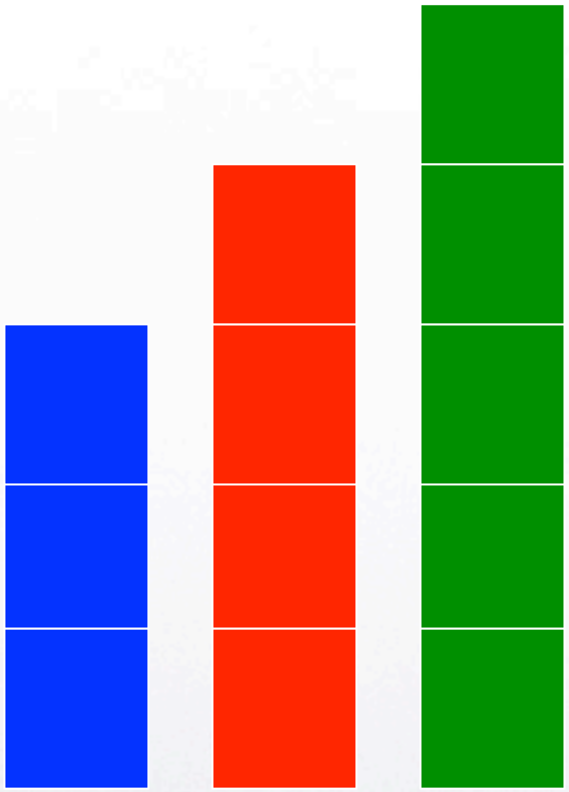
Or I might ask

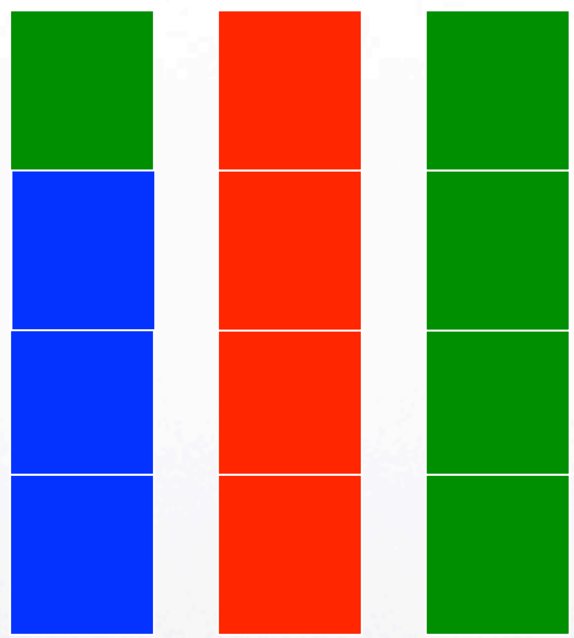
- Choose three consecutive whole numbers. Add them.
- Repeat.
- What do you notice?
- How could you describe this algebraically?

For example

- $5 + 6 + 7 = 18$
- $20 + 21 + 22 = 63$
- $49 + 50 + 51 = 150$

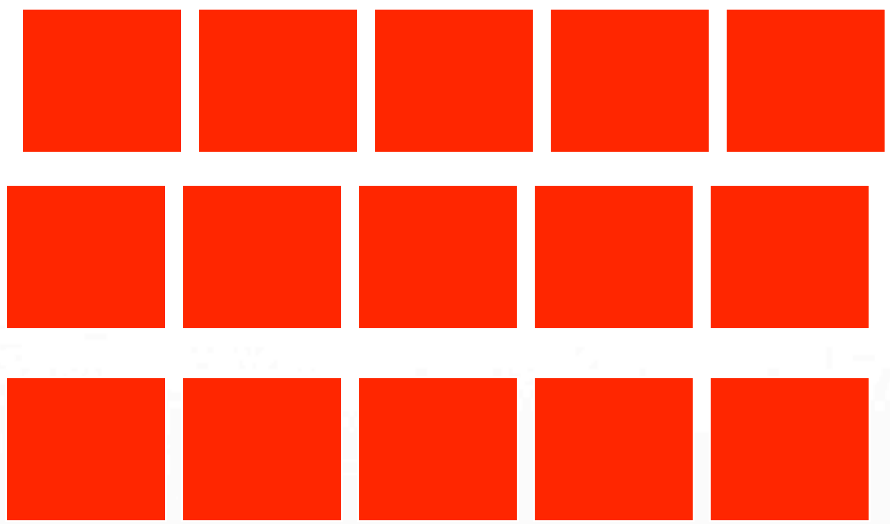
- $n + (n+1) + (n+2) = 3(n+1)$ OR
- $(n - 1) + n + (n + 1) = 3n$

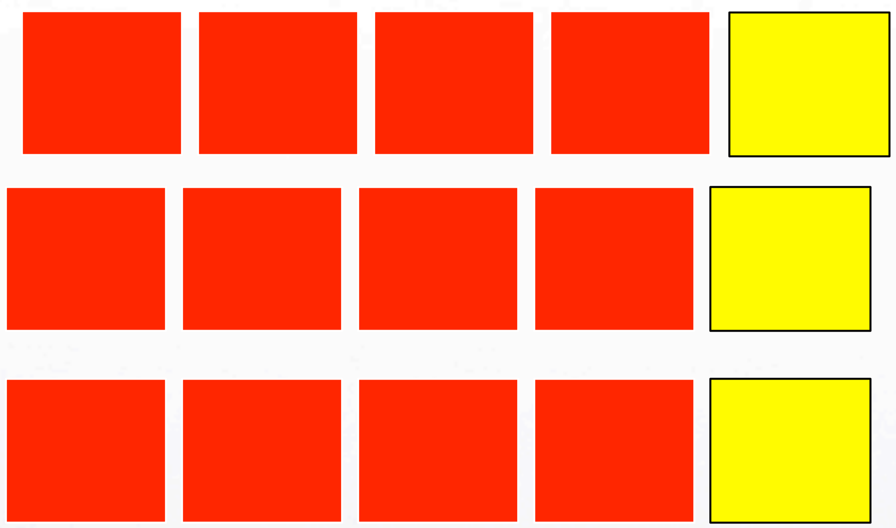


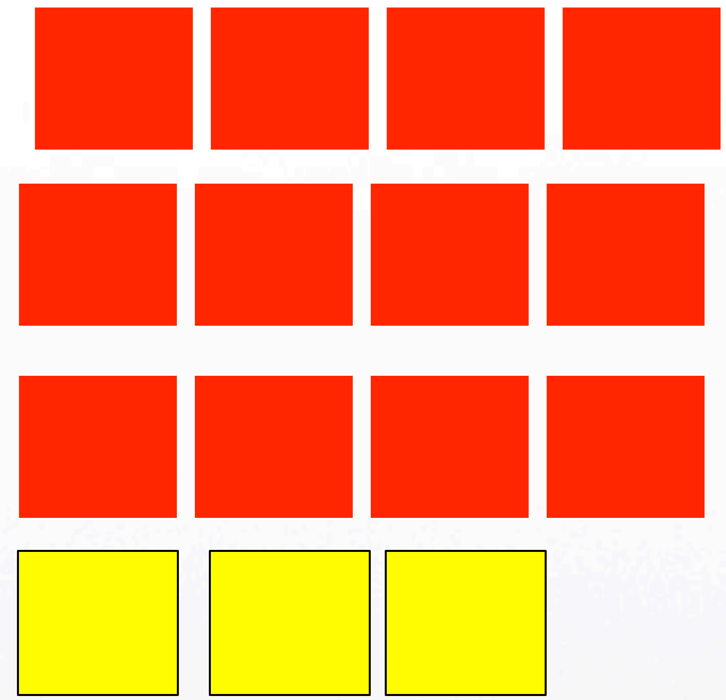


Or

- Multiply a whole number by the number two greater.
- Square the number between those two values.
- What do you notice?
- How can you say this algebraically?







For example

- $5 \times 7 = 6^2 – 1$
- $8 \times 10 = 9^2 – 1$
- $99 \times 101 = 100^2 – 1$
- $n(n + 2) = (n + 1)^2 – 1$ OR
- $(n-1)(n+1) = n^2 - 1$

Once you decide

- what is important, that should be the focus of both the kind of instructional situations and the kind of assessment questions you ask.

You probably noticed

- The key, for me, is posing good questions.

Maybe

- There are some questions that require repeating what you've shown them.

Some of

- The straightforward is okay, but it can't be the majority of what you do.
- Otherwise students are not really learning anything new.

Knowledge vs understanding

- I could ask:
- What is the slope of the line that goes through (3,2) and (4, 5)?

OR

Knowledge vs understanding

- Two lines go through (3,2).
- One has a greater slope than the other.
- How do the intercepts relate?

Knowledge vs understanding

- A batch of cookies needs 2 ¼ cups of flour for every ¾ cup sugar.
- How much flour would you need if you only use ½ cup sugar?

OR

Knowledge vs understanding

- A batch of cookies uses 2 ¼ cups of flour for every ¾ cup sugar.
- You have a measuring cup with no markings on it. You fill it with sugar. Can you be sure how much flour to pour in? Explain.

Knowledge vs understanding

- Solve $100\,x + 6 = 87x + 2$

OR

Knowledge vs understanding

- WITHOUT SOLVING, tell why the solution to $100x + 6 = 87x + 2$ HAS TO be negative.

Knowledge vs understanding

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Knowledge vs understanding

- What is $(3x + 2) - (2x^2 - 8)$?

OR

Knowledge vs understanding

- When you subtract two binomials, show that your answer could be a monomial, a binomial, or a trinomial.

Knowledge vs understanding

- Where do the lines $y = 3x + 5$ and $y = 2x - 8$ intersect?

OR

Knowledge vs understanding

- How can you predict that the lines $y = 3x + 5$ and $y = 2x - 8$ intersect when x is negative without actually figuring out where?

You try

- Start with a knowledge question you typically ask.
- Try to change it into an understanding question.

Thinking questions

- These may be understanding questions or may be "thinking" questions.
- Either way, the student needs to think.

Here are some examples

- A cylinder and a prism have the same volume.
- Choose a volume and choose dimensions to make it happen.

Here are some examples

- A rectangle has a length two times its width.
- Another rectangle with the same perimeter has a length five times its width.
- Can you be sure which has more area without knowing the dimensions? Explain.

Here are some examples

- A shape made up of two trapezoids, two triangles and a parallelogram has an area of 50 cm^2. What might all the dimensions be?

Here are some examples

- A right triangle is on a coordinate grid.
- One side is on the line $y = -2$.
- On what lines might the other sides be?

Here are some examples

- A line goes through Quadrants II, III and IV.
- Tell everything you know about the line.

Here are some examples

- You bought a number of $5 items and a number of $2 items and spent $80.
- Your friend did the same.
- Could you have bought one more $5 item than her?
- Could you have bought 6 more $5 items?

Here are some examples

- You saved 40% on a jacket and ended up paying the same amount as you did for a pair of shoes that was 20% off.
- How did the original prices compare?

For example

- Which equation does not belong? Why?

$3x - 4 = 2x - 7$

$6/x = -2$

$2x = -8$

$5x + 8 = -7$

For example

- If you know 20% of an amount, what other percents of it would be quick for you to figure out?
- Would be possible to figure out, but not quick?

For example

- You know that a line goes through the point $(1, -1)$ and slants down to the right.

- Tell other things you are sure are true about that line.

For example

- You know that a line is of the form $y = mx + m$.
- What is true about that line?
- Would the same thing be true of lines of the form $y = mx + 2m$?

For example

- An equation is REALLY easy to solve.
- What could it be?

For example

- When working with linear relations, when would you find the table of values the best thing to have?
- When the graph?
- When the equation?

For example

- You have a table of values.
- It is REALLY EASY to figure out the equation that goes with it.
- What might the table look like?

For example

- When subtracting polynomials, when would it be easier NOT to add the opposite?

Your chance

- Try writing a “thinking” question on a topic you teach.

Before I come next

- It would be great if you tried:
- 1) changing some knowledge questions into understanding questions
- 2) changing some knowledge questions into thinking questions

Before I come next

- 3) planned a topic based on big idea thinking

Download

- www.onetwoinfinity.ca
- LouisRielHS2